

## DERIVATIVES AND INDEFINITE INTEGRALS

In these formulas,  $u$ ,  $v$ , and  $w$  represent functions of  $x$ . Also,  $a$ ,  $c$ , and  $n$  represent constants. All arguments of the trigonometric functions are in radians. A constant of integration should be added to the integrals. To avoid terminology difficulty, the following definitions are followed:  $\arcsin u = \sin^{-1} u$ ,  $(\sin u)^{-1} = 1/\sin u$ .

1.  $dc/dx = 0$
2.  $dx/dx = 1$
3.  $d(cu)/dx = c du/dx$
4.  $d(u + v - w)/dx = du/dx + dv/dx - dw/dx$
5.  $d(uv)/dx = u dv/dx + v du/dx$
6.  $d(uvw)/dx = uv dw/dx + uw dv/dx + vw du/dx$
7.  $\frac{d(u/v)}{dx} = \frac{v du/dx - u dv/dx}{v^2}$
8.  $d(u^n)/dx = nu^{n-1} du/dx$
9.  $d[f(u)]/dx = \{d[f(u)]/du\} du/dx$
10.  $du/dx = 1/(dx/du)$
11.  $\frac{d(\log_a u)}{dx} = (\log_a e) \frac{1}{u} \frac{du}{dx}$
12.  $\frac{d(\ln u)}{dx} = \frac{1}{u} \frac{du}{dx}$
13.  $\frac{d(a^u)}{dx} = (\ln a) a^u \frac{du}{dx}$
14.  $d(e^u)/dx = e^u du/dx$
15.  $d(u^v)/dx = vu^{v-1} du/dx + (\ln u) u^v dv/dx$
16.  $d(\sin u)/dx = \cos u du/dx$
17.  $d(\cos u)/dx = -\sin u du/dx$
18.  $d(\tan u)/dx = \sec^2 u du/dx$
19.  $d(\cot u)/dx = -\csc^2 u du/dx$
20.  $d(\sec u)/dx = \sec u \tan u du/dx$
21.  $d(\csc u)/dx = -\csc u \cot u du/dx$
22.  $\frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad (-\pi/2 \leq \sin^{-1} u \leq \pi/2)$
23.  $\frac{d(\cos^{-1} u)}{dx} = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad (0 \leq \cos^{-1} u \leq \pi)$
24.  $\frac{d(\tan^{-1} u)}{dx} = \frac{1}{1+u^2} \frac{du}{dx} \quad (-\pi/2 < \tan^{-1} u < \pi/2)$
25.  $\frac{d(\cot^{-1} u)}{dx} = -\frac{1}{1+u^2} \frac{du}{dx} \quad (0 < \cot^{-1} u < \pi)$
26.  $\frac{d(\sec^{-1} u)}{dx} = \frac{1}{u\sqrt{u^2-1}} \frac{du}{dx} \quad (0 < \sec^{-1} u < \pi/2) \quad (-\pi \leq \sec^{-1} u < -\pi/2)$
27.  $\frac{d(\csc^{-1} u)}{dx} = -\frac{1}{u\sqrt{u^2-1}} \frac{du}{dx} \quad (0 < \csc^{-1} u \leq \pi/2) \quad (-\pi < \csc^{-1} u \leq -\pi/2)$
1.  $\int df(x) = f(x)$
2.  $\int dx = x$
3.  $\int a f(x) dx = a \int f(x) dx$
4.  $\int [u(x) \pm v(x)] dx = \int u(x) dx \pm \int v(x) dx$
5.  $\int x^m dx = \frac{x^{m+1}}{m+1} \quad (m \neq -1)$
6.  $\int u(x) dv(x) = u(x) v(x) - \int v(x) du(x)$
7.  $\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b|$
8.  $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$
9.  $\int a^x dx = \frac{a^x}{\ln a}$
10.  $\int \sin x dx = -\cos x$
11.  $\int \cos x dx = \sin x$
12.  $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4}$
13.  $\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4}$
14.  $\int x \sin x dx = \sin x - x \cos x$
15.  $\int x \cos x dx = \cos x + x \sin x$
16.  $\int \sin x \cos x dx = (\sin^2 x)/2$
17.  $\int \sin ax \cos bx dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$
18.  $\int \tan x dx = -\ln |\cos x| = \ln |\sec x|$
19.  $\int \cot x dx = -\ln |\csc x| = \ln |\sin x|$
20.  $\int \tan^2 x dx = \tan x - x$
21.  $\int \cot^2 x dx = -\cot x - x$
22.  $\int e^{ax} dx = (1/a) e^{ax}$
23.  $\int x e^{ax} dx = (e^{ax}/a^2)(ax-1)$
24.  $\int \ln x dx = x [\ln(x)-1] \quad (x > 0)$
25.  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} \quad (a \neq 0)$
26.  $\int \frac{dx}{ax^2+c} = \frac{1}{\sqrt{ac}} \tan^{-1} \left( x \sqrt{\frac{a}{c}} \right), \quad (a > 0, c > 0)$
- 27a.  $\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \quad (4ac-b^2 > 0)$
- 27b.  $\int \frac{dx}{ax^2+bx+c} = \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| \quad (b^2-4ac > 0)$
- 27c.  $\int \frac{dx}{ax^2+bx+c} = -\frac{2}{2ax+b}, \quad (b^2-4ac = 0)$