

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \alpha \nabla^2 T + S \quad (\text{Energy})$$

$$\frac{\partial C_A}{\partial t} + \vec{v} \cdot \nabla C_A = D \nabla^2 C_A - (-r_A) \quad (\text{Material})$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = \nu \nabla^2 \vec{v} - \frac{1}{\rho} \nabla p + \vec{g} \quad (\text{Momentum})$$

$$\vec{v} \cdot \nabla T = v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \quad (\text{Cartesian})$$

$$= v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \quad (\text{Cylindrical})$$

$$= v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \quad (\text{Spherical})$$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \quad (\text{Cartesian})$$

$$= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \quad (\text{Cylindrical})$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \quad (\text{Spherical})$$

Note : ρ, c_p, k, D, μ are constants

$\nu = \mu / \rho = \text{Kinematic viscosity}$, $\vec{g} = \text{Gravitational acceleration}$

$-r_A = \text{Reaction term e.g. } -r_A = kC_A \text{ for first order reaction}$

$S = \text{Heat generation term} = Q / (\rho c_p)$ where Q is heat generation / (volume.time)

$\alpha = k / (\rho c_p)$, Thermal diffusivity

$D = \text{diffusivity}$