CS122 Algorithms and Data Structures

MW 11:00 am - 12:15 pm, MSEC 101
Instructor: Xiao Qin
Lecture 17: Graphs (4)

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Topological Sort

- Ordering of vertices in a directed acyclic graph(DAG)
- If there is a path from u to v, then v appears after u in the ordering
- Edge (v, w) indicates that activity v must be completed before w
- Course prerequisites
- Topological order of courses is any sequence that does not violate the prereq requirements

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Algorithm

- Find any vertex v with no incoming edges
- Print and logically remove it
- Apply to the rest of the graph
- Do this by
- Compute the indegrees of all vertices
- Logically remove means that we lower the count of incoming edges for each vertex adjacent to v

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Algorithm (cont.)

- choose any vertex with indegree 0 as next
- All acyclic graphs have a topological order
- This algorithm will find one
- If there are no 0 nodes, there must be a cycle

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Alg analysis

- find next vertex is O(|V|)
- since there are O(|V|) such calls
- algorithm is O(|V|*|V|)
- Can lower to O(|V|+|E|) if we keep a structure of all nodes which have degree 0

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Network Flow Problems

- Consider the graph *G* to be a network, and the costs on edges to be "flow capacities".
- A network is a directed graph with two special vertices: a source *s* and a sink *t*.
- The flow through an edge *e* cannot be greater than its capacity
- Total flow coming to a vertex *v* must equal the flow going out.
- Maximum flow problem: find the maximum amount of flow that can pass from *s* to *t*.

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Maximum Flow Algorithm

- We use three graphs, the original graph G, a flow graph G_f and a residual graph $G_r = G - G_f$.
- We proceed in stages. Each stage we choose a path in G_r from s to t. The minimum edge on this path is the amount of flow that can be added to every edge on that path.
- We do this by adjusting G_f and recomputing G_r .
- We continue until there are no paths from s to t.
- We can't follow any edges that have capacity 0.

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Discussion

- In the prior example we in fact obtained the maximum flow. However, how did I choose the paths?
- If we choose a greedy algorithm, we'd be tempted to choose paths that allows the maximum amount of flow to be added at each step. This might not work.
- For example, let's choose (s, a, d, t) first, because this allows 3 units of flow to be added.

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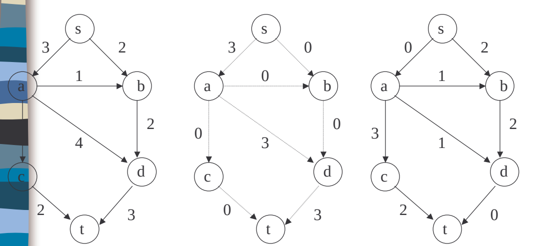
Example



There are no paths from s to t in the residual graph, so we are done, but we have not obtained the maximum possible flow.

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A Better Algorithm

- We can make the algorithm work by allowing the algorithm to change its mind.
- In effect we allow the algorithm to undo its decisions by sending flow back in the opposite direction. This is best seen by example. We have to modify the residual graph.

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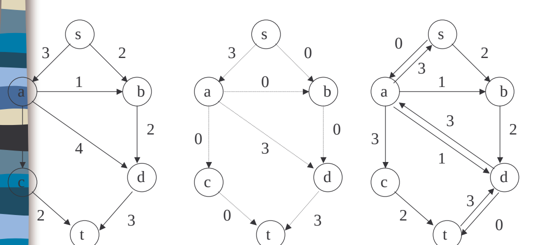
Example



We again choose the path (s, a, d, t). But note we now allow the flow to backup in the residual graph.

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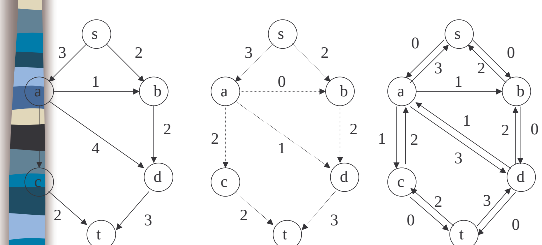
Example



We can now follow the path (s, b, d, a, c, t). The minimum flow along this path is 2. Note the flow from d to a is now 1 = 3 – 2. We are done. This is the maximum flow solution.

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Conclusions

- This better greedy algorithm will always find the maximum flow solution if the edge capacities are rational numbers.
- Although we have used an acyclic graph in the example, the algorithm works on arbitrary graphs!

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