Second Order ODE Reducible to Bessel's Equation

$$x^2 y''+x(a+2bx^r)y'+[c+dx^{2q}+b(a+r-1)x^r+b^2x^{2r}]y=0$$

If $(1-a)^2 \geq 4c$ and neither d, r, nor q is zero, then except in the special cases where it reduces to Euler's equation the ODE has a complete solution of:

$$y = x^{\alpha} e^{-\beta x^r}[c_1 Z_p(\lambda x^q) + c_2 Z_{-p}(\lambda x^q)]$$

Where $\alpha = \frac{1-a}{2}$, $\beta = \frac{b}{r}$, $\lambda = \frac{\sqrt{|d|}}{q}$, $p = \frac{\sqrt{(1-a)^2 - 4c}}{2q}$

Z_p denotes one of the Bessel functions as follows.

If $d \geq 0$ and p is not zero nor an integer,

$Z_p = J_p$

$Z_{-p} = J_{-p}$

If $d \geq 0$ and p is zero or an integer,

$Z_p = J_p$

$Z_{-p} = Y_p$

If $d < 0$ and p is not zero nor an integer,

$Z_p = I_p$

$Z_{-p} = I_{-p}$

If $d < 0$ and p is zero or an integer,

$Z_p = I_p$

$Z_{-p} = K_p$