# ELEC 3400

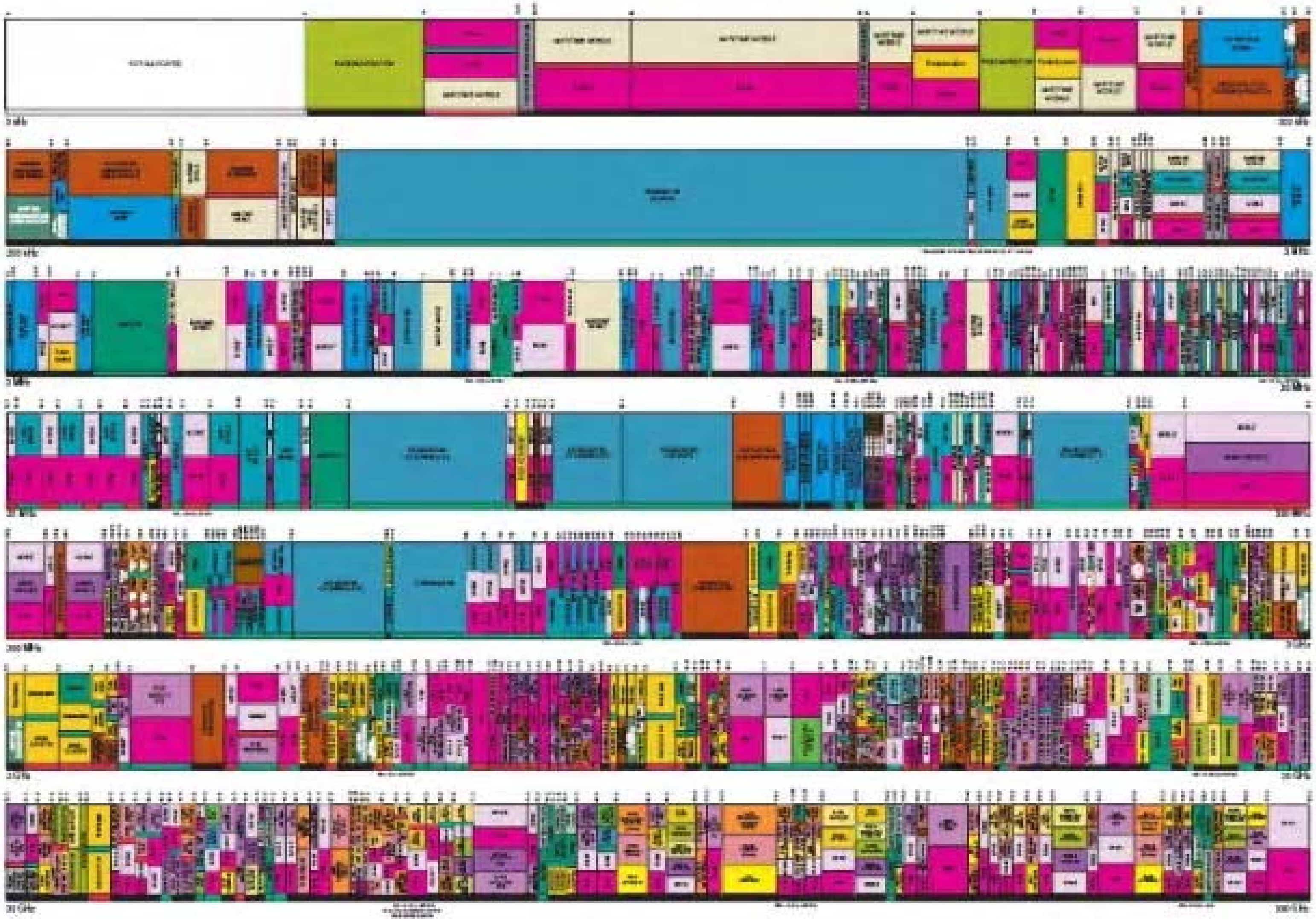
# Communication Systems

## Chapter 8: Baseband Digital Transmission

# Introduction

- Transmission of digital signal
  - Over a baseband channel (Chapter 8) → local communications
  - Over a band-pass channel using modulation (Chapter 9) → network
- Channel-induced transmission impairments
  - Channel noise, or receiver noise
    - Interference: sometimes treated as noise
  - Intersymbol interference (ISI)
    - Digital data has a broad bandwidth with a significant low-frequency content
    - Many channels are bandwidth limited: dispersive, unlike low-pass filter
    - Each received pulse is affected by neighboring pulses → ISI
    - Major source of bit errors in many cases
- Solutions to be studied in this chapter
  - Noise: matched filter → maximize the signal noise level at the receiver
  - ISI:
    - Pulse shaping → minimize the ISI at the sampling points
    - Equalization → compensate the residual distortion for ISI

# US Spectrum Allocation



# Transmission Impairment: Noise

- Thermal noise: generated by the equilibrium fluctuations of the electric current inside the receiver circuit
  - Due to the random thermal motion of the electrons
  - Modeled as an Additive white Gaussian noise (AWGN)
    - Noise spectral density: $N_o = KT$ (watts per hertz), where $K$ is the Boltzmann's constant $K = 1.380\times10^{-23}$, and $T$ is the receiver system noise temperature in kelvins ([K] = [°C] + 273.15)
    - If bandwidth is $B$ Hz, then the noise power is $N = BKT$
- Always exists
- Other sources of noise: interference

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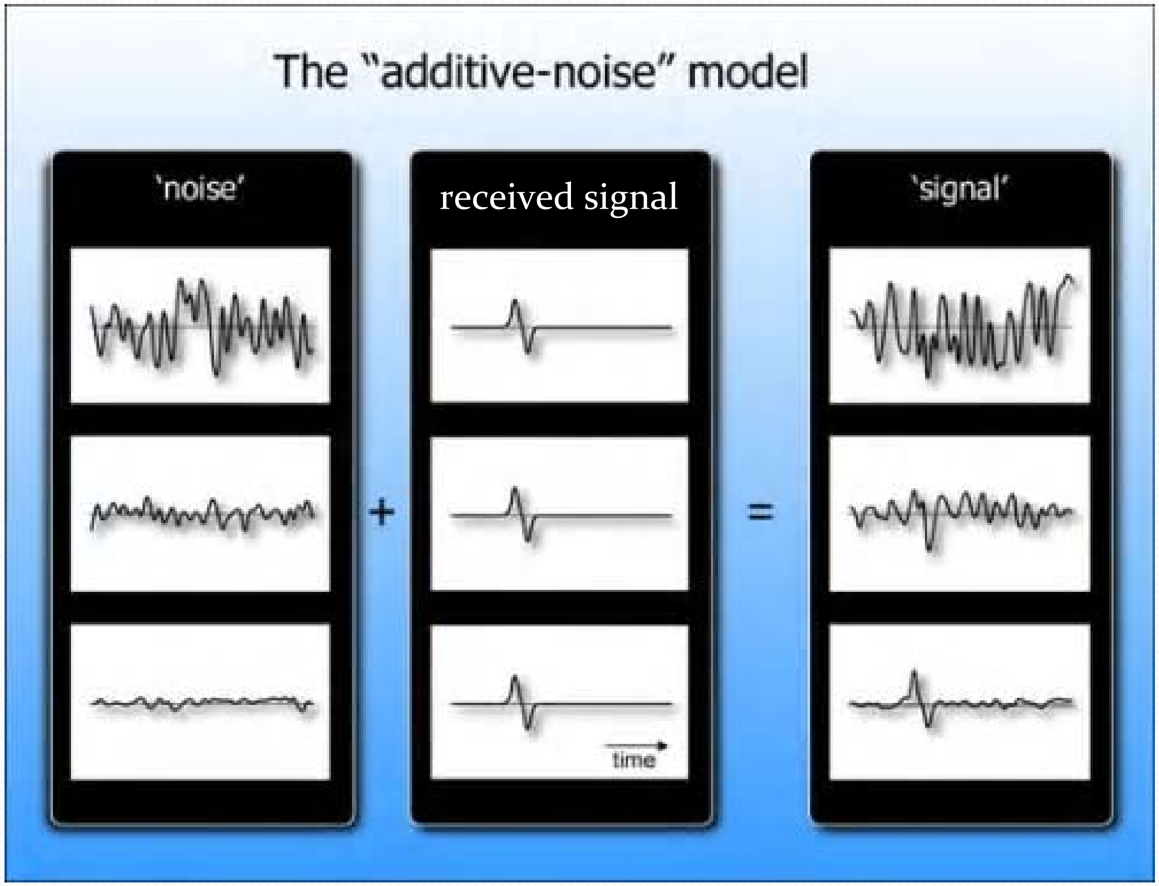
# Additive Noise




http://files.amouraux.webnode.com/200000047-3fd9440d34/resint_eeg2.jpg

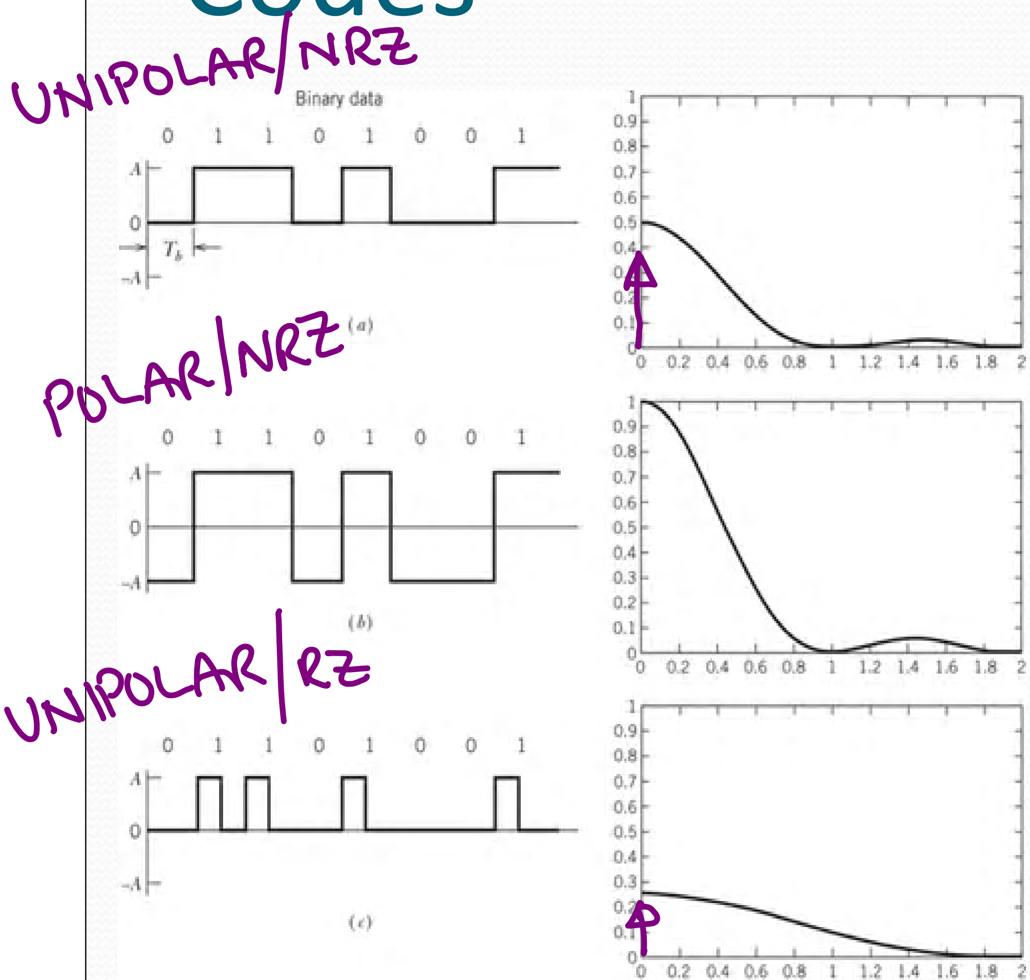
# Transmission Impairment: ISI

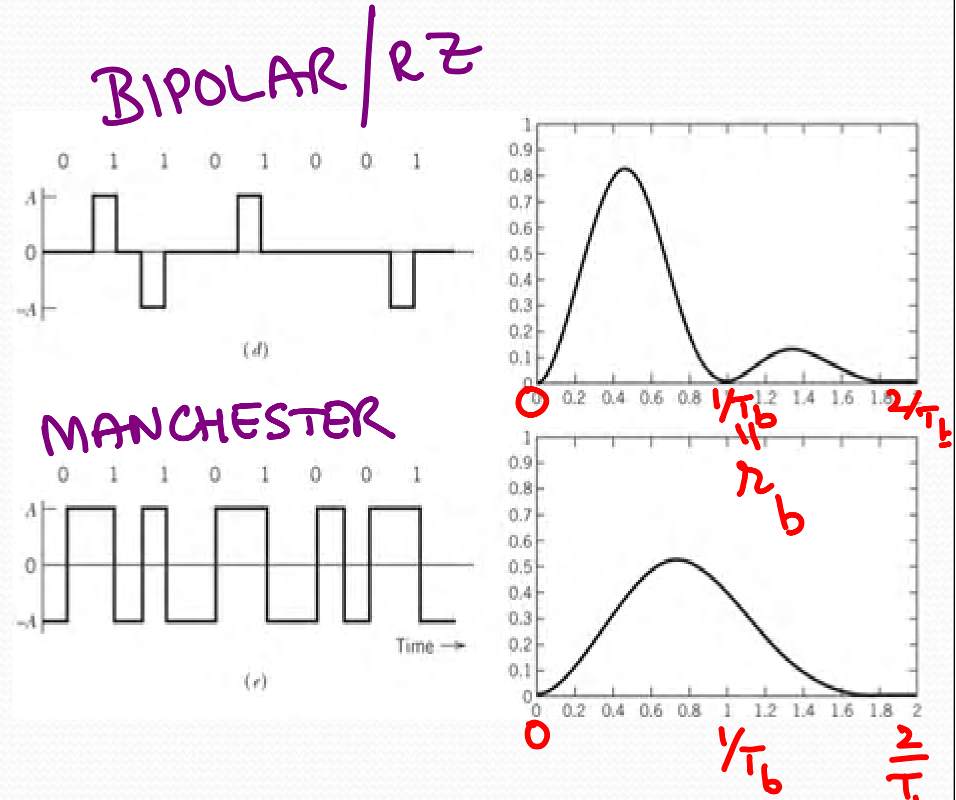
- Line codes
  - Mapping 1's and 0's to symbols
  - Random process, since 1's and 0's are random
- Power spectrum (Section 5.8)
  - Representation in the frequency domain
  - The nominal bandwidth of the signal is the same order of magnitude as $1/T_b$ and is centered around the origin

baseband

- Mismatch between signal bandwidth $B_s$ and channel bandwidth $B_c$
  - If $B_c \geq B_s$, no problem
  - If $B_c < B_s$, the channel is *dispersive*, the pulse shape will be changed and there will be ISI

# Power Spectra of Several Line Codes



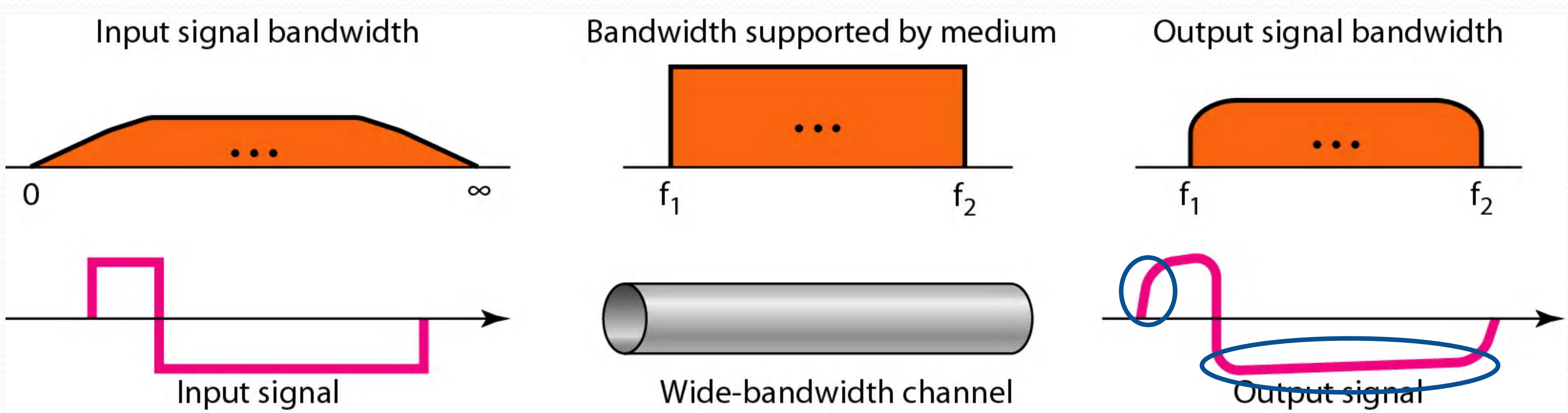




- Frequency axis normalized with $T_b$
- Average power is normalized to unity

[redacted]

# Transmission Impairments due to Limited Channel Bandwidth

- Each received symbol may be wider than the transmitted one, due to loss of high frequency components
- Overlap between adjacent symbols: ISI → INTERSYMBOL INTERFERENCE
- Limit on data rate: use guard time between adjacent symbols
- Or need to shape the pulses to cancel ISI at sampling points



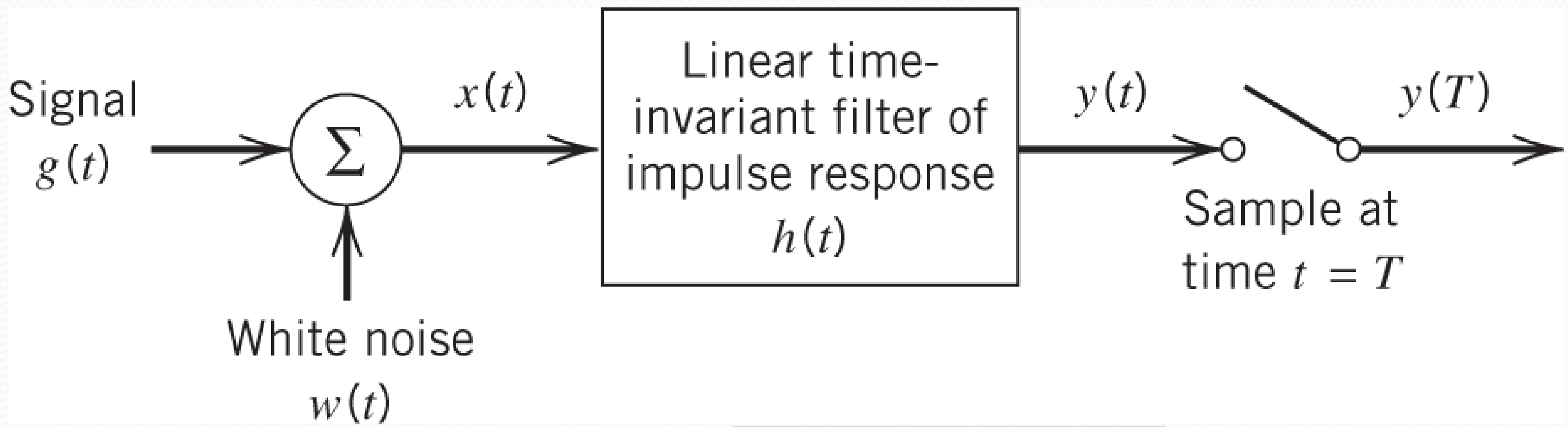

From *Data Communications and Networking*, Behrouz A. Forouzan

# Matched Filter – The Problem

- Methodology:
  - Cope with the two types of impairments separately
- First assume an ideal channel and only consider noise
  - e.g., low data rate over a short range cable
    - No problem of ISI
  - Transmitted pulse $g(t)$ for each bit is unaffected by the transmission except for *the additive white nose at the receiver front end*

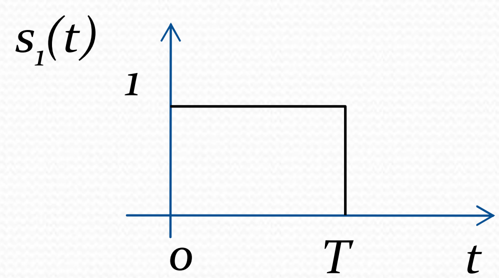
**Basic problem** of detecting a pulse transmitted over a channel that is corrupted by additive white noise at the receiver front end

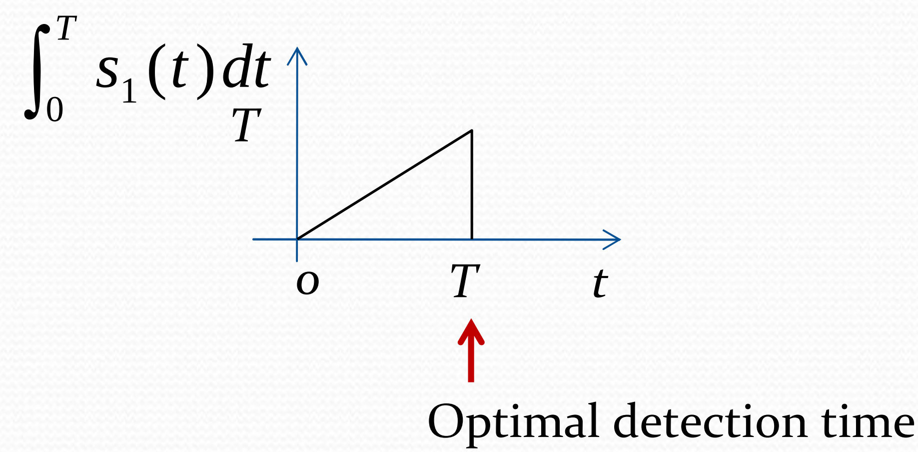
# Matched Filter

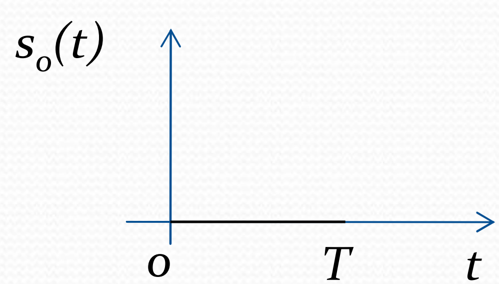


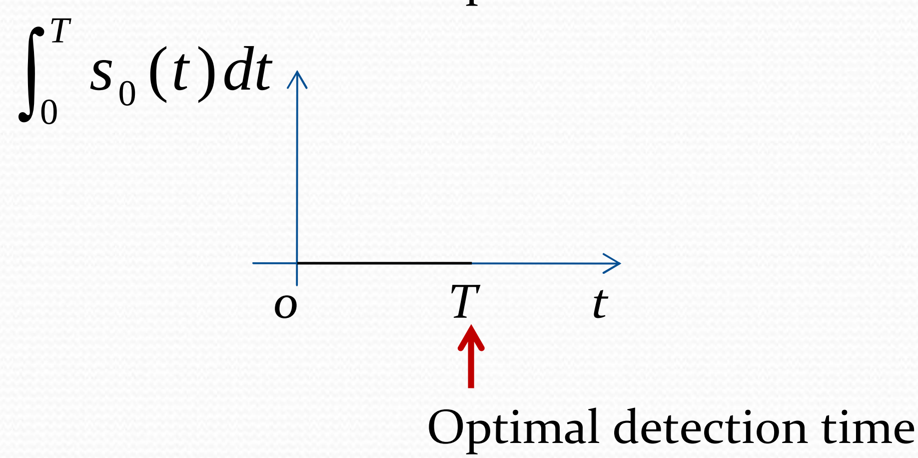

- Received (or, input) signal: $\boxed{x(t) = g(t) + w(t)}$, $0 \le t \le T$
  - $T$: an arbitrary observation interval
  - $g(t)$: represents a binary symbol 1 or 0
  - $w(t)$: white Gaussian noise process of zero mean and power spectrum density $N_0/2$
- Output signal: $\boxed{y(t) = x(t) \otimes h(t) = g_0(t) + n(t)}$

# Detection of Received Signal

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# Matched Filter (contd.)

Instantaneous power

- Problem
  - Find $h(t)$ to maximize the peak pulse signal-to-noise ratio at the sampling instant $t=T$.

Why square?

$$\eta = \frac{|g_0(T)|^2}{E[n^2(t)]} = \frac{\text{Instantaneous power in the output signal}}{\text{Average output noise power}}$$

- If $G(f) \leftrightarrow g(t)$, $H(f) \leftrightarrow h(t)$, then $g_o(t) \leftrightarrow H(f)G(f)$. We can derive $g_o(t)$ by inverse Fourier transform:

$$g_0(t) = \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi ft)df$$

- The instantaneous signal power at $t=T$ is:

$$|g_0(T)|^2 = \left| \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)df \right|^2$$

# Matched Filter (contd.)

- The average noise power
  - The power spectral density of the output noise $n(t)$ is

$$S_N(f) = \frac{N_0}{2}|H(f)|^2$$

  - The average noise power is

$$\mathrm{E}\left[n^2(t)\right] = \int_{-\infty}^{\infty} S_N(f)df = \frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df$$

- The peak pulse signal-to-noise ratio is

$$\eta = \frac{\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}$$

**Find $H(f) \leftarrow\rightarrow h(t)$ that maximizes $\eta$**

[redacted]

# Matched Filter (contd.)

- Schwarz's inequality

$$\boxed{\left|\int_{-\infty}^{\infty} \phi_1(x)\phi_2(x)\,dx\right|^2 \le \int_{-\infty}^{\infty} |\phi_1(x)|^2\,dx \int_{-\infty}^{\infty} |\phi_2(x)|^2\,dx}$$

If $\int_{-\infty}^{\infty} |\phi_1(x)|^2\,dx < \infty$ and $\int_{-\infty}^{\infty} |\phi_2(x)|^2\,dx < \infty$

Complex conjugation

The equality holds if and only if: $\boxed{\phi_1(x) = k\phi_2^*(x)}$

- Therefore, we have

$$\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)\,df\right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2\,df \int_{-\infty}^{\infty} |G(f)|^2\,df$$

$$\eta \le \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2\,df$$

$$\eta_{\max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2\,df \quad \text{when} \quad H_{\text{opt}}(f) = kG^*(f)\exp(-j2\pi fT)$$

# Matched Filter (contd.)

- Except for the factor $k \cdot \exp(-j2\pi fT)$, the transfer function of the optimal filter is the same as the complex conjugate of the spectrum of the input signal
  - $k$: scales the amplitude
  - $\exp(-j2\pi fT)$: time shift
  - For real signal $g(t)$, we have $G^*(f)=G(-f)$: time inversed
- The optimal filter is found by inverse Fourier transform

$$h_{\text{opt}}(t) = k \int_{-\infty}^{\infty} G^*(f) \exp[-j2\pi f(T-t)]\, df$$

$$h_{\text{opt}}(t) = k \int_{-\infty}^{\infty} G(-f) \exp[-j2\pi f(T-t)]\, df$$
$$= kg(T-t)$$

- Matched filter: *matched to the signal*
  - A <u>time-inversed</u> and <u>delayed</u> version of the input signal $g(t)$

# Properties of Matched Filters

- Matched filter:

$$\boxed{\begin{aligned} h_{\text{opt}}(t) &= kg(T-t) \\ H_{\text{opt}}(f) &= kG^*(f)\exp(-j2\pi fT) \end{aligned}}$$

- Received signal:

$$G_o(f) = H_{\text{opt}}(f)G(f) = kG^*(f)G(f)\exp(-j2\pi fT) = k|G(f)|^2\exp(-j2\pi fT)$$

$$g_o(T) = \int_{-\infty}^{\infty} G_o(f)\exp(j2\pi fT)\,df = k\int_{-\infty}^{\infty} |G(f)|^2\,df$$

$$\boxed{g_o(T) = kE}$$

- Noise power:

$$\mathbf{E}[n^2(t)] = \frac{k^2N_0}{2}\int_{-\infty}^{\infty} |G(f)|^2 df = k^2N_0E/2$$

# Properties (contd.)

- Maximum peak pulse signal-to-noise ratio

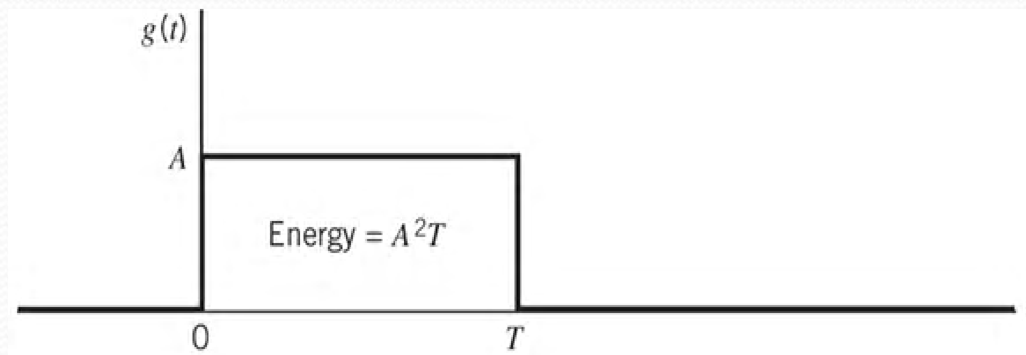
$$\eta_{\max} = \frac{(kE)^2}{(k^2 N_0 E/2)} = \frac{2E}{N_0}$$

- Observations
  - Independent of $g(t)$: removed by the matched filter
  - Signal energy (or, transmit power) matters
    - For combating additive white Gaussian noise, all signals that have the same energy are equally effective
  - Not true for ISI, where the signal wave form matters
  - $E/N_o$: signal energy-to-noise spectral density ratio

# Example 8.1 Matched Filter for Rectangular Pulse

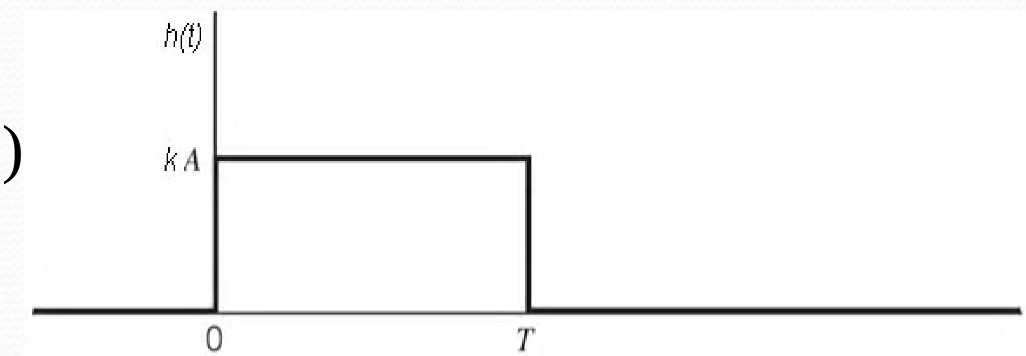
- Rectangular pulse for $g(t)$

$$g(t) = A\mathrm{rect}\left(\frac{t}{T} - \frac{1}{2}\right)$$




- Matched filter

$$h(t) = kg(T - t)$$
$$= kA\mathrm{rect}\left(\frac{T-t}{T} - \frac{1}{2}\right)$$
$$= kA\mathrm{rect}\left(\frac{1}{2} - \frac{t}{T}\right)$$

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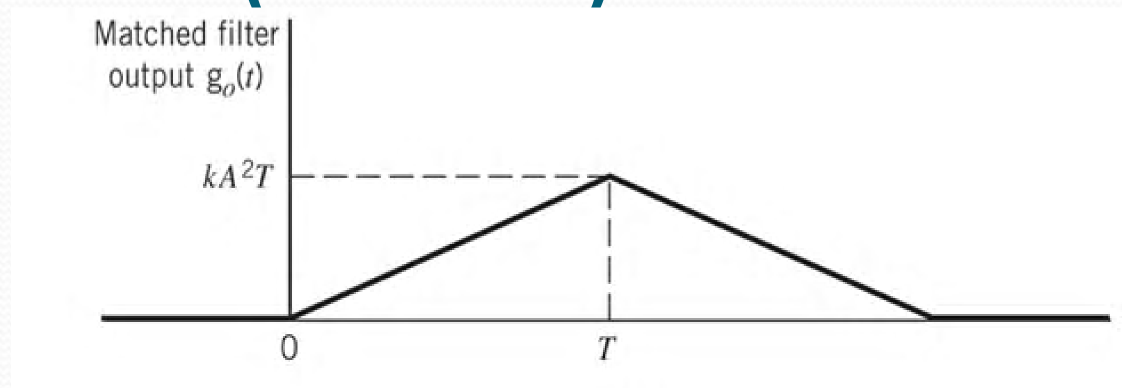
# Example 8.1 Matched Filter for Rectangular Pulse (contd.)

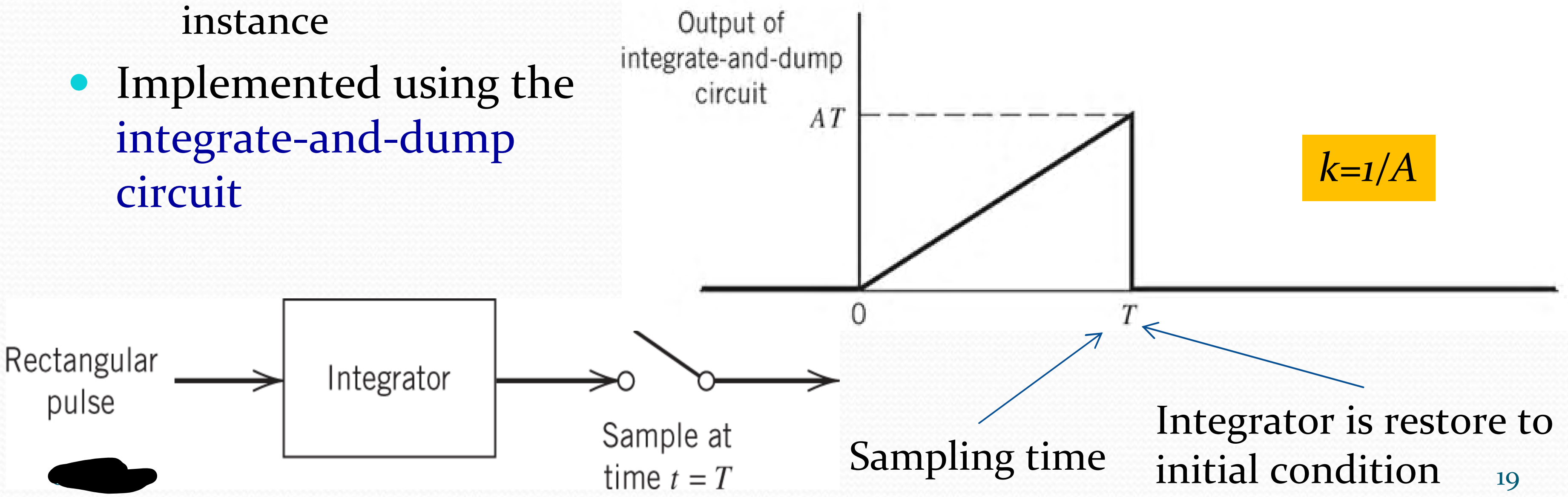
- Output $g_o(t)$

$$g_o(t) = g(t) * h(t)$$

- Max output $kA^2T$ occurs at $t=T$
  - Optimal sampling instance
- Implemented using the integrate-and-dump circuit



(b)




$k=1/A$

Sampling time

Integrator is restore to initial condition

# Effect of Noise

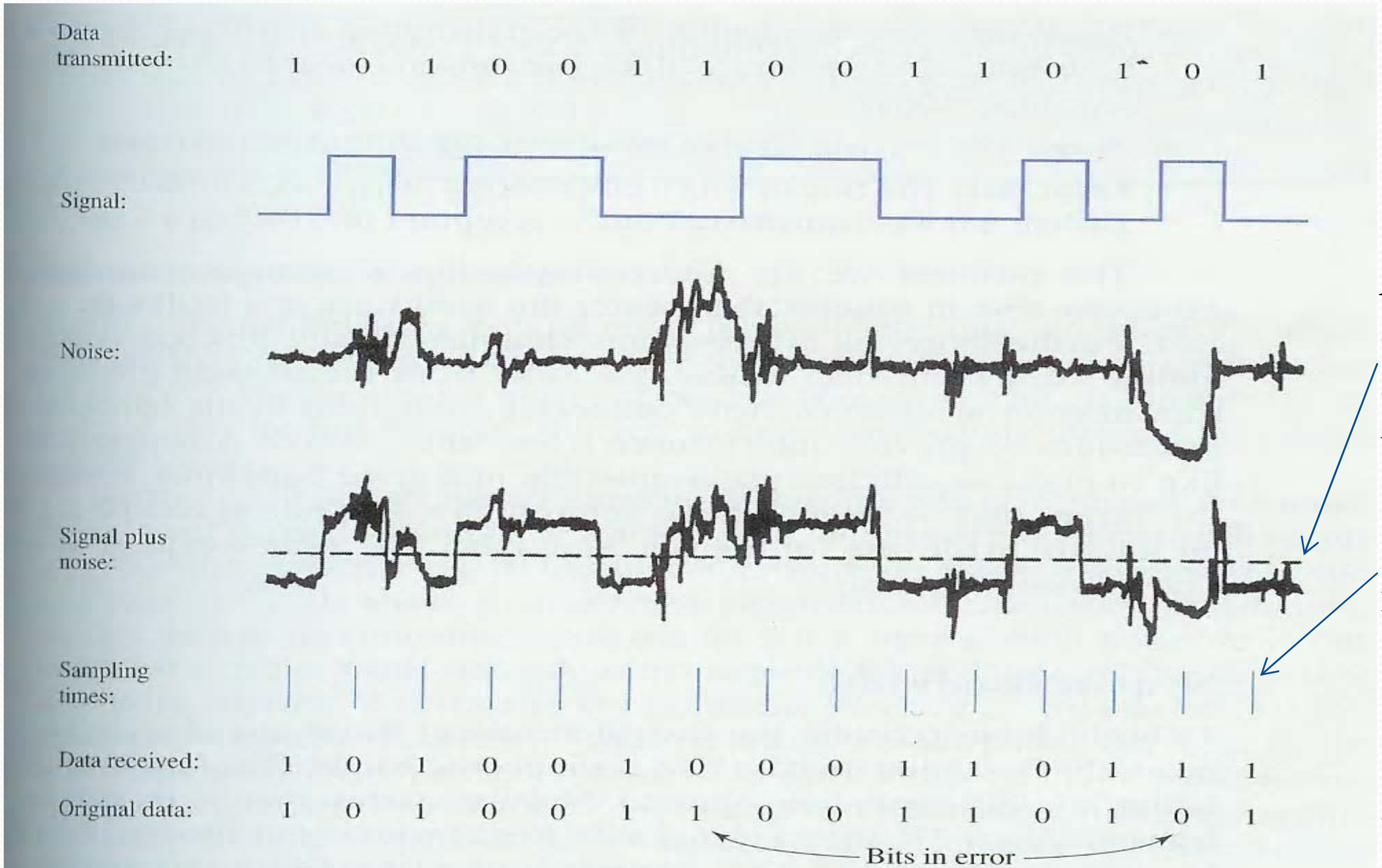



**Figure 3.16** Effect of Noise on a Digital Signal

Line coding:
- "1": -5 volts
- "0": 5 volts

A properly chosen decision threshold

Sampling instance

Bit error rate (BER) = 2/15=13.3%

[redacted]

William Stallings, Data and Computer Communications, 8/E, Prentice Hall, 2007.

# Probability of Error due to Noise

- Assume polar nonreturn-to-zero (NRZ) signaling
  - 1: positive rectangular pulse, $+A$
  - 0: negative rectangular pulse, $-A$
- Additive white Gaussian Noise $w(t)$ of zero mean and power spectral density $N_o/2$
- Received signal is

$$x(t) = \begin{cases} +A + w(t), & \text{symbol 1 was sent} \\ -A + w(t), & \text{symbol 0 was sent} \end{cases}$$

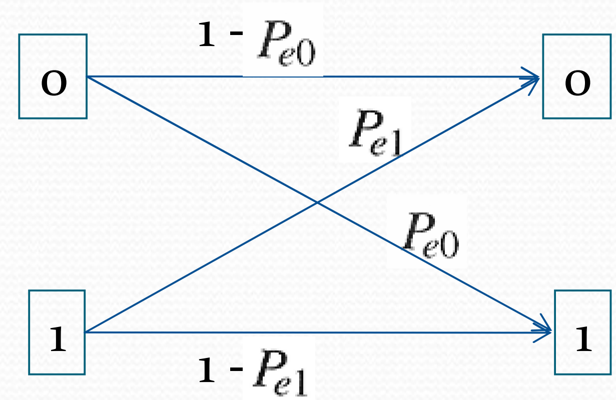
- The receiver has prior knowledge of the pulse shape, need to decide 1 or 0 for a received amplitude in each signaling interval $0 \le t \le t_b$

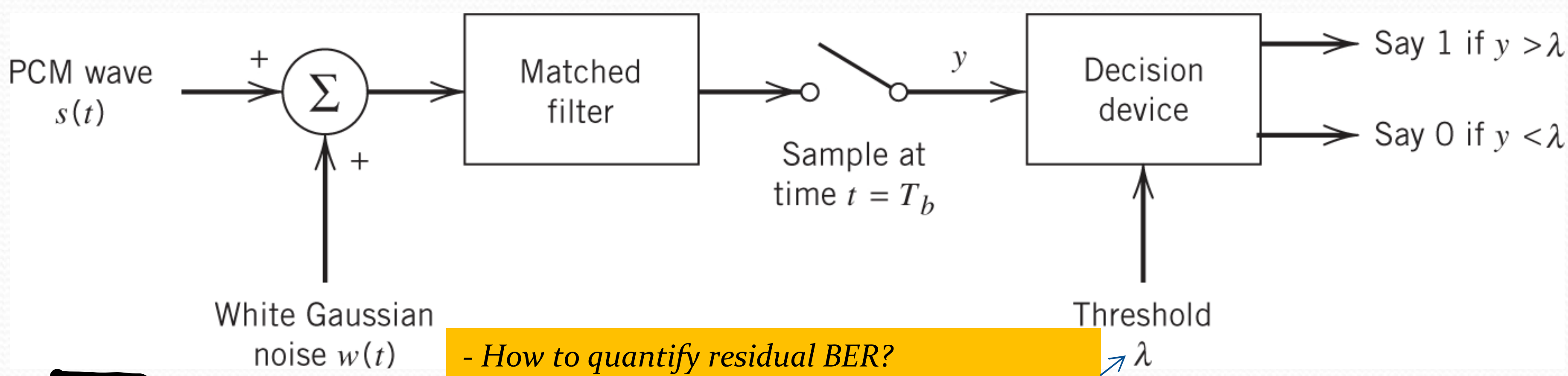
# Probability of Error due to Noise (contd.)

- Sampled value y, with threshold $\lambda$,

$$\begin{cases} \text{if } y > \lambda, \text{symbol 1 received} \\ \text{if } y \le \lambda, \text{symbol 0 received} \end{cases}$$

- Two kinds of errors

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*- How to quantify residual BER?*
*- How to choose threshold to minimize BER?*

# Consider the Case When Symbol 0 Was Transmitted

- Receiver gets $x(t) = -A + w(t)$, for $0 \le t \le T_b$

See Page 19

- The matched filter output, sampled at $t=T_b$, is the sampled value of a random variable $Y$

$$y = \frac{1}{T_b}\int_0^{T_b} x(t)\, dt = -A + \frac{1}{T_b}\int_0^{T_b} w(t)\, dt$$

- Since $w(t)$ is white and Gaussian, $Y$ is also Gaussian with mean $E[Y]=-A$, and variance

$$\sigma_Y^2 = \mathbf{E}[(Y+A)^2] = \frac{1}{T_b^2}\mathbf{E}\left[\int_0^{T_b}\int_0^{T_b} w(t)w(u)\, dt\, du\right]$$

$$= \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} \mathbf{E}[w(t)w(u)]\, dt\, du = \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} R_W(t,u)\, dt\, du$$

*Gaussian distribution can be completely determined by it mean and variance*

[redacted]

Sampled voltage

$$Y = \begin{cases} +A + N & : \text{Transmit "1"} \\ -A + N & : \quad \text{" \quad "0"} \end{cases}$$

where

$$f_N(n) = \frac{1}{\sqrt{2\pi}\,\sigma_n} \exp\left[-\frac{n^2}{2\sigma_n^2}\right], \quad -\infty < n < \infty$$

Detection Rule (at the Receiver)

Decode as "1" if $Y \geq 0$ THRESHOLD VOLTAGE

" " "0" if $Y < 0$

| Transmit \ Decode as | "1" | "0" |
|---|---|---|
| "1" | ✓ | ✗ |
| "0" | ✗ | ✓ |

Probability of Error

$$P_e = P\Big( \underbrace{\{T_x \text{"1"}, \text{Decode "0"}\}}_{\tilde{A}} \cup \underbrace{\{T_x \text{"0"}, \text{Decode "1"}\}}_{\tilde{B}} \Big)$$

UNION (OR)

AND ($\cap$)

$$P(\tilde{A} \cup \tilde{B}) = P(\tilde{A}) + P(\tilde{B}) \quad \text{if } \tilde{A} \cap \tilde{B} = \phi$$

$$\Rightarrow P_e = P(\underbrace{T_x \text{"1"}}_{A}, \underbrace{\text{Decode "0"}}_{B}) + P(T_x \text{"0"}, \text{Decode "1"})$$

$$P(A,B) = P(A \cap B) = P(AB)$$

$$= \underbrace{P(A|B)}_{\text{Conditional prob. of A given B}} P(B) = \underbrace{P(B|A)\,P(A)}$$

$$\{\text{Decode "0"}\} = \{Y < 0\}$$

$$\{\text{Decode "1"}\} = \{Y \geq 0\}$$

$$P(T_x\text{"1"}, \text{Decode "0"}) = P(T_x\text{"1"}, Y < 0)$$

$$= P(Y < 0 \mid T_x\text{"1"})\,P(T_x\text{"1"})$$

$$= \underbrace{P(A+N < 0)}_{P(N < -A)}\ \underbrace{P(T_x\text{"1"})}_{P_1}$$

By symmetry of $f_N(n)$ about $n=0$,

$$\overbrace{\underbrace{P(N<-A)}_{\int_{-\infty}^{-A} f_N(n)\,dn}}^{P(-\infty<N<-A)} = \overbrace{\underbrace{P(N>A)}_{\int_{A}^{\infty} f_N(n)\,dn}}^{P(A<N<\infty)}$$

$$P(N>A) = \int_{n=A}^{n=\infty} \frac{1}{\sqrt{2\pi}\,\sigma_n} \exp\left(-\frac{n^2}{2\sigma_n^2}\right) dn$$

Set $x = n/\sigma_n$

$\Rightarrow dx = \frac{dn}{\sigma_n}$

$$\int_{x=\frac{A}{\sigma_n}}^{x=\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = Q\left(\frac{A}{\sigma_n}\right)$$

Def. $Q(d) = \int_{d}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \quad d \geq 0$

$*$ $Q(-d) = 1 - Q(d)$ , $d \geq 0$.

$P(T_n\text{"1"}) + P(T_n\text{"0"}) = 1$

MATLAB : qfunc(·)

qfuncinv(·)

$$\Rightarrow \boxed{P(T_n\text{"1"}, \text{Decode "0"}) = p_1 \, Q\left(\frac{A}{\sigma_n}\right)}$$

$$P(T_n\text{"0"}, \text{Decode "1"}) = P(T_n\text{"0"}, Y \geq 0)$$

$$= \underbrace{\underbrace{P(Y \geq 0 \mid T_n\text{"0"})}_{P(-A+N \geq 0)}}_{P(N \geq A) = Q\left(\frac{A}{\sigma_n}\right)} \underbrace{P(T_n\text{"0"})}_{1-p_1} = (1-p_1)\, Q\left(\frac{A}{\sigma_n}\right)$$

$$\Rightarrow P_e = p_1 Q\left(\frac{A}{\sigma_n}\right) + (1-p_1)\, Q\left(\frac{A}{\sigma_n}\right)$$

$$= Q\left(\frac{A}{\sigma_n}\right)$$

Power spectral density of $w(t)$ is $S_w(f) = \frac{N_0}{2}$, $-\infty < f < \infty$

# When 0 Was Transmitted (contd.)

- Since *w(t)* is white Gaussian, $R_W(t,u) = \frac{N_0}{2}\delta(t-u)$
- The variance is

$$\sigma_Y^2 = \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b}\frac{N_0}{2}\delta(t-u)\,dt\,du = \frac{N_0}{2T_b}$$

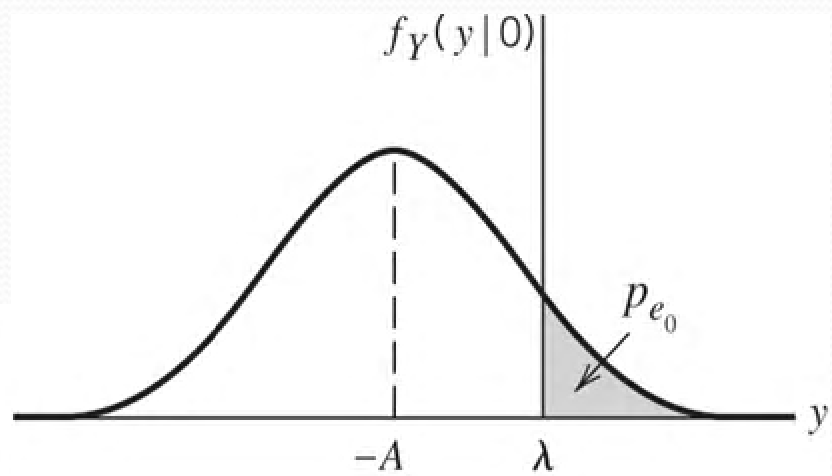
- $Y$ is Gaussian with mean $\mu_Y = -A$, variance $\sigma_Y^2 = N_o/(2T_b)$
  - The conditional probability density function (PDF) of *Y*, conditioned on that symbol 0 was transmitted, is

$$\underbrace{f_Y(y\,|\,0) = \frac{1}{\sqrt{2\pi}\sigma_Y}\exp\left(-\frac{(y-\mu_Y)^2}{2\sigma_Y^2}\right)}_{\text{Standard PDF of Gaussian r.v., with mean } \mu_Y \text{ and variance } \sigma_Y^2} = \frac{1}{\sqrt{\pi N_0/T_b}}\exp\left(-\frac{(y+A)^2}{N_0/T}\right)$$

[redacted]

# When 0 Was Transmitted (contd.)

- When no noise, $Y$=-$A$
- With noise, drifts away from –$A$
  - If less than $\lambda$, output 0 (no bit error)
  - If larger than $\lambda$, output 1 (bit error occurs)




$$P_{e0} = P(y > \lambda | \text{symbol 0 was sent})$$
$$= \int_{\lambda}^{\infty} f_Y(y|0)\, dy$$
$$= \frac{1}{\sqrt{\pi N_0/T_b}} \int_{\lambda}^{\infty} \exp\left(-\frac{(y+A)^2}{N_0/T_b}\right) dy$$

# When 0 Was Transmitted (contd.)

- Assume symbol 1 and 0 are equal likely to be transmitted, we choose $\underline{\lambda=0}$, due to symmetry

$$P_{e0} = \frac{1}{\sqrt{\pi N_0/T_b}} \int_0^{\infty} \exp\left(-\frac{(y+A)^2}{N_0/T_b}\right) dy$$

- Define $z = \frac{y+A}{\sqrt{N_0/2T_b}}$ we have

$$P_{e0} = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2E_b/N_0}}^{\infty} \exp(-z^2/2)\, dz$$

$E_b$: the transmitted signal energy per bit $E_b = A^2 T_b$

$E_b/(N_o/2)$ : (signal power per bit)/(noise power per Hz)

# When 0 Was Transmitted (contd.)

- Define $Q$-Function:

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} \exp(-z^2/2)\, dz$$

- The conditional bit error probability when 0 was transmitted is

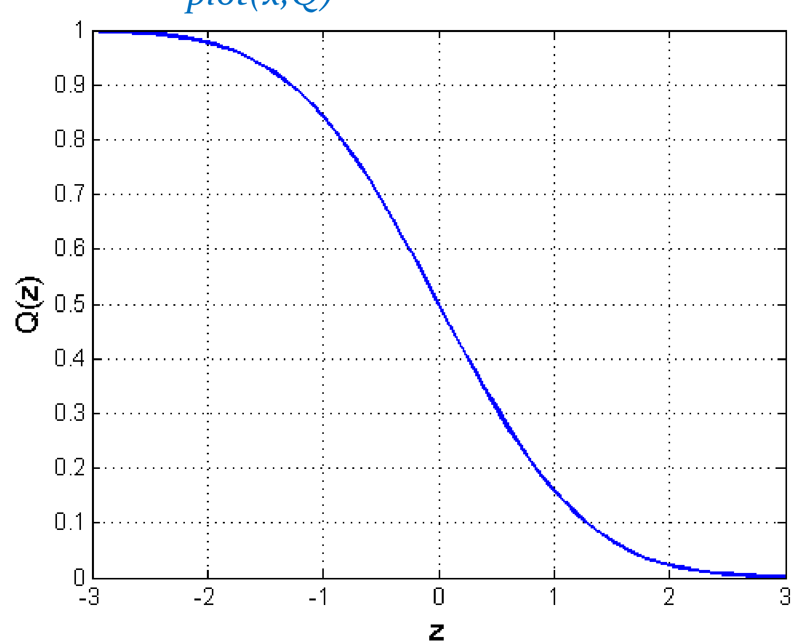
$u$

$$P_{e0} = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2E_b/N_0}}^{\infty} \exp(-z^2/2)\, dz$$

$$\boxed{P_{e0} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

Q-function, see Page 401

```
x=[-3:0.1:3];
for i=1:length(x)
Q(i)=0.5*erfc(x(i)/sqrt(2));
end
plot(x,Q)
```

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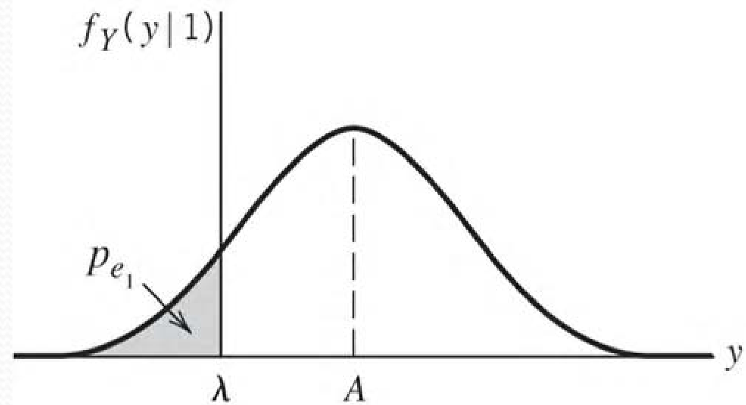
# When 1 Was Transmitted

- Receives: $x(t)=A+w(t),\ 0\le t\le T_b$
- $Y$ is Gaussian with $\mu_Y=A$, $\sigma_Y^2=N_0/(2T_b)$
- We have

$$f_Y(y|1) = \frac{1}{\sqrt{\pi N_0/T_b}} \exp\left(-\frac{(y-A)^2}{N_0/T_b}\right)$$




- The conditional bit error rate is

$$P_{e1} = P(y<\lambda|\text{symbol 1 was sent}) = \int_{-\infty}^{\lambda} f_Y(y|1)\,dy$$
$$= \frac{1}{\sqrt{\pi N_0/T_b}} \int_{-\infty}^{\lambda} \exp\left(-\frac{(y-A)^2}{N_0/T_b}\right) dy$$

- Choosing $\lambda=0$, and defining $\frac{y-A}{\sqrt{N_0/2T_b}} = -z$, we have

$$\boxed{P_{e1} = P_{e0} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

# Bit Error Probability (or, Bit Error Rate – BER)

- Bit Error Rate (BER) is

$$P_e = \Pr\{0 \text{ is transmitted}\} \times P_{e0} + \Pr\{1 \text{ is transmitted}\} \times P_{e1}$$

$$= p_0 \times P_{e0} + p_1 \times P_{e1} = \frac{1}{2} \times Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \frac{1}{2} \times Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- Depends only on $E_b/N_o$, the ratio of the transmitted signal energy per bit to the noise spectral density
  - Noise is usually fixed for a given temperature
  - Energy plays the crucial role → transmit power

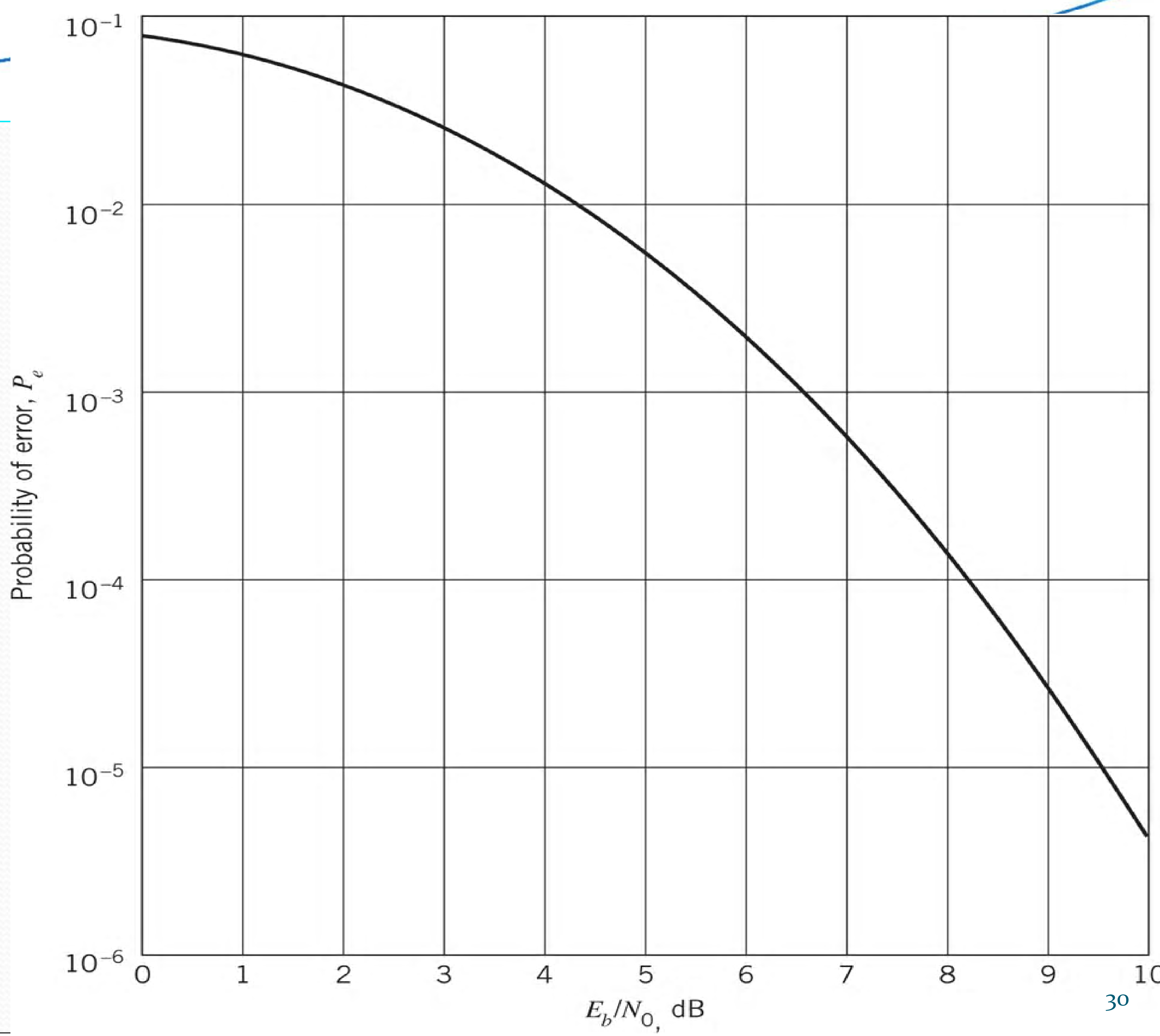
$$E_b = A^2 T_b$$

Wider pulse
Higher amplitude

- What are the limiting factors?
  - Battery life, interference to others, data rate requirement

[redacted]

# BER



Probability of error, $P_e$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
0 1 2 3 4 5 6 7 8 9 10
$E_b/N_0$, dB

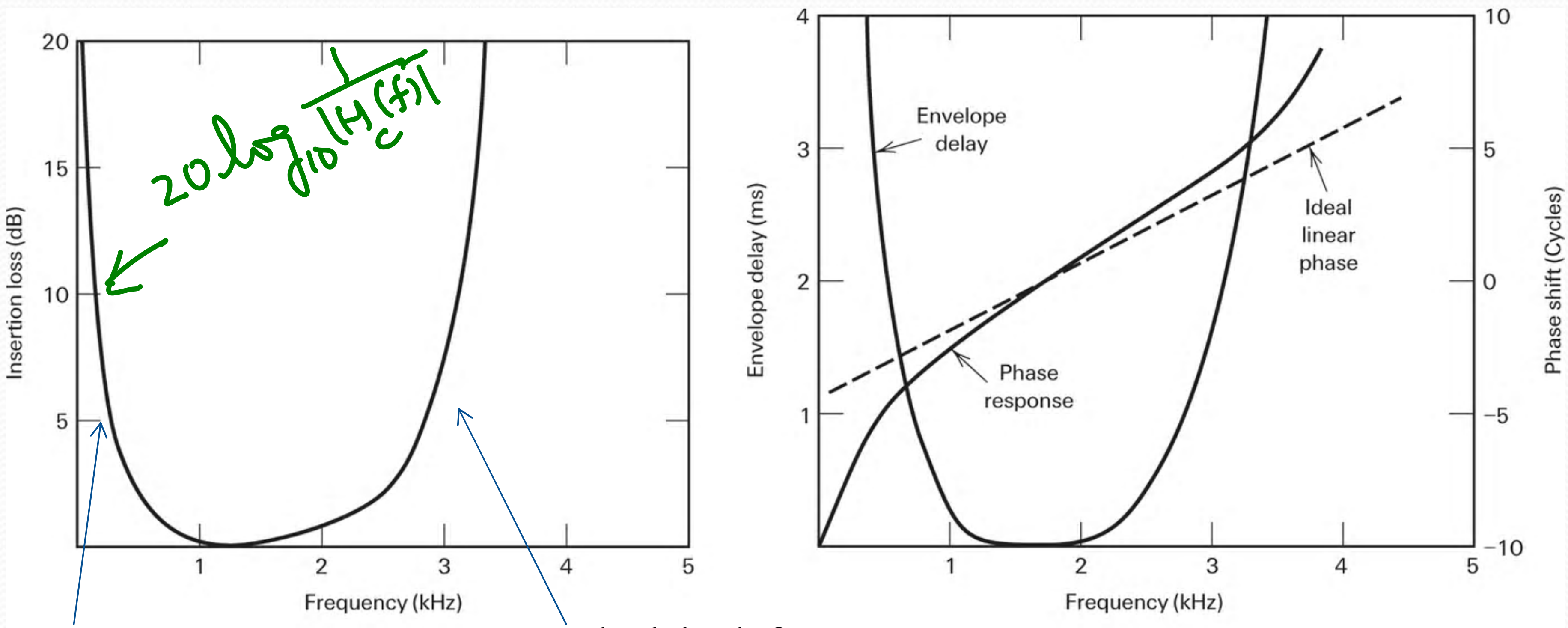
# Intersymbol Interference

- The next source of bit errors to be addressed
- Happens when the channel is dispersive
  - The channel has a frequency-dependent (or, frequency-selective) amplitude spectrum
    - e.g., band-limited channel:
      - passes all frequencies $|f|<W$ without distortion
      - Blocks all frequencies $|f|>W$
- Use discrete pulse-amplitude modulation (PAM) as example
  - First examine binary data
  - Then consider the more general case of $M$-ary data

[redacted]

# Example 8.2: The Dispersive Nature of a Telephone Channel

- Band-limited and dispersive



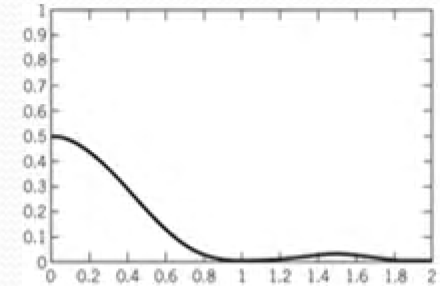

*Block dc* (*a*)

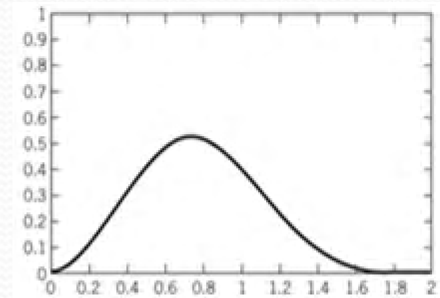
*Block high frequencies: cut-off at 3.5 kHz* (*b*)

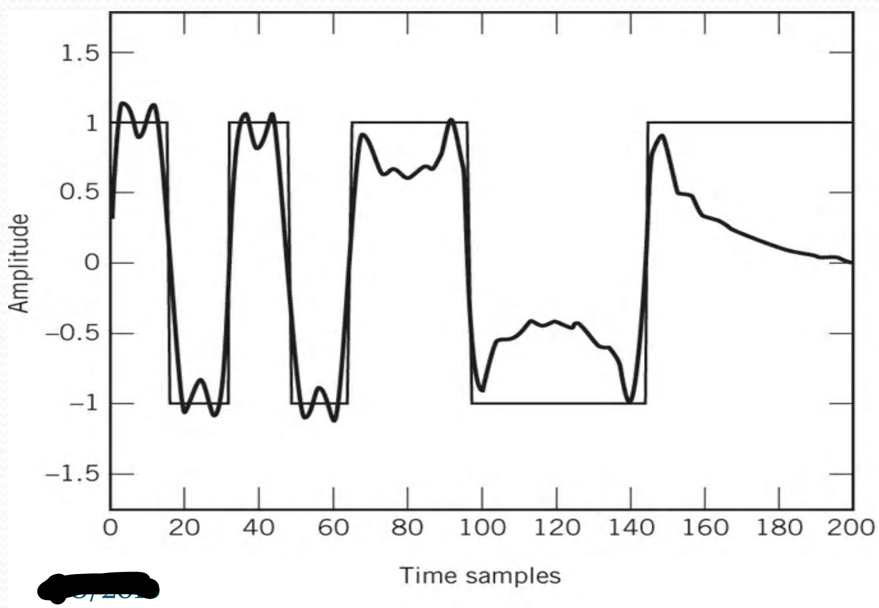
[redacted]

# Example 8.2: The Dispersive nature of a Telephone Channel

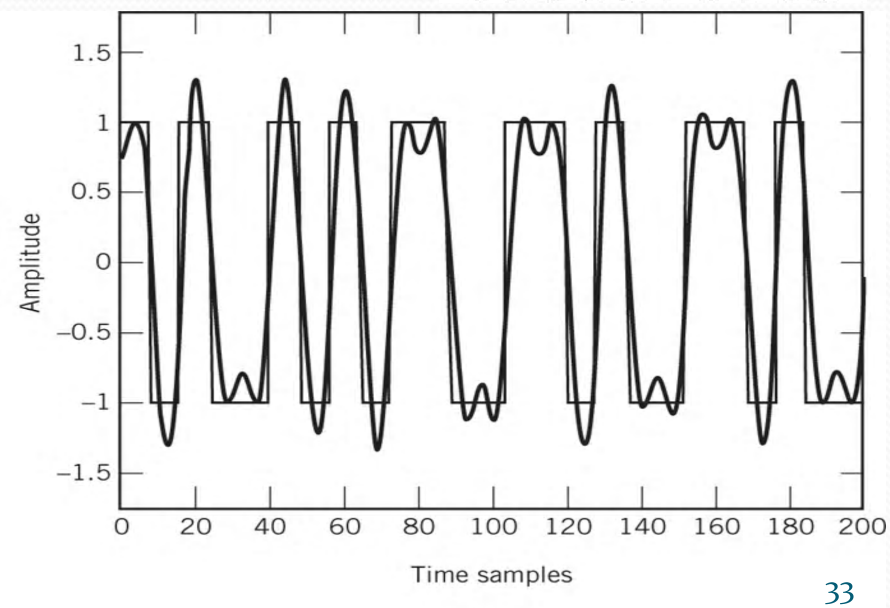
- Conflicting requirements for line coding
  - High frequencies blocked →need a line code with a narrow spectrum → polar NRZ
    - But polar NRZ has *dc*
  - Low frequencies blocked → need a line code that has no *dc* → Manchester code
    - But Manchester code has high frequency







(*a*)

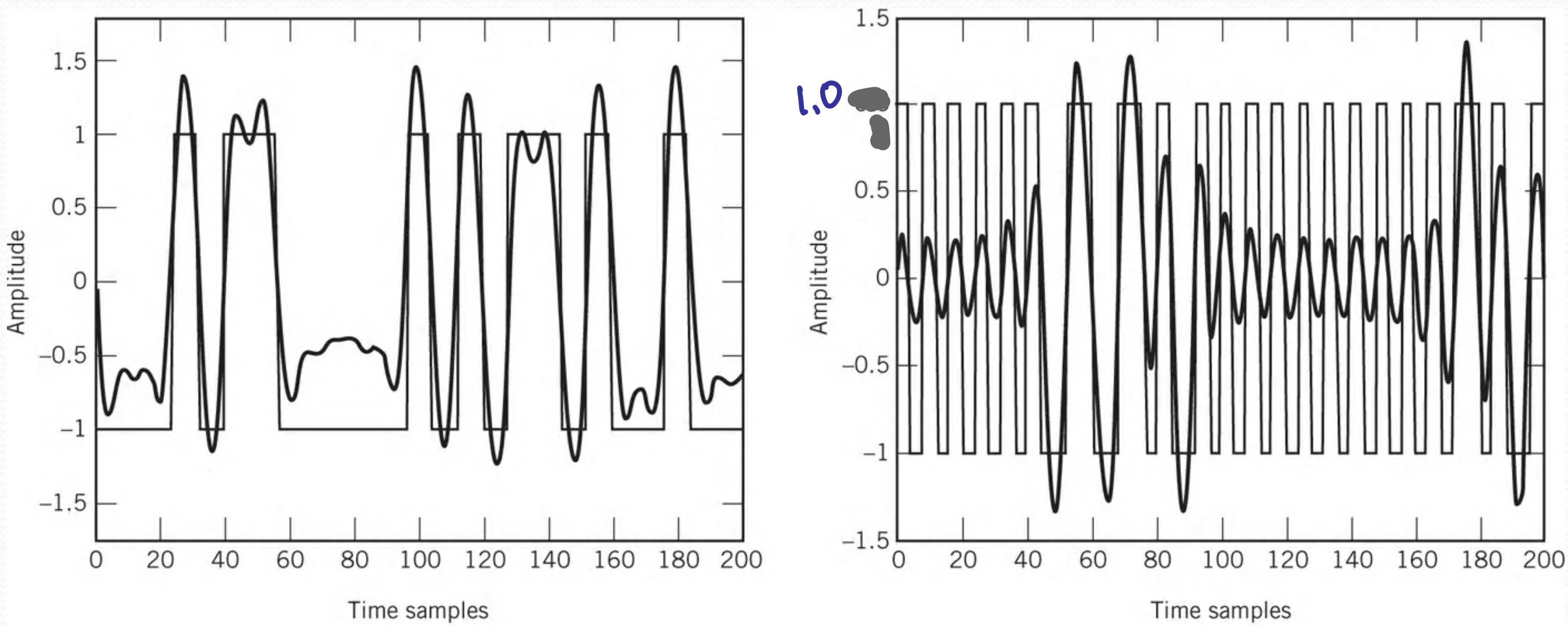


(*b*)

[redacted]

# Example 8.2: The Dispersive nature of a Telephone Channel (contd.)

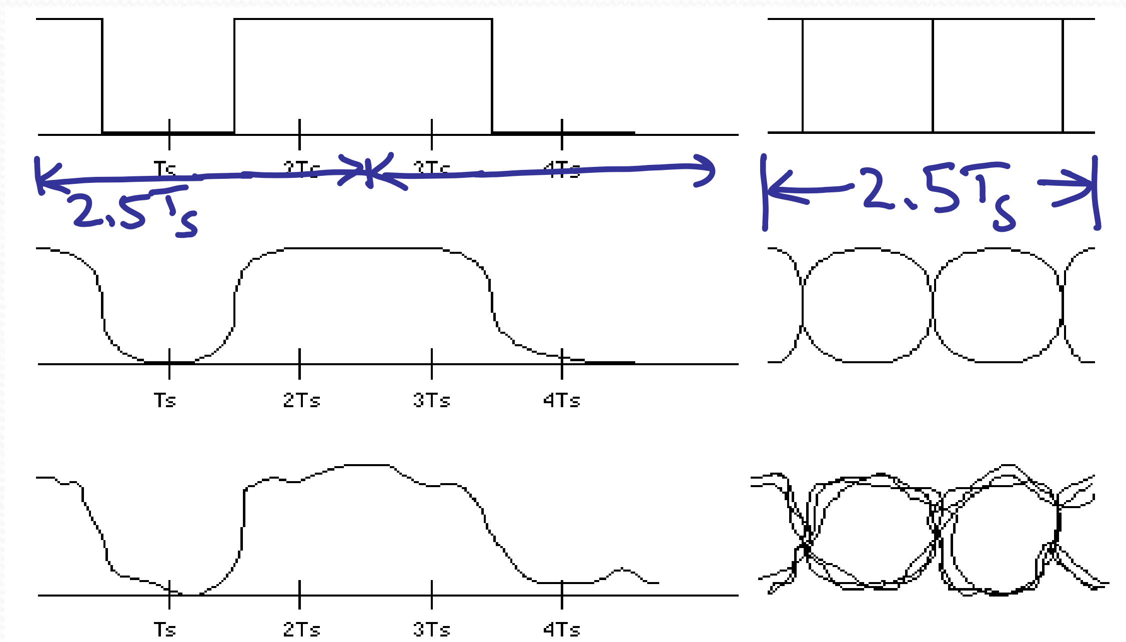
- Previous page: data rate at 1600 bps
- This page: data rate at 3200 bps




(a) (b)

# Eye Pattern

- An operational tool for evaluating the effects of ISI
- Synchronized superposition of **all** possible realizations of the signal viewed within a particular signaling interval

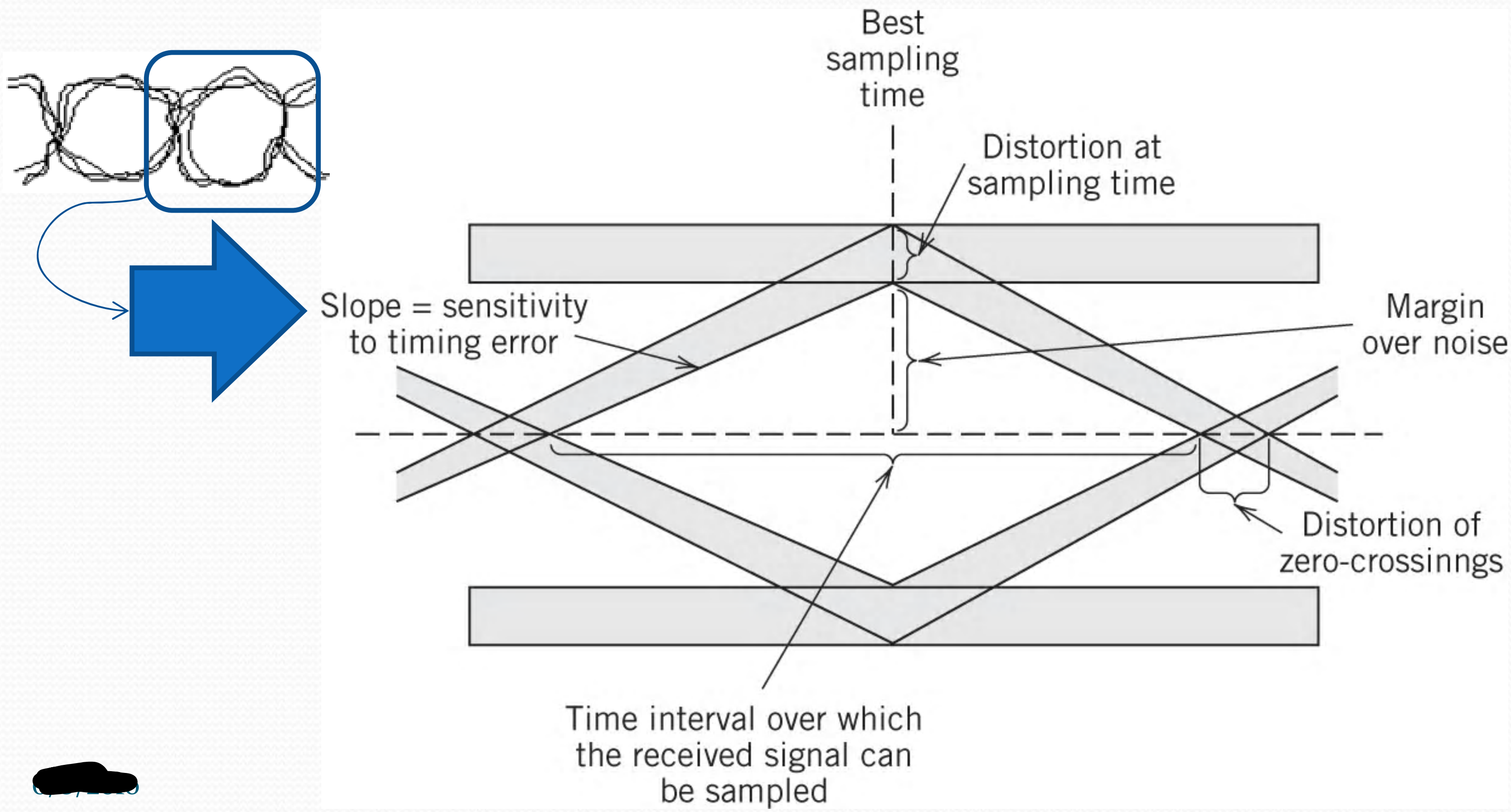





http://members.chello.nl/~m.heijligers/DAChtml/digcom/digcom.html

# Eye Pattern (contd.)

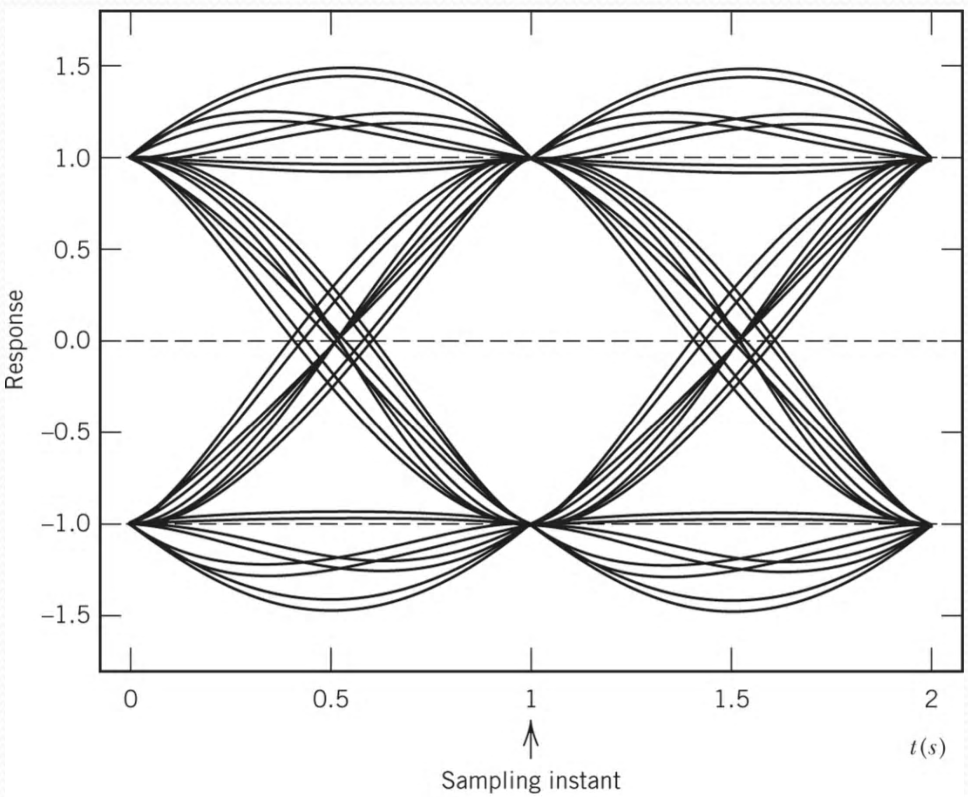
- Eye opening: the interior region of the eye pattern



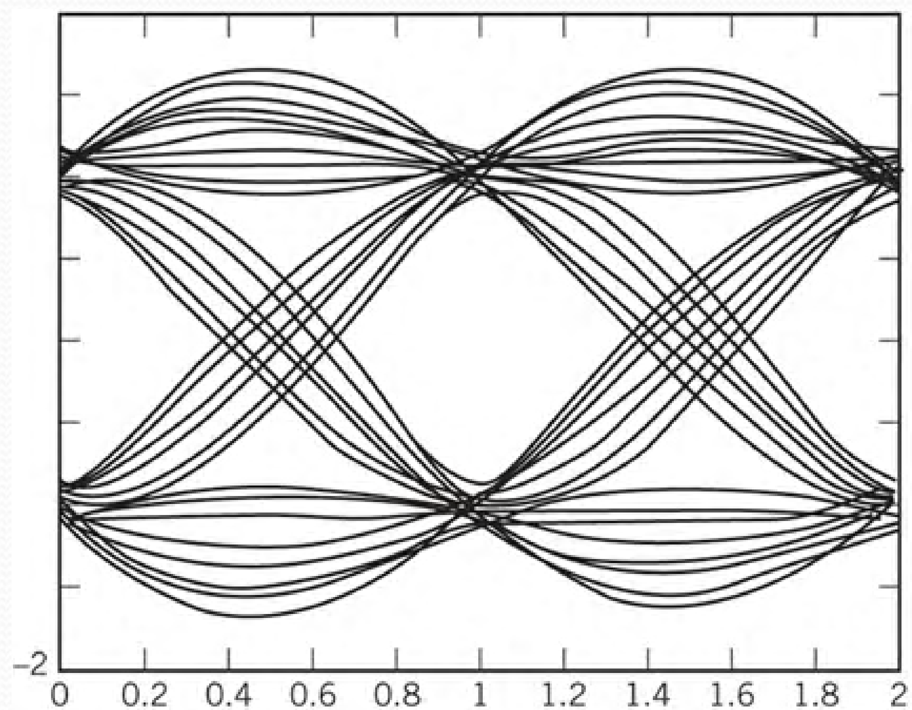
# Interpreting Eye Pattern

- The width of the eye opening
  - Defines the time interval over which the received signal can be sampled without error from ISI
  - The best sampling time: when the eye is open the widest
- The slope
  - The sensitivity of the system to timing errors
  - The rate of closure of the eye as the sampling time is varied
- The height of the eye opening
  - Noise margin of the system
- Under severe ISI: the eye may be completely closed
  - Impossible to avoid errors due to ISI

# Example 8.3




(a) $M = 2$




(a)

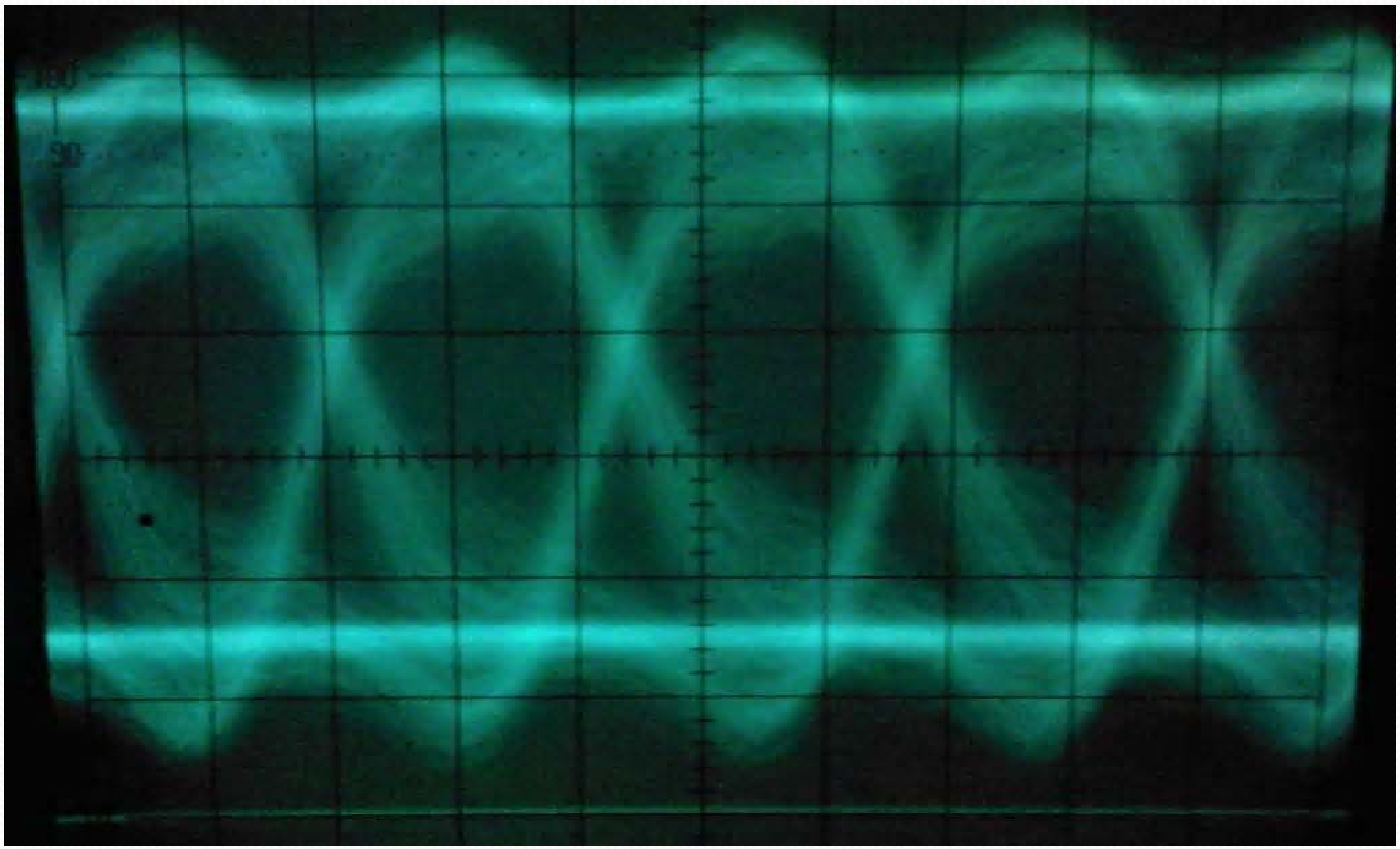
m = 2

The channel has no bandwidth limitation: the eyes are open

Band-limited channel: blurred region at sampling time

[redacted]

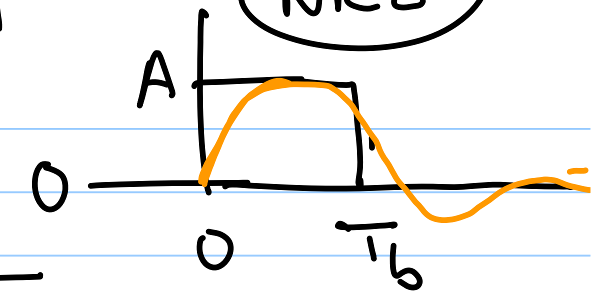
# Eye Pattern on Oscilloscope



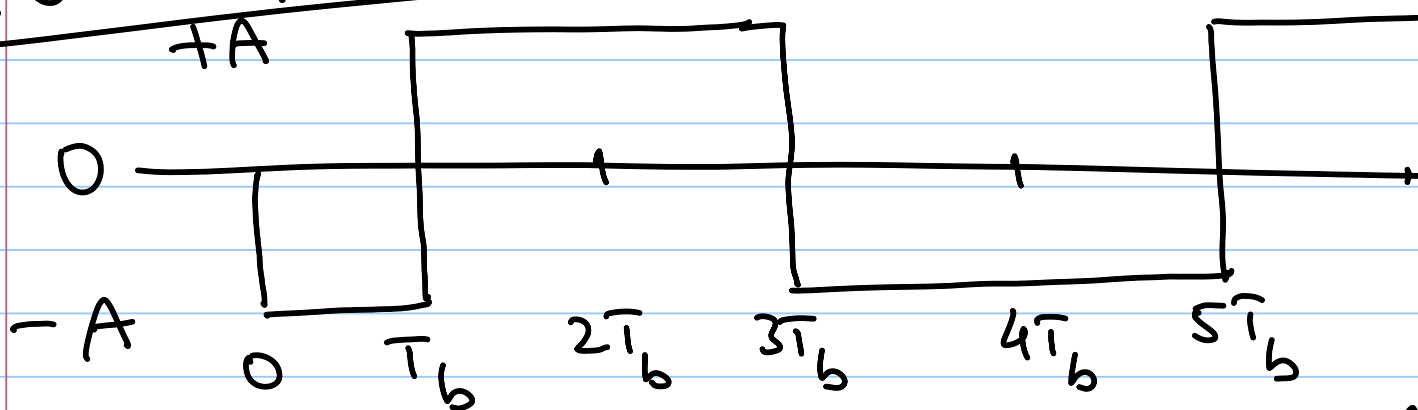
http://www.myprius.co.za/pcmx2_processor1.ht
m

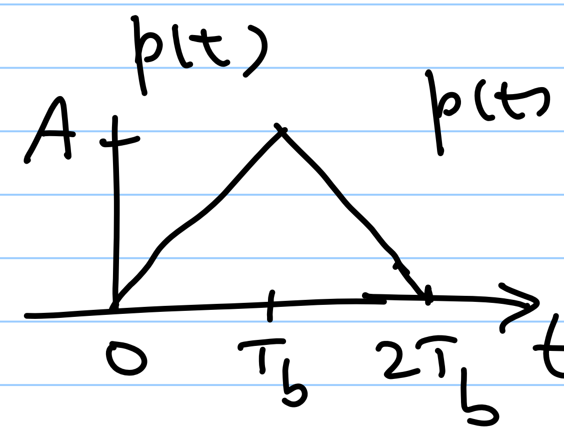
0 1 1 0 0 1

NRZ



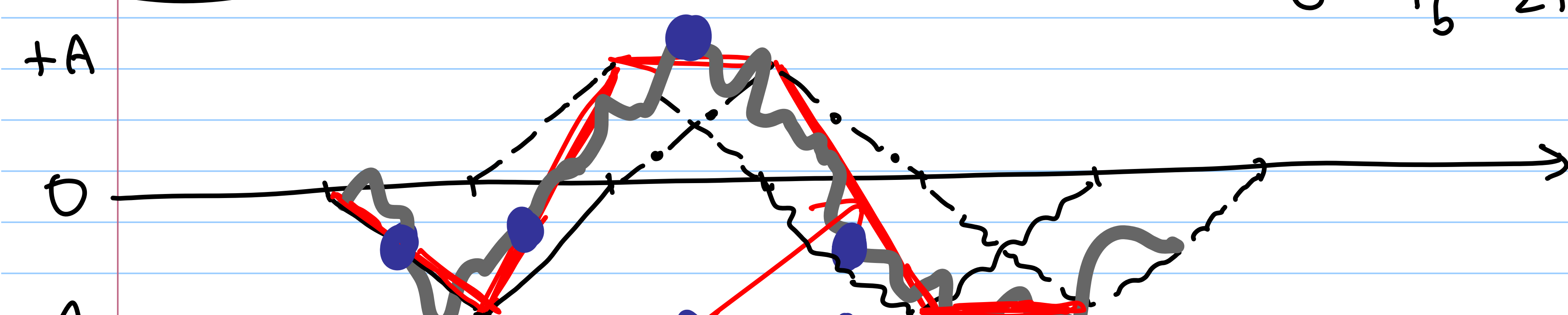

POLAR / NRZ



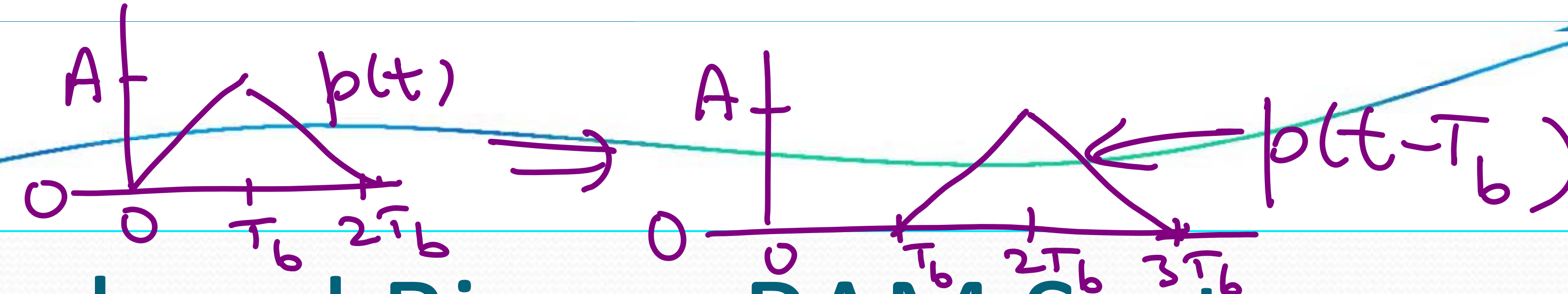




POLAR / p(t)

0 1 1 0 0




"Distorted" line coded waveform




# Baseband Binary PAM System

- Input sequence: $\{b_k \mid b_k = 0 \text{ or } 1\}$
- Transmitted signal: $s(t) = \sum_k a_k g(t - kT_b)$
- The receiver filter output: $y(t) = \mu \sum_k a_k p(t - kT_b) + n(t)$

$iT_b$

NOISE

  - where:

$t = iT_b,$

$i = 0, \pm 1, \pm 2, \ldots$

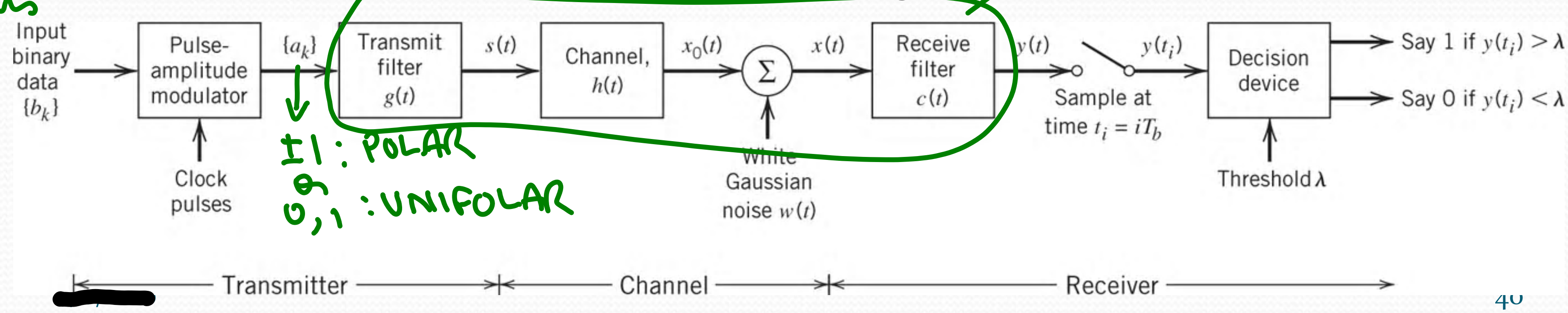
$$\mu \times p(t) = g(t) * h(t) * c(t)$$

$$\mu \times P(f) = G(f) \times H(f) \times C(f)$$

p(t)

  - Assume $p(t)$ is normalized, $p(0)=1$, using $\mu$ as a scaling factor to account for amplitude change during transmission

Bits



# Baseband Binary PAM System (contd.)

- For the $i$-th received symbol, sample the output $y(t)$ at $t_i = iT_b$, yielding

$t = iT_b$

$t_i = t$

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k\, p[(i-k)T_b] + \cancel{n(t_i)}$$

$k = i$

$$= \mu a_i\, p(0) + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k\, p[(i-k)T_b] + \cancel{n(t_i)}$$

Contribution of the $i$-th transmitted bit

Residual effect due to the occurrence of pulses before and after the sampling instant $t_i$: the ISI

~~Effect of noise, taken care of by *matched filter*~~

In the absence of both ISI and noise: $y(t_i) = \mu a_i$

# Nyquist's Criterion for ~~Distortionless Transmission~~ ZERO ISI

- Recall that:

$$y(t_i) = \underbrace{\mu a_i}_{p[0]} + \mu \sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} a_k p[(i-k)T_b] + n(t_i)$$

$$\mu \times p(t) = g(t) * h(t) * c(t)$$

- The problem:
  - Usually the transfer function of the channel *h(t)* and the transmitted pulse shape are specified (e.g., p32, telephone channel)
  - to determine the transfer functions of the transmit and receive filters so as to reconstruct the input binary $\{b_k\}$
- The receiver performs
  - Extraction: sampling *y(t)* at time $t=iT_b$
  - Decoding:
    - requires the ISI to be zero at the sampling instance:

$$\boxed{p[(i-k)T_b] = \begin{cases} 1, & i = k \\ 0, & i \neq k. \end{cases}}$$

$p(\pm mT_b) = 0$
for $m = 1, 2, 3, \ldots$

# Nyquist's Criterion for Distortionless Transmission (contd.)

- Sampling $p(t)$ at $nT_b$, $n$=0, ±1, ±2, … → $\{p(nT_b)\}$
- Sampling in the time domain produces periodicity in the frequency domain, we have

$$P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b)$$

- On the other hand, the sampled signal is

$$p_\delta(t) = \sum_{n=-\infty}^{\infty} p(nT_b)\delta(t - nT_b)$$

Its Fourier transform is

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} [p(nT_b)\delta(t - nT_b)]\exp(-2\pi ft)dt$$

If the condition is satisfied $\leftarrow$ $\boxed{p[(i-k)T_b] = \begin{cases} 1, & i = k \\ 0, & i \neq k. \end{cases}}$

$$P_\delta(f) = \int_{-\infty}^{\infty} p(0)\delta(t)\exp(-2\pi ft)dt = p(0) = 1$$

# Nyquist's Criterion for Distortionless Transmission (contd.)

$P(f)$: for the overall system, including the transmit filter, the channel, and the receive filter

$p(t) = \begin{cases} \neq 0, & t = 0 \\ 0, & t = \pm mT_b, \; m = 1, 2, \ldots \end{cases}$

- Finally we have

$$P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b) = 1$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f - nR_b) = 1/R_b = T_b$$

ZERO ISI

- The Nyquist Criterion for ~~distortionless baseband~~ transmission in the absence of noise:

FREQ. DOMAIN VERSION

The frequency function $P(f)$ eliminates ISI for samples taken at intervals $T_b$ provided that it satisfies

$$\boxed{\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b}$$

$c \neq 0$ , $R_b = \frac{1}{T_b}$ = Bit rate

$p(t) \leftrightarrow P(f)$

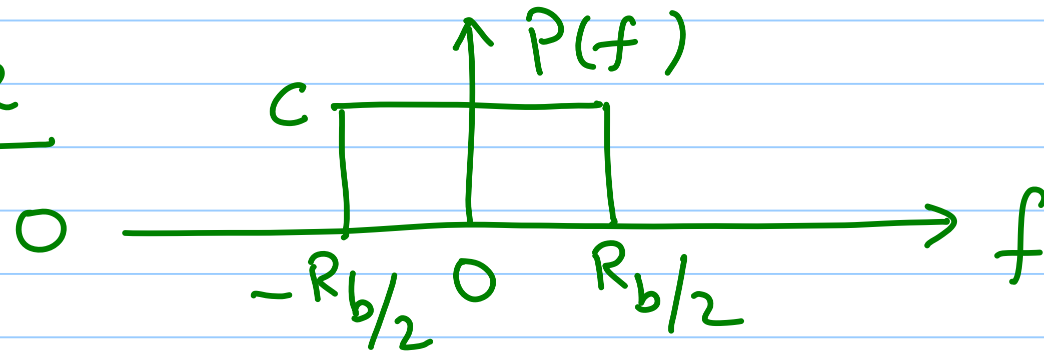
$$\sum_{n=-\infty}^{\infty} P(f-nR_b) = C \,, \quad -\infty < f < \infty \equiv -R_b/2 \le f \le \frac{R_b}{2}$$

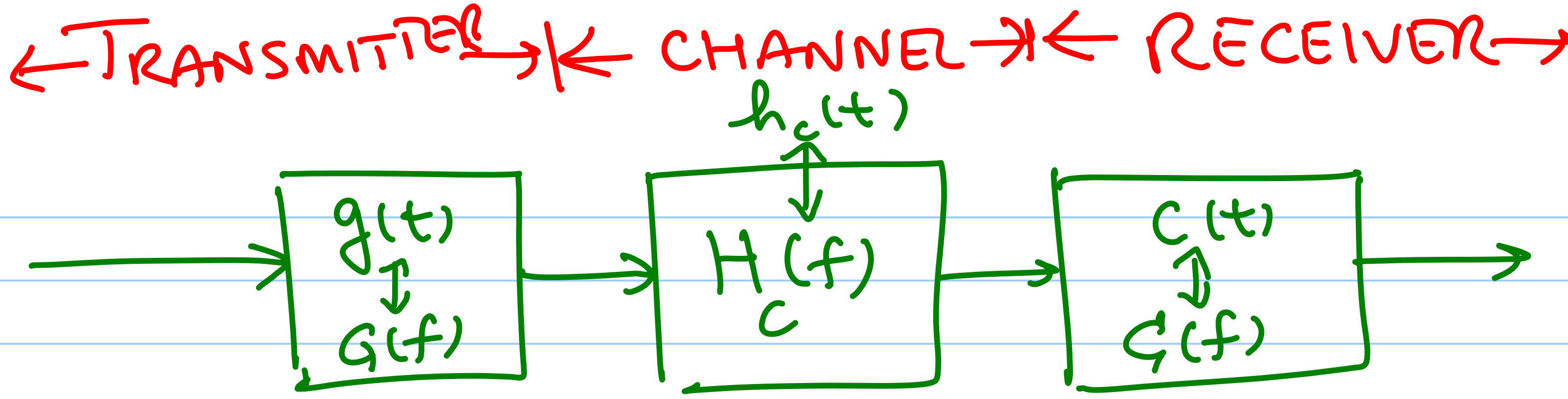
"in practice" $\Longrightarrow$

$$P(f)+P(f-R_b)+P(f+R_b) = C \text{ for } |f| \le \frac{R_b}{2}$$

$$P(f)+P(f-R_b) \stackrel{?}{=} C \,, \quad 0 \le f \le \frac{R_b}{2}$$

"Candidate" Pulse





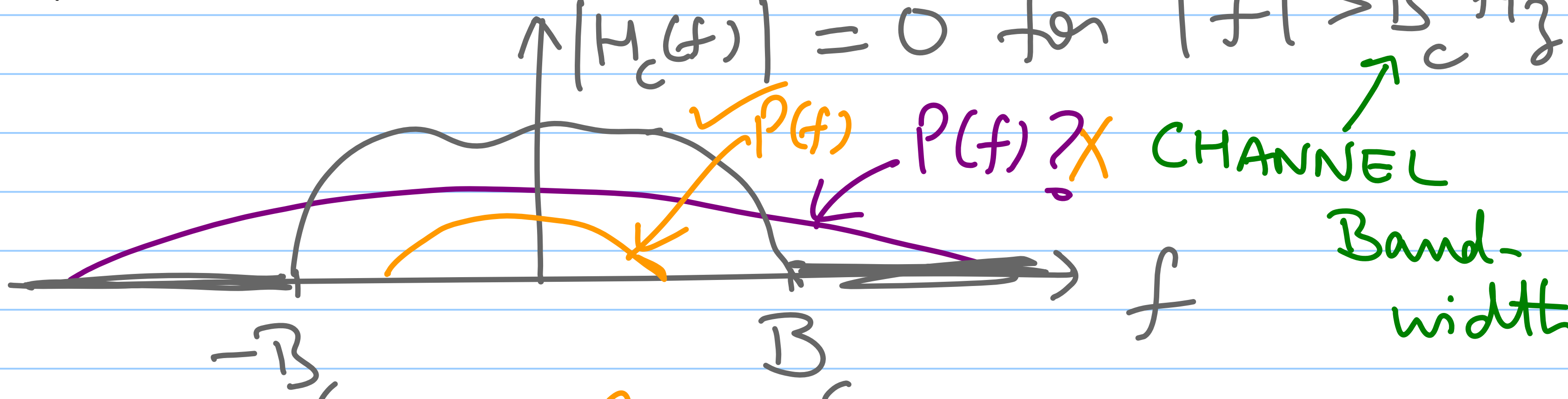

$p(t) \leftrightarrow P(f)$

$$P(f) = G(f)\, H_c(f)\, C(f) \Rightarrow \boxed{C(f) = \frac{P(f)}{G(f) H_c(f)} \text{ for } P(f) \neq 0.}$$

$$\Updownarrow$$

$$p(t) = g(t) * h_c(t) * c(t)$$

$|H_c(f)| = 0$ for $|f| > B_c$ Hz




$\Rightarrow$ [Bandwidth of $P(f)$] $\leq$ [Bandwidth of $H_c(f)$]

$\Rightarrow$ Bandwidth of $P(f) \leq B_c$

$$\Rightarrow \underbrace{\frac{R_b}{2}}_{\leq \text{Bandwidth of } P(f)} \leq B_c \Rightarrow R_b \leq 2B_c$$

1. NO SOLUTION if $R_b > 2B_c$
2. UNIQUE solution if $R_b = 2B_c$:

$$P(f) = \begin{cases} c, & -\frac{R_b}{2} \leq f \leq \frac{R_b}{2} \\ 0, & \text{elsewhere} \end{cases}$$

3 $\frac{R_b}{2} < B_c$ :

Infinitely many solutions !!

# Ideal Nyquist Channel

- The simples way of satisfying The Nyquist Criterion is

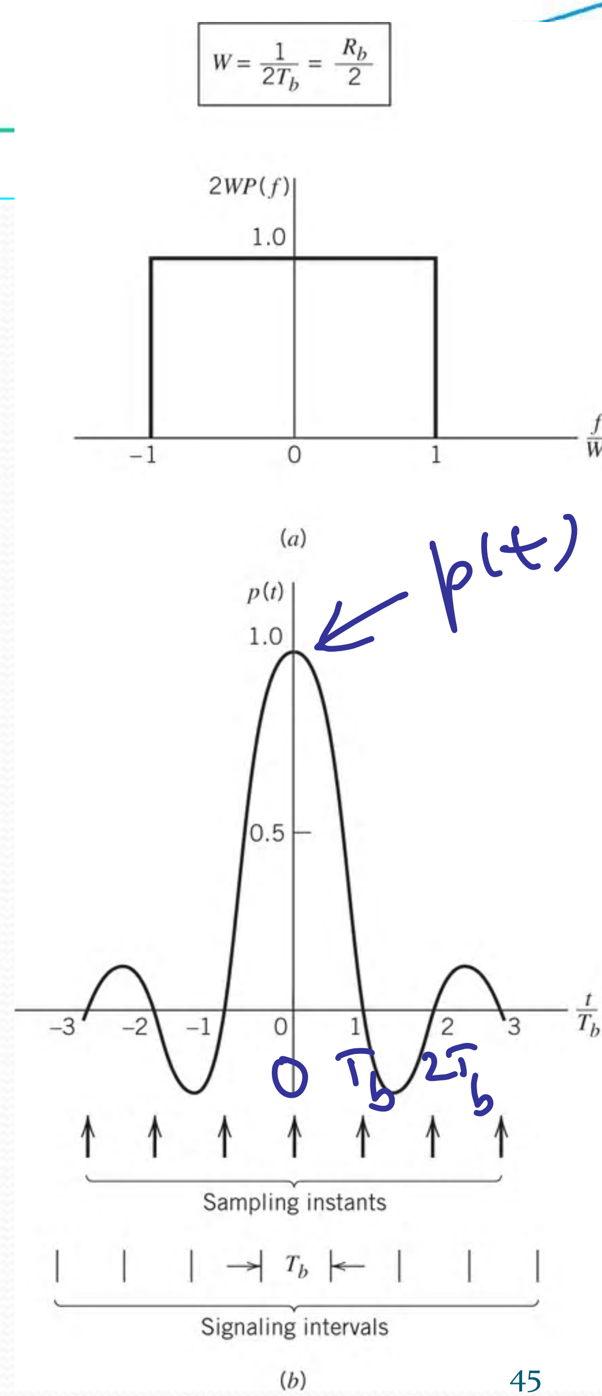
$$P(f) = \frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right) = \begin{cases} \dfrac{1}{2W}, & -W < f < W \\ 0, & |f| > W \end{cases}$$

Where $W = R_b/2 = 1/(2T_b)$.

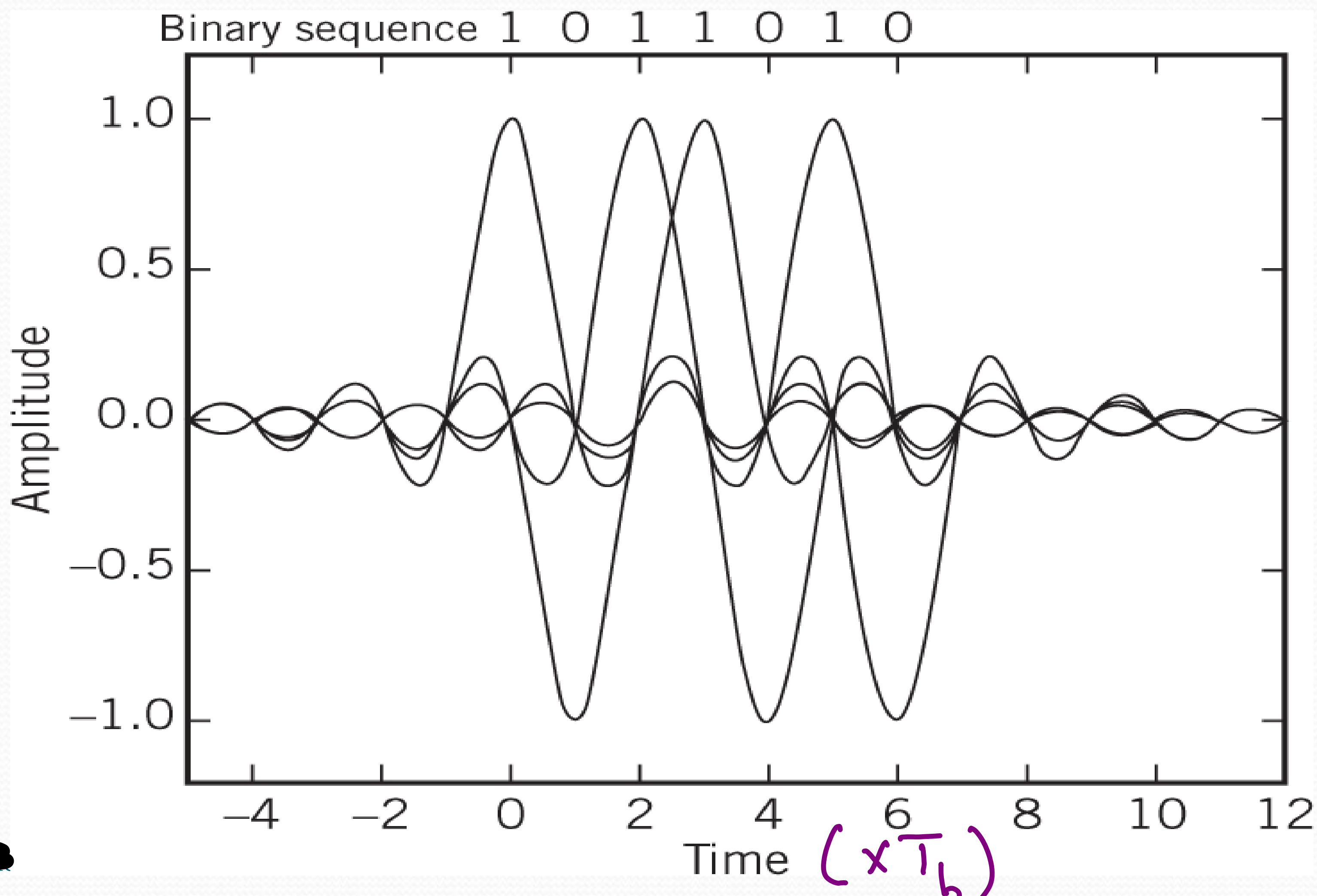
- The signal that produces zero ISI is the *sinc* function

$$p(t) = \frac{\sin(2\pi Wt)}{2\pi Wt} = \text{sinc}(2Wt) = \text{sinc}(R_b t) = \text{sinc}(t/T_b)$$

- Nyquist bandwidth: $W$
- Nyquist rate: $R_b = 2W$



# Ideal Nyquist Channel (contd.)




[redacted]

# Raised Cosine Spectrum

$W = \frac{R_b}{2}$

- $P(f)$ is physically unrealizable
  - No filter can have the abrupt transitions at $f = \pm W$
  - $p(t)$ decays at rate $1/|t|$: too slow, no margin for sampling time error (see Fig. 8.16)
- Use raised cosine spectrum: a flat top + a rolloff portion

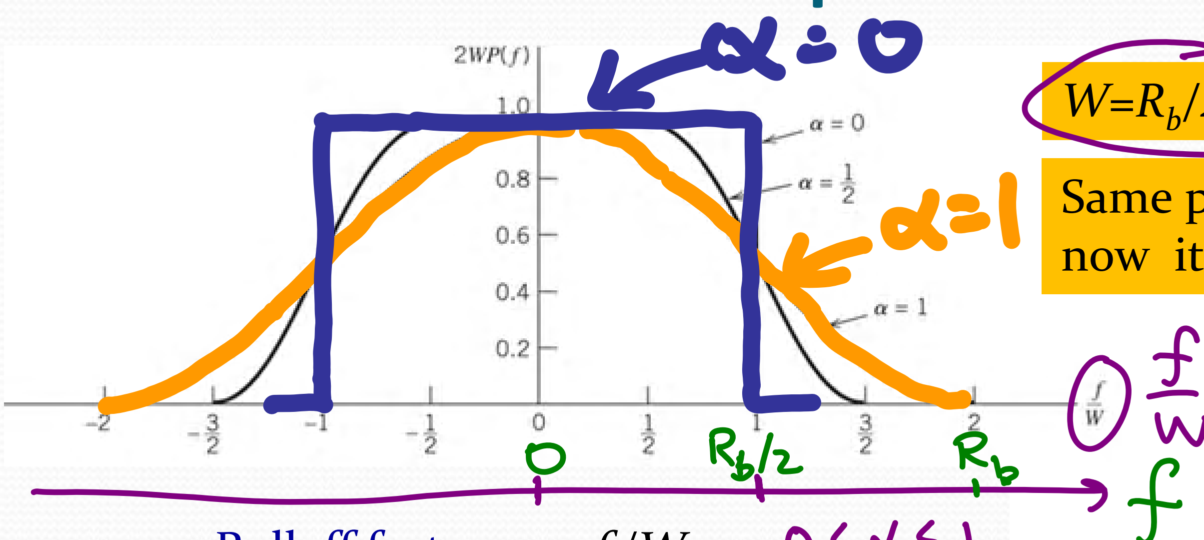
$$P(f) = \begin{cases} \frac{1}{2W}, & 0 \le |f| < f_1 \\ \frac{1}{4W}\left\{1 - \sin\left[\frac{\pi(|f| - W)}{2W - 2f_1}\right]\right\}, & f_1 \le |f| < 2W - f_1 \\ 0, & |f| \ge 2W - f_1 \end{cases}$$

$$\Rightarrow \quad p(t) = \left[\operatorname{sinc}(2Wt)\right]\left(\frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2 W^2 t^2}\right)$$

[redacted]

$\alpha$ = ROLL-OFF FACTOR | $0 \le \alpha \le 1$

# Raised Cosine Spectrum (contd.)




$W=R_b/2=1/(2T_b)$.

Same property as $\mathrm{sinc}(2Wt)$, but now it is practical

Rolloff factor: $\alpha=1-f_1/W$, $0 \le \alpha \le 1$

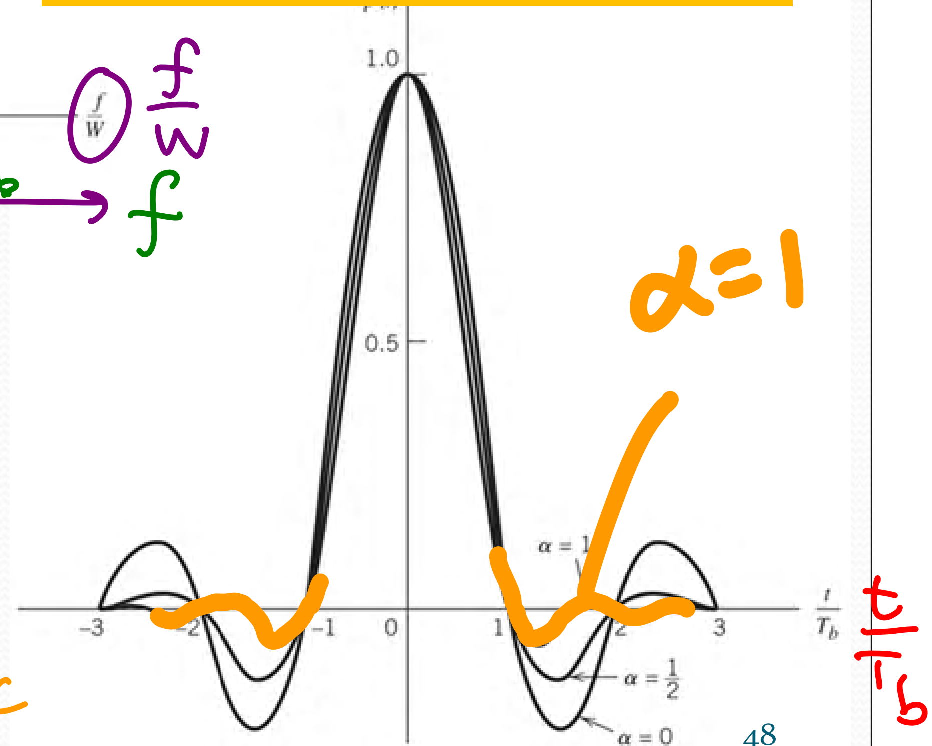
Indicates the excess bandwidth over the ideal solution, $W$

Bandwidth of P(f)

= "Transmission BW"

$$B_T = (2W - f_1) = W(1+1-\frac{f_1}{W})$$

$$= W(1+\alpha) = R_b\left(\frac{1+\alpha}{2}\right) \le B_c$$




[redacted]

# NYQUIST FIRST-CRITERION ZERO ISI

pulse $p(t)$:

**Time Domain**

$$p(t) = \begin{cases} c \neq 0, & t = t_0 \ (=0) \\ 0, & t = t_0 \pm mT_b, \quad m = 1, 2, 3, \ldots \end{cases}$$

$P(f) \leftrightarrow p(t)$

**Frequency Domain** $(t_0 = 0)$

$$\sum_{n=-\infty}^{\infty} P(f + nR_b) = \bar{c} \ (\neq 0), \quad -\frac{R_b}{2} \leq f \leq \frac{R_b}{2}$$

<u>M-ary Signals</u>: $\frac{1}{T_b} = R_b \rightarrow R_s$

$T_b \rightarrow T_s$

$$\overbrace{\text{TRANSMISSION BANDWITH}}^{\text{BW of } p(t)}$$

**of Raised-cosine family of pulses:**

$$B_T = \begin{cases} R_b \dfrac{1+\alpha}{2} & : \text{Binary} \\ R_s \dfrac{1+\alpha}{2} = R_b\left(\dfrac{1+\alpha}{2}\right) \cdot \dfrac{1}{\log_2 M} & : \text{M-ary} \end{cases}$$

where $\alpha$ = roll-off factor $(0 \le \alpha \le 1)$

$$B_T \le B_c = \text{Channel BW}$$

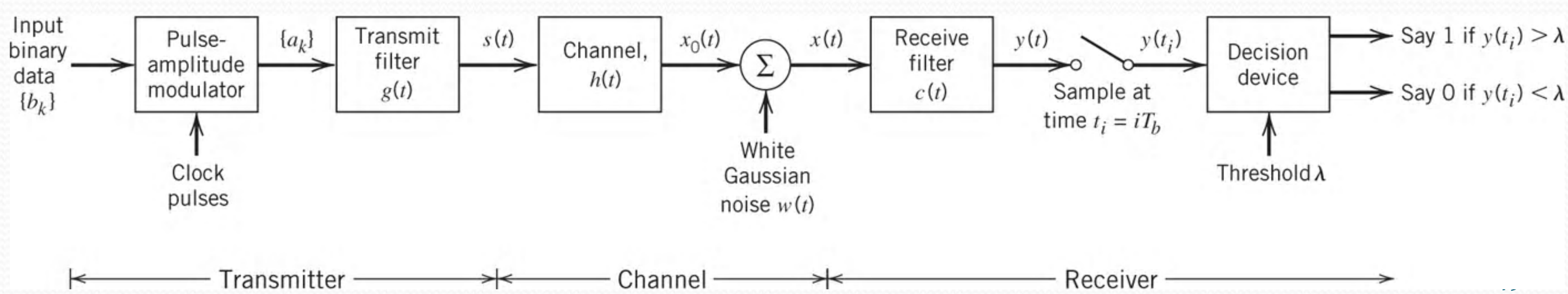
# How to Design the Transceiver

- Nyquist Criterion ➔ $P(f)$ = raised cosine spectrum
- Study the channel ➔ find $h(t)$
- Matched filter (to cope with noise) ➔ $c(t)$ and $g(t)$ are symmetric ➔ solve for $c(t)$ and $g(t)$

$$C(f) = kG(-f)\exp(-j2\pi fT)$$

$$\mu \times p(t) = g(t) * h(t) * c(t)$$

$$\mu \times P(f) = G(f) \times H(f) \times C(f)$$



# Example 8.4 Bandwidth Requirement of the T1 System

- T1 system: multiplexing 24 voice calls, each 4 kHz, based on 8-bit PCM word, $T_b$=0.647 $\mu s$
- Assuming an ideal Nyquist channel, the minimum required bandwidth is

$$B_T = W = R_b / 2 = 1/(2T_b) = 773\,\text{kHz}$$

- In practice, a full-cosine rolloff spectrum is used with $\alpha$=1. The minimum transmission bandwidth is

$$B_T = W(1+\alpha) = 2W = 1.544\,\text{MHz}$$

- In Chapter 3, if use SSB and FDM, the bandwidth is

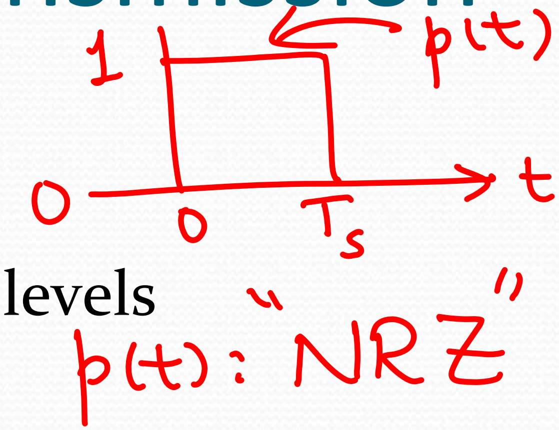
$$B_T = 24 \times 4 = 96\,\text{kHz}$$

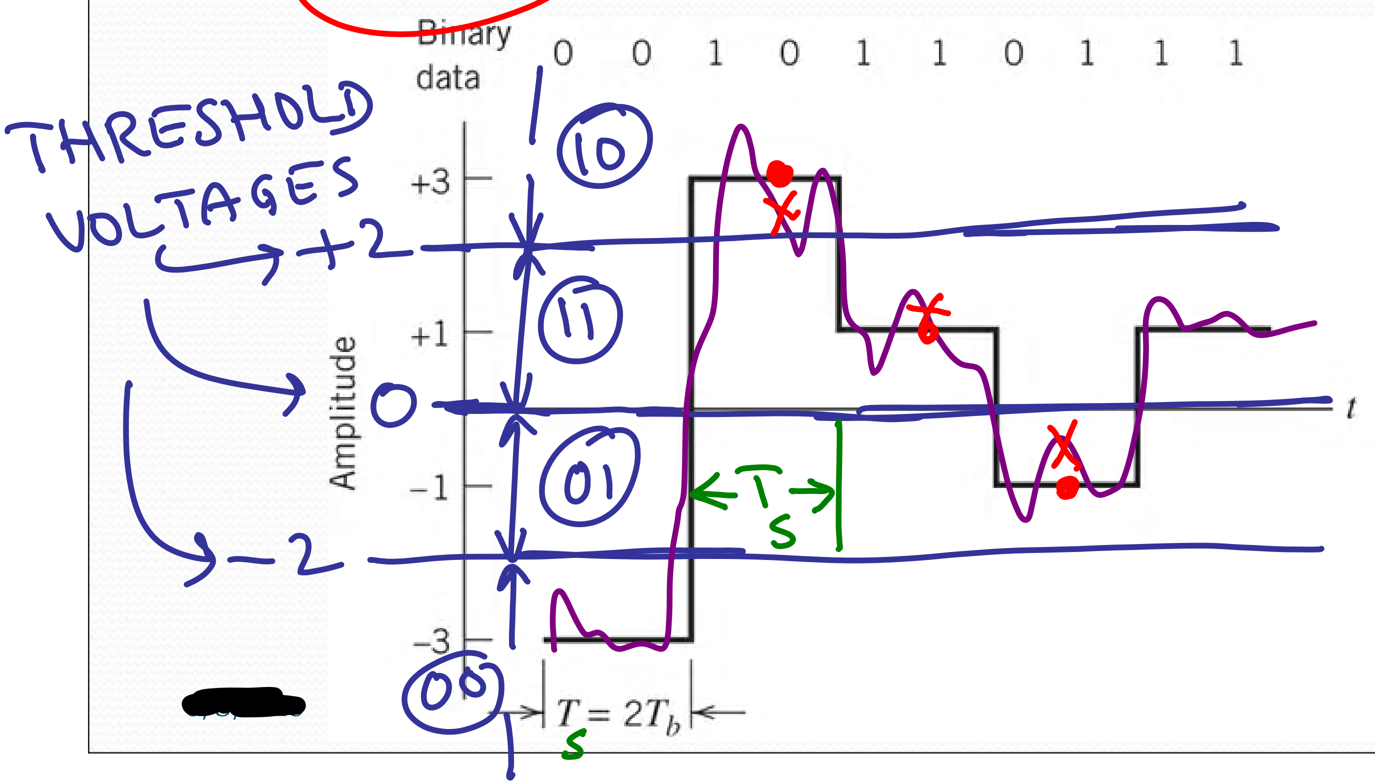
Digital transmission is not bandwidth efficient

# of bits / symbol

# Baseband M-ary PAM Transmission

- Baseband M-ary PAM system
  - $M$ possible amplitude levels, with $M$>2
  - $\log_2(M)$ bits are mapped to one of the levels




| Dibit | Amplitude |
|---|---|
| 00 | −3 |
| 01 | −1 |
| 11 | +1 |
| 10 | +3 |

[redacted]

# Baseband M-ary PAM Transmission (contd.)

- Symbol duration: $T_s$
- Signaling rate: $R_s$=1/$T_s$, in symbols per second, or bauds
- Binary symbol duration: $T_b$
- Binary data rate: $R_b$=1/$T_b$

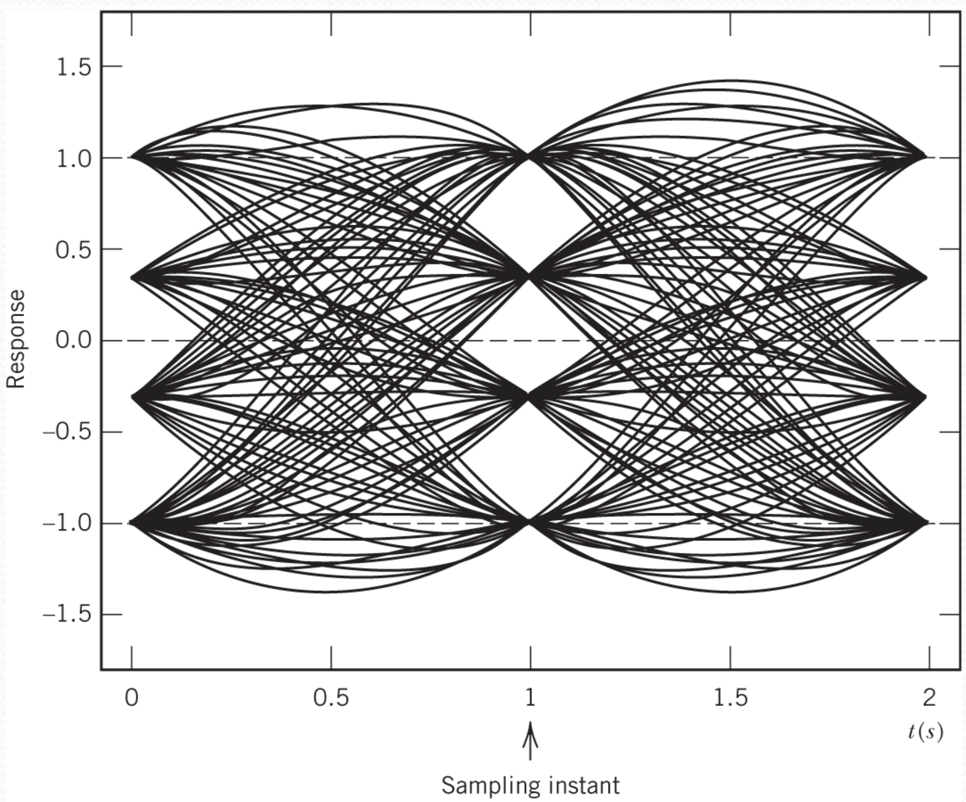
$$\boxed{T_s = T_b \log_2 M} \qquad \boxed{R_b = R_s \underbrace{\log_2 M}_{\text{\# of bits/symbol}}}$$

- Similar procedure used for the design of the filters as in the binary data case

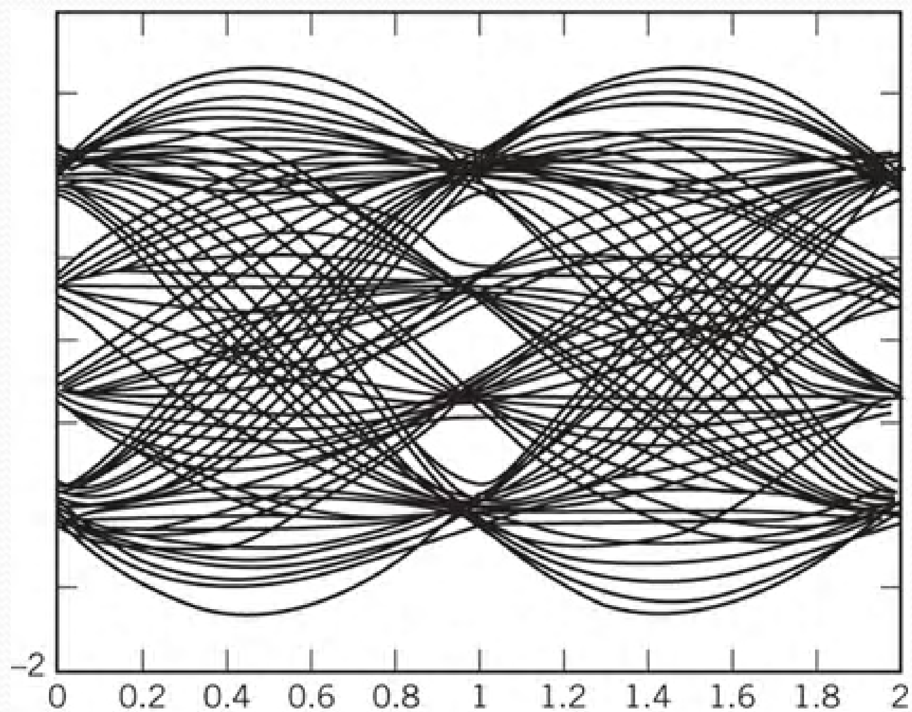
[redacted]

# Eye Pattern for M-ary Data

- Contains ($M$-1) eye openings stacked up vertically




($b$) $M = 4$




(b)
m = 4

# Tapped-Delay-Line Equalization

- ISI is the major cause of bit error in baseband transmissions
- If channel $h(t)$ or $H(f)$ is known precisely, one can design transmit and receiver to make ISI arbitrarily small
  - Find $P(f)$ ➔ find $G(f)$ ➔ find $C(f)$
- However in practice, $h(t)$ may not be known, or be known with errors (i.e., time-varying channels)
  - Cause residual distortion
  - A limiting factor for data rates
- Use a process, equalization, to compensate for the intrinsic residual distortion
  - Equalizer: the filter used for such process

$$\longrightarrow \boxed{G_T(f)} \longrightarrow \boxed{H_C(f)} \longrightarrow \boxed{G(f)} \longrightarrow$$

<<— Transmitter —>|<— Channel —>|<— Receiver —>>

$$\underbrace{P(f)}_{\text{Pick this}} = \underbrace{G_T(f)\,H_C(f)}_{\text{"Given"}}\,G(f)$$

$$\Rightarrow \quad G(f) = \begin{cases} \dfrac{P(f)}{G_T(f)\,H_C(f)}, & \text{for } f \text{ over signal BW} \\ 0, & \text{elsewhere} \end{cases}$$

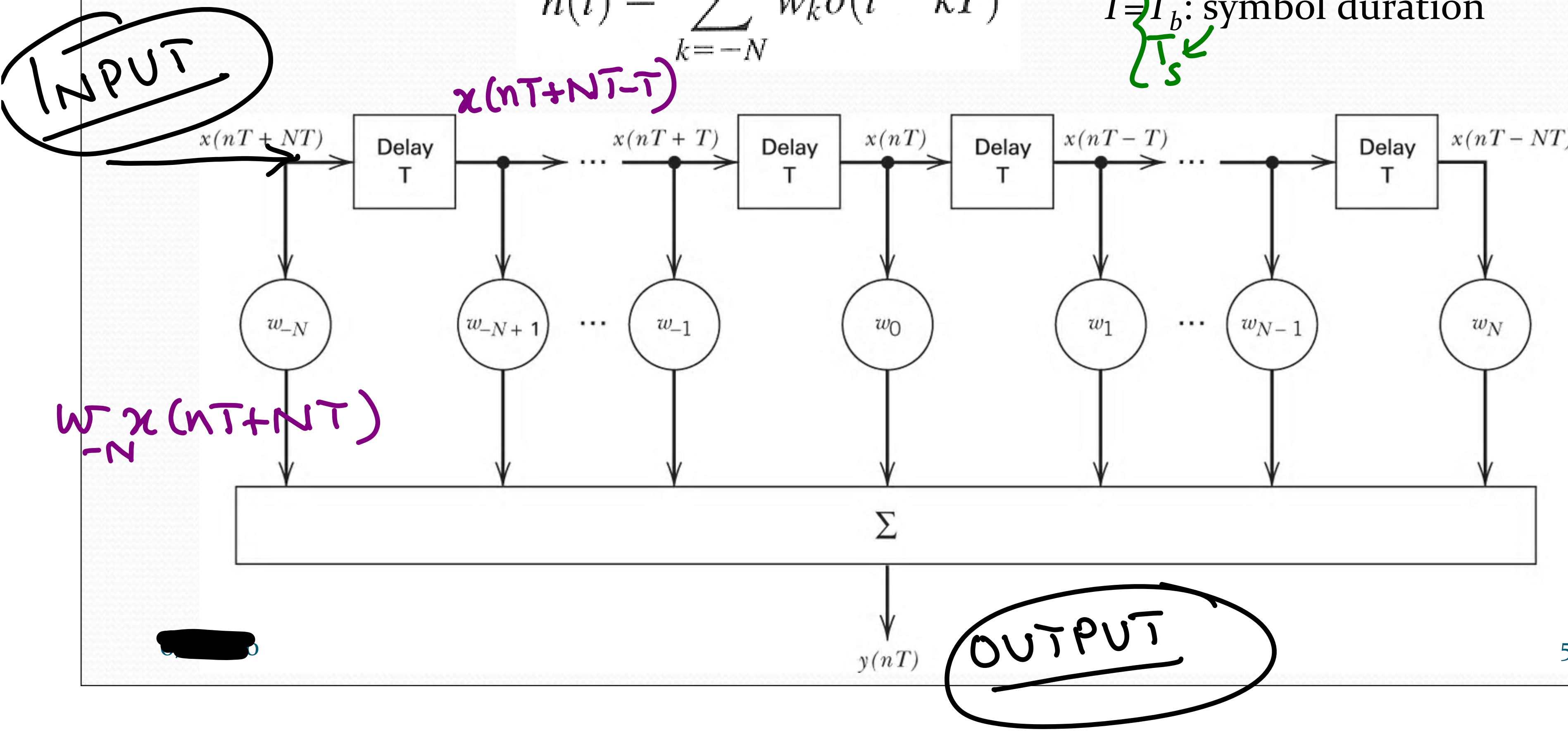
for freq. $f$ for which $P(f) \neq 0$

# Tapped-Delay-Line Filter

- Totally $(2N+1)$ taps, with weights $w_{-N}, \ldots w_1, w_0, w_1, \ldots w_N$

$$h(t) = \sum_{k=-N}^{N} w_k \delta(t - kT)$$

$T = T_b$: symbol duration $T_s$




[redacted]

# Tapped-Delay-Line Filter (contd.)

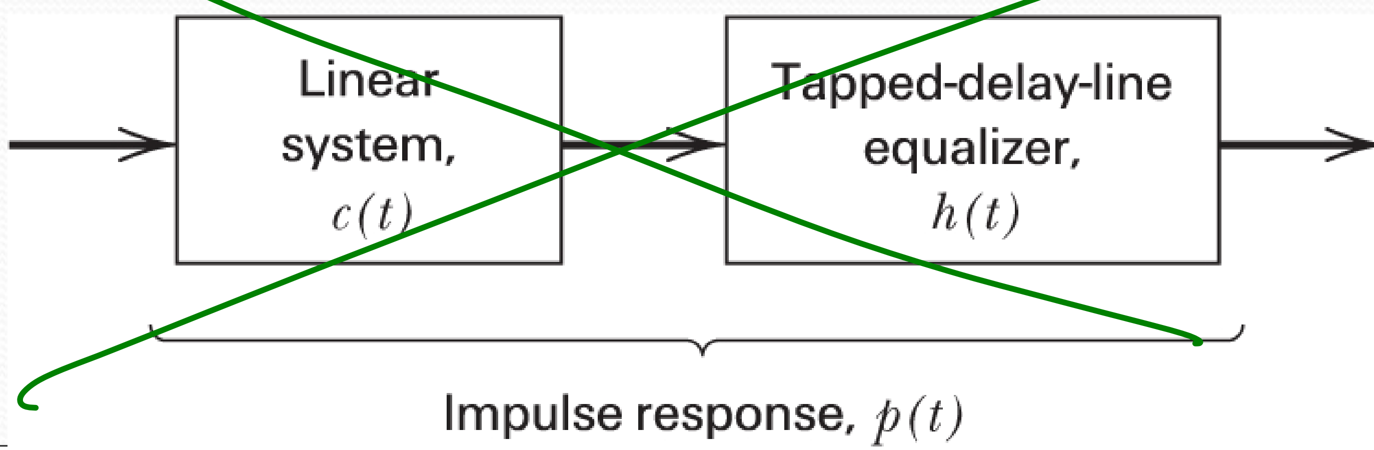
- We have

$$p(t) = c(t) \star h(t) = c(t) \star \sum_{k=-N}^{N} w_k \, \delta(t - kT)$$

$$= \sum_{k=-N}^{N} w_k c(t) \star \delta(t - kT) = \sum_{k=-N}^{N} w_k c(t - kT)$$

and

$$p(nT) = \sum_{k=-N}^{N} w_k \, c((n-k)T)$$



# Tapped-Delay-Line Filter (contd.)

- The Nyquist criterion must be satisfied. We have

$$p(nT) = \begin{cases} 1, & n = 0 \\ 0, & n = \pm 1, \pm 2, \ldots, \pm N, \pm(N+1), \pm(N+2), \ldots \end{cases}$$

DESIRED OUTPUT

- Denote $c_n = c(nT)$, we have

$$\boxed{p_n = \sum_{k=-N}^{N} w_k\, c_{n-k}} = \begin{cases} 1, & n = 0 \\ 0, & n = \pm 1, \pm 2, \ldots, \pm N \end{cases}$$

$n=-N$

$$\begin{bmatrix} c_0 & \cdots & c_{-N+1} & c_{-N} & c_{-N-1} & \cdots & c_{-2N} \\ c_{N-1} & \cdots & c_0 & c_{-1} & c_{-2} & \cdots & c_{-N-1} \\ c_N & \cdots & c_1 & c_0 & c_{-1} & \cdots & c_{-N} \\ c_{N+1} & \cdots & c_2 & c_1 & c_0 & \cdots & c_{-N+1} \\ c_{2N} & \cdots & c_{N+1} & c_N & c_{N-1} & \cdots & c_0 \end{bmatrix} \begin{bmatrix} w_{-N} \\ \vdots \\ w_{-1} \\ w_0 \\ w_1 \\ \vdots \\ w_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$n = -N$, $-1$, $0$, $1$, $N$

[redacted]

**Example:** Received sampled signal

$c_0 = 1, \quad c_1 = -\frac{1}{2}, \quad c_{-1} = \frac{1}{4},$

$c_n = 0$ for other values of $n$

Design 3-tap zero-forcing equalizer:

$\underbrace{2N+1}=3 \Rightarrow N=1$

**Solution:**

$$\sum_{k=-1}^{1} w_k c_{n-k} = p_n$$

$$\Rightarrow \boxed{w_{-1} c_{n+1} + w_0 c_n + w_1 c_{n-1} = p_n} = \underbrace{\begin{cases} 1, & n=0 \\ 0, & n=\pm1, \pm2, \pm3, \ldots \end{cases}}_{\text{DESIRED OUTPUT}}$$

$n \leq -3$

$n=-3$: $w_{-1}\cancelto{0}{c_{-2}} + w_0 \cancelto{0}{c_{-3}} + w_1 \cancelto{0}{c_{-4}} = p_{-3} = 0$ ✓

$n=-2$: $w_{-1}\underbrace{c_{-1}}_{1/4} + w_0 \cancelto{0}{c_{-2}} + w_1 \cancelto{0}{c_{-3}} = p_{-2} = 0$

$n=-1$: $w_{-1}\underbrace{c_0}_{1} + w_0\underbrace{c_{-1}}_{1/4} + w_1 \cancelto{0}{c_{-2}} = p_{-1} = 0$

$n=0$: $w_{-1}\underbrace{c_1}_{-1/2} + w_0\underbrace{c_0}_{1} + w_1\underbrace{c_{-1}}_{1/4} = p_0 = 1$

$n=1$: $w_{-1}\cancelto{0}{c_2} + w_0\underbrace{c_1}_{-1/2} + w_1\underbrace{c_0}_{1} = p_1 = 0$

$n=2$: $w_{-1}\cancelto{0}{c_3} + w_0\cancelto{0}{c_2} + w_1\underbrace{c_1}_{-1/2} = p_2 = 0$

$n\geq 3$: $p_n = 0$

DESIRED OUTPUT

$$
\boxed{
\begin{aligned}
\tfrac{1}{4}w_{-1} & & &= 0\\
w_{-1} &+ \tfrac{1}{4}w_0 & &= 0\\
-\tfrac{1}{2}w_{-1} &+ w_0 &+ \tfrac{1}{4}w_1 &= 1\\
&-\tfrac{1}{2}w_0 &+ w_1 &= 0\\
& &-\tfrac{1}{2}w_1 &= 0
\end{aligned}
}
$$

keep this

$$w_{-1} = -\frac{1}{4} w_0$$

$$w_1 = \frac{1}{2} w_0$$

$$-\frac{1}{2}\left(-\frac{1}{4} w_0\right) + w_0 + \frac{1}{4}\left(\frac{1}{2} w_0\right) = 1$$

$$\Rightarrow \underbrace{\left[\frac{1}{8} + 1 + \frac{1}{8}\right]}_{\frac{1+8+1}{8} = \frac{10}{8} = \frac{5}{4}} w_0 = 1 \Rightarrow w_0 = \frac{4}{5} = 0.8$$

$$w_{-1} = -\frac{1}{4} w_0 = -\frac{1}{5} = -0.2$$

$$w_1 = \frac{1}{2} w_0 = \frac{2}{5} = 0.4$$

Actual Output: $p_n = 0$ for $n \le -3$ & $n \ge 3$

$p_{-1} = p_1 = 0, \quad p_0 = 1$

$p_{-2} = \frac{1}{4} w_{-1} = -\frac{1}{20} \neq 0$

$p_{+2} = -\frac{1}{2} w_1 = -0.2 = -\frac{1}{5} \neq 0$

# Tapped-Delay-Line Filter (contd.)

- Remarks
  - Referred to as a *zero-forcing equalizer*
  - Optimum in the sense that it minimizes peak distortion (ISI)
  - Simple to implement
  - The longer, the better, i.e., the closer to the ideal condition as specified by the Nyquist criterion
- For time-varying channels
  - Training
  - Adaptive equalization: adjusts the weights

# Theme Example – 100Base-TX – Transmission of 100 Mbps over Twisted Pair

- Fast Ethernet: 100BASE-TX
  - Up to 100Mbps
  - Using two pairs of twisted copper wires → Category 5 cable
    - One pair for each direction
  - Maximum distance: 100 meters
- First stage: NRZ 4B5B → provide clocking information
- Second stage: NRZI
- Third stage: three-level signaling MLT-3
- With tapped-delay-line equalization

# Summary

- Two transmission impairments
  - Noise
  - ISI
- Impact of the two transmission impairments
- How to mitigate the effects of transmission impairments
  - Matched filter
  - Evaluating the BER
  - Eye pattern
  - Nyquist criterions for distortionless criterion
  - Tapped-delay-line equalization
- Binary transmissions and M-ary transmissions