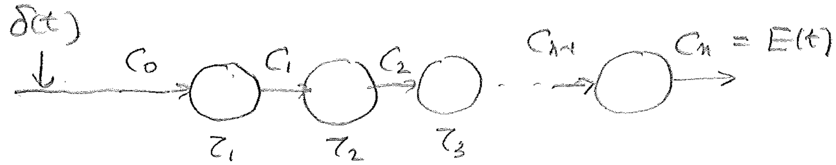
$\boxed{\text{TIS model}}$



$$\tau_1 = \tau_2 = \tau_3 \cdots = \tau_i$$

$$\frac{\bar{C}_1}{\bar{C}_0} = \frac{1}{1+\tau_i s}$$

$$\frac{\bar{C}_2}{\bar{C}_0} = \frac{1}{1+\tau_i s}$$

$$\vdots$$

$$\frac{\bar{C}_n}{\bar{C}_{n-1}} = \frac{1}{1+\tau_i s}$$

(multiply)

$$\bar{C}_n = \frac{\bar{C}_n}{\bar{C}_0} = \left[\frac{1}{1+\tau_i s}\right]^n = \left[\frac{1/\tau_i}{1/\tau_i + s}\right]^n$$

$$\bar{C}_0 = \mathcal{L}\{\delta(t)\} = 1$$

Inversion Formula (From $\mathcal{L}$T Table):

$$\mathcal{L}^{-1}\left[\frac{1}{(s+a)^n}\right] = \frac{t^{n-1}}{(n-1)!} e^{-at}$$

$$E(t) = C_n = \mathcal{L}^{-1}\left[\frac{1/\tau_i}{1/\tau_i + s}\right] = \frac{1}{\tau_i}\left(\frac{t}{\tau_i}\right)^{n-1} \frac{1}{(n-1)!} e^{-t/\tau_i}$$

$$= \frac{1}{\tau_i^{\,n}} \frac{t^{n-1}}{(n-1)!} e^{-t/\tau_i}$$

$$\left(\tau = n\tau_i, \quad t/\tau = \theta; \quad t/\tau_i = \theta_i = \frac{t}{(\tau/n)} = n\left(\frac{t}{\tau}\right) = n\theta\right)$$

$$= \frac{1}{\tau_i}(\theta_i)^{n-1} \frac{1}{(n-1)!} e^{-\theta_i}$$

$$= \left(\frac{n}{\tau}\right)(n\theta)^{n-1} \frac{1}{(n-1)!} e^{-n\theta} = \frac{1}{\tau} \frac{n(n\theta)^{n-1}}{(n-1)!} e^{-n\theta}$$

$$\boxed{\tau E(t) = E(\theta) = \frac{n^n}{(n-1)!} \theta^{n-1} e^{-n\theta}} \qquad (14\text{-}8)$$

$$E(\theta) = \frac{n^n}{(n-1)!}\,\theta^{n-1} e^{-n\theta}$$

$$\sigma^2 = \int_0^\infty t^2 E(t)\,dt - \tau^2$$

$$\sigma_\theta^2 = \frac{\sigma^2}{\tau^2} = \frac{1}{\tau^2}\int_0^\infty t^2 E(t)\,dt - 1$$

$$= \frac{1}{\tau^2}\int_0^\infty \tau^3 \frac{\theta^2}{\tau} E(\theta)\,d\theta - 1$$

$\tau E(t) = E(\theta)$

$t = \tau\theta$

$$= \int_0^\infty \theta^2 E(\theta)\,d\theta - 1$$

$$\sigma_\theta^2 = \frac{n^n}{(n-1)!}\underbrace{\int_0^\infty \theta^{n+1} e^{-n\theta}\,d\theta}_{= I} - 1$$

Let $n\theta = x$

$$I = \int_0^\infty \left(\frac{x}{n}\right)^{n+1} e^{-x}\left(\frac{1}{n}\right) dx$$

$$= \left(\frac{1}{n}\right)^{n+2}\int_0^\infty x^{n+1} e^{-x}\,dx$$

$$= \left(\frac{1}{n}\right)^{n+2}(n+1)!$$

$$\boxed{\int_0^\infty x^n e^{-x}\,dx = n!}$$

$$\sigma_\theta^2 = \frac{n^n}{(n-1)!}\left(\frac{1}{n^{n+2}}\right)(n+1)! - 1$$

$$= \frac{(n+1)(n)}{n^2} - 1 = \frac{n+1}{n} - 1 = 1 + \frac{1}{n} - 1 = \frac{1}{n}$$