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Multi-commodity *k*-splittable survivable network design problems with relays

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Abstract The network design problem is a well known optimization problem with applications in telecommunication, infrastructure designs and military operations. This paper devises the first formulation and solution methodology for the multi-commodity *k*-splittable two-edge disjoint survivable network design problem with capacitated edges and relays. This problem realistically portrays telecommunications network design but has not been solved previously due to its computational difficulty. Edge capacity is considered as either a discrete or a continuous variable. An exact method and a practical heuristic method are presented, and computational results are discussed.

Keywords OR in telecommunications · Reliability · Survivability · Networks · Heuristics · Capacitated edges

1 Introduction

The network design problem is a well known optimization problem which has many applications in telecommunication networks, military applications and infrastructure designs. In this paper, the main focus is telecommunication networks. A telecommunication network can be defined as a graph G, whose elements are nodes, edges and commodities. Cabral et al. [7] presented the mathematical notation of a telecommunication network as a node set defined as $V = \{1, 2, \ldots, n\}$ and an edge set defined as $E = \{(i, j) : i, j \in V, i < j\}$. The set of commodities is defined as $K = \{s(k), t(k)\}$ where $s(k), t(k) \in V$ are the origin (source) and destination (sink) nodes, respectively. Additionally, each edge $(i, j) \in E$ has a cost c_{ij} and a distance d_{ij}.

Cabral et al. [7] specified a class of network design problems where a subset of possible edges is included and relays are located at some of the nodes. A relay (or regenerator) is used to regenerate the signal of the commodity (flow) along the telecommunication network. In digital telecommunication networks, relays are required to regenerate the signal because it loses fidelity according to distance along its transmission path [23]. Relays are commonly used in long distance transculent optical telecommunications networks [19]. Accordingly, a relay point is a node where a signal regenerator is located and incurs a fixed cost [24]. If the cumulative travel distance of a flow exceeds a predetermined length value, a relay is required [8]. Relays can only be located at nodes and there can be more than one relay along a path.

The network design problem with relays (NDPR) is also known as the regenerator placement problem in the optical networks field. An optical signal needs to be regenerated (usually by optical-electrical-optical conversion) when its quality of transmission (QoT) degrades due to physical impairments [29]. The regeneration process reamplifies, reshapes and retimes (i.e., 3R regeneration) an optical signal to send it over long distances. Optical networks have been evolving from opaque to transparent (all-optical) architectures [5]. In transparent networks no opto-electronic (optical–electrical–optical) regeneration is needed, however, some connections may fail in a large geographical area due to physical layer impairments of the optical signal [29]. Using regenerators enables optical networks to

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span over large geographical areas without disrupting QoT [36]. In opaque networks, all nodes are capable of opto-electronic regeneration to increase flexibility in network control and facilitate network design but this incurs a higher cost [15]. Translucent networks lie between opaque and transparent networks in which some nodes are capable of opto-electronic regeneration. Regenerator nodes in translucent optical networks need to be strategically placed in order to minimize the network cost (i.e., capital expenditure or CAPEX). The problem solved in this paper is applied to regenerator placement problem in translucent optical networks.

In this paper, we extend the survivable NDPR in two ways. First, we use k-splittable flow which permits split of the total flow of a commodity among up to k different paths. Relays are considered independently for each split path and each survivable path. Second, we include capacitated edges in the formulation as either continuous or discrete decision variables to better represent real life applications. These extensions, k-splittable flow and capacitated edges, have not been considered before in the survivable NDPR.

The paper is organized as follows. Section 2 summarizes prior relevant work. Section 3 explains the motivation of this paper and states the problem. The mathematical model and the heuristic method are presented in Sect. 4. Section 5 summarizes the computational results. Discussion is provided in Sect. 6.

2 Literature review

The relay concept was first introduced for the network design problem by [7]. They applied a column generation method to the NDPR. Their objective function was to minimize the total cost of the multi-commodity network design problem with unsplittable flow. They developed four simple constructive heuristics (where the solution is generated for one commodity at a time). Konak et al. [23] extended the formulation of [7]. Their major contribution was survivability of the network and they termed the problem the “two-edge connected network design problem with relays”. Survivability is an important issue in telecommunications networks because high capacity fiber-optic links make communication networks less dense [23]. Therefore, survivability helps to maintain connectivity in the event of failure of an edge and can be achieved by incorporating redundant paths in the network [33]. For survivability, Konak et al. [23] assumed that commodity flow is rerouted over a redundant path in case of edge failure. Therefore, survivability increases the reliability of the network by introducing a certain level of redundancy. (A similar survivability requirement was proposed by Dahl and Stoer [11] who used a cutting plane algorithm to solve the multicommodity survivable network design problem without relays.) Konak et al. [23] proposed a mixed integer programming (MIP) model, as well as an efficient genetic algorithm procedure. Additionally, they developed three constructive heuristics and showed that their genetic algorithm model outperforms both the constructive heuristics and the MIP formulation in terms of best objective value and CPU time. In another paper [16], failure modes of edges are considered along with survivability constraints of a multicommodity unsplittable network without relays. They defined failure modes for simultaneous failures of multiple edges.

Many researchers have studied the splittable flow problem which is known as the “k-splittable flow problem”. Obviously, it is a generalization of the unsplittable flow problem where k equals 1. Split flows are commonly used in Internet Protocol (IP) networks using Open Shortest Path First (OSPF) or Intermediate System–Intermediate System (IS–IS) routing protocols. OSPF allows routers to check their adjacent links in IP networks, share this information with other routers to get a complete view of the network topology and use the shortest paths from source to destination permitting flows to split equally if there are more than one shortest path [37]. OSPF was designed to split traffic equally among multiple shortest paths according to the equal-cost multipath (ECMP) principle considering link capacities [28]. Xu et al. [37] suggested a different technique, Distributed Exponentially-weighted Flow SpliTting (DEFT), to direct split traffic on non-shortest paths as well. Similarly, the multipath label switching (MPLS) protocol also allows splitting traffic among non-shortest paths [1]. In one influential paper on the hub location problem [25], several MIP formulations for the node constrained splittable flow problem were summarized and a new formulation was proposed. Truffot and Duhamel [35] proposed a branch-and-price algorithm for the single commodity k-splittable maximum flow problem. They used the column generation method to obtain the lower bound for the branching process.

Another aspect of network design problems, capacitated edges, have been included in several studies. Xu at al. [38] worked on the dynamic routing of a telecommunication network with capacitated links. In this problem, the alternate paths are changed dynamically from hour to hour as the traffic between pairs of nodes varies with time of day. To solve this problem efficiently, they proposed a tabu search algorithm. Ramirez-Marquez et al. [31] proposed an evolutionary approach to solve stochastic network interdiction problems where an interdictor tries to minimize the flow of a network by stopping flow on some edges. The objective of their model is to minimize total cost without violating reliability requirements. Atamturk and Rajan [2] worked on the splittable/unsplittable multi-commodity edge constrained network design problem. They developed valid inequalities and a branch-and-cut algorithm to solve this problem. Costa et al. [10] also used valid inequalities to ensure that there is enough capacity on each edge to satisfy demand. They compared the efficiency of Benders, metric and cutset inequalities as solution approaches. Rajan and Atamturk [30] studied survivability on capacitated network design problems and presented a column-and-cut generation method to solve the problem. In a later paper, Atamturk and Rajan [3] worked on the capacitated survivable network design problem based on *p*-cycles (that is, a cycle having at least three edges). In their approach, the flow is rerouted along an undirected *p*-cycle in case of an edge failure. Frangioni and Gendron [13] solved the multicommodity capacitated network design problem using a MIP model in which capacity is defined as the number of facilities on each edge.

There are many studies on network design with relays in optical networks, which is also known as the regenerator placement problem. Regenerators are deployed to satisfy some QoT requirements of optical lightpaths [e.g., optical signal to noise ratio or bit error rates (BER)] that are determined by the optical reach (i.e., the maximum distance of an optical signal to travel without regeneration) which is usually between 2000 and 4000 km in optical networks [26,29].

With the regenerator placement problem, routing of the traffic flows must also be optimized. Without having wavelength conversion capability, a single wavelength is assigned to each lightpath so that the wavelength does not change on fiber links that the lightpath traverses [5]. The routing and wavelength assignment (RWA) problem has to satisfy this wavelength continuity constraint. The wavelength continuity constraint is eliminated when all switching nodes have wavelength conversion capability, however, the total number of available wavelengths must not be exceeded when assigning wavelengths on a link. Technically, opto-electronic regeneration enables wavelength conversion without any additional cost [4,29]. Therefore, similar to [17], we assume the traffic can be switched between different wavelengths with full wavelength conversion at every node. Also, it is assumed that there are sufficiently many wavelengths available so that wavelength assignment problem is not considered in our mathematical model.

Physical impairment issues are important in optical networks. Azoldolmolky et al. [5] summarized the types of physical impairments (linear or nonlinear) and different approaches (analytical and hybrid/simulation methods) to model them in RWA problems in optical networks. Linear impairments affect individual wavelengths, however, nonlinear impairments may also disturb and interfere with other wavelengths [29]. There are two ways to include physical impairments in the model. The first one [6,15] is to calculate linear and/or nonlinear physical impairments using simulation or analytical methods and consider this in the routing problem. The second way [29] is to include the impairment issue indirectly by specifying an optical reach and solve the routing problem accordingly. In our paper, the optical reach is specified and the routing problem is solved with that constraint.

In regenerator placement problems traffic demands can be static (offline) or dynamic (online). Static network demands are known a-priori, whereas dynamic demands have stochastic arrival times and lifetimes [5]. In the static demand case, the routing problem is solved for network planning [14]. According to Varvarigos and Christodoulopoulos [36], static routing problems in optical networks are harder to solve than dynamic ones. In some dynamic traffic studies, the location of regenerators are given and only the routing problem is solved according to a given traffic matrix and network topology. For example, Dey and Adhya [12] solved the routing problem in wavelength-division multiplexing (WDM) networks with a limited number of regenerator facilities without considering CAPEX. In our paper, demands are assumed to be static.

Garcia-Manrubia et al. [15] worked on the regenerator placement and wavelength assignment problem with offline routing in translucent optical networks considering linear and nonlinear physical impairments. They modeled the problem as Mixed Integer Linear Programming (MILP) model and tested it on NSFNET and Internet2 networks. They minimized the number of regenerators and the number of blocked lightpath requests. They also developed a heuristic to decompose the problem into smaller problems and then solved them sequentially. Their heuristic first solves the routing problem, then assigns wavelengths, and finally places regenerators. Klinkowski [21] developed a heuristic algorithm to solve spectrum allocation and regenerator placement in translucent elastic optical networks. Martinelli et al. [26] worked on the all-optical regenerator placement problem in Wavelength Switched Optical Networks (WSON). They proposed an MILP model and applied their method on Deutsche Telekom and Pacific Bell networks. Martinelli et al. [27] solved the same problem proposed by [26] using Genetic Algorithms (GA).

According to Azodolmolky et al. [5], there are very few studies addressing resilience and protection in routing with physical layer impairment problems. Beshir et al. [6] worked on the regenerator placement problem on two-edge disjoint survivable optical networks. They investigated two versions of this problem: (1) dedicated regenerators where sharing of regenerators are not allowed between edge disjoint paths, and (2) shared regenerators where sharing of regenerators are allowed between edge disjoint paths. They proposed an MILP model and a heuristic method to solve each variant. In their model, the objective function is to minimize the total number of regenerators. They also assigned a wavelength for each link, which was not considered in our model. They considered the impairment level of lightpaths, but they did not consider splits. Cherubini et al. [9] proposed a linear programming

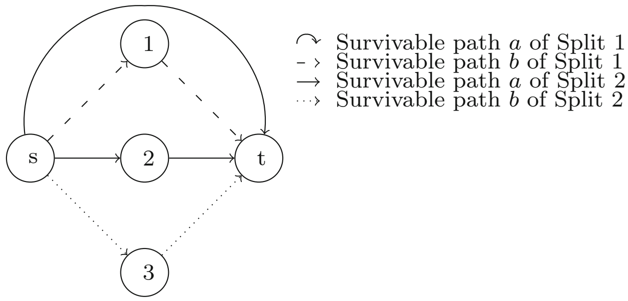


Fig. 1 Two-edge disjoint survivability of a network with two splits (nodes s and t are the source and sink nodes, respectively)

Fig. 2 NSFNET backbone network topology (from [18])

model to minimize the maximum link utilization in telecommunication networks considering survivability for link failures. They explained the split mechanism in Interior Gateway Protocols, such as IS–IS and OSPF. In these protocols, the traffic can be split into equal flows between the origin and the destination nodes. In our model, splits do not have to be equal. They also defined link restoration constraints as their survivability requirements. They tested their proposed method on random and real networks. Considering only routing and splits with survivability, they did not consider regenerator placement or impairment issues. In our problem, regenerator locations are determined as well as the survivable offline splittable routing problem. Huang et al. [17] proposed a routing scheme, differential-delay-constrained disjoint paths, in mesh networks employing WDM considering survivability. They use a backup path with a reserved bandwith (equal to the primary path) as the survivability constraint allowing sharing of the backup capacity. They worked on the dynamic traffic case without considering regenerators or split flows. They took into account a differential-delay constraint, which is based on the distance of the path and the number of links it traverses, when considering survivable paths.

3 Motivation and problem definition

The base problem herein is similar to that of [23], a multicommodity survivable NDPR. The problem is also known as the regenerator placement problem in the optical networks literature and only a few studies considered survivability in regenerator placement problem for optical networks [5,6,9,17]. Also, the survivable network with k-splittable flow has not been considered in the literature, and this is one of the major differences of this paper with that of [23]. In this paper, an edge disjoint survivable path is assigned to each split path. In other words, there are two-edge disjoint paths for each split. Figure 1 shows the survivability requirements of a network with one commodity. In this example, there are two splits ($k = 2$) and two-edge disjoint paths are assigned for each split. As the other major distinction, capacitated edges are used herein. Considering edge capacity is an important decision in real life applications [20]. In telecommunication networks, edge capacity is usually defined in terms of Gbps (gigabit per second) [32]. As technology matures, higher rate connections become available. In WDM networks, connections moved from 10 to 40 and 100 Gbps became available recently, and therefore mixed-line-rate WDM systems (having mixed capacities) with lower costs than single-line-rate WDM systems are in use [36]. Note that, optical reach gets significantly lower as bit rate increases [4]. Although capacitated edges [2,10,13,30,31,38] and k-splittable flows [1,25,28,35,37] are not new in network design problem as reviewed in the previous section, survivable network design with the properties of capacitated edges, relays and k-splittable flow has not previously been formulated nor solved. The original problem is NP-hard [5–7], and both the capacitated edges and k-splittable flow introduce new decision variables and constraints to the mathematical programming formulation, therefore the problem in this paper is expected to be harder to solve than the original problem. Tomaszewski et al. [34] showed that the two-edge disjoint capacitated survivable network design problem is NP-hard.

Relays (or regenerators) are used in real life applications. For example, National Science Foundation Network (NSFNET) in North America uses regenerators (Fig. 2). Katrinis and Tzanakaki [18] and Garcia-Manrubia et al. [15] studied the regenerator location problem on NSFNET to satisfy optical reach requirements. For example, consider an optical signal from Houston to Seattle where the optical reach of the network is 3000 km. The optical signal from Houston cannot reach Seattle without a regenerator because the shortest distance exceeds 3000 km. If the primary path is Houston–San Diego–Seattle, the signal must be regenerated at San Diego. Similarly, Salt Lake City (or Boulder) must also be a regenerator node if the edge disjoint survivable path is selected as Houston–Boulder–Salt Lake City–Palo Alto–Seattle.

To address this problem class, an MIP formulation and a heuristic method are presented in the next section.

4 Methodology

4.1 Mathematical model

The mathematical model of the Multi-Commodity k-splittable Two-Edge Disjoint Survivable Network Design Problem with Relays (where edge capacities are known) is given below:

Notation

k = Commodity
s = Split
p = Survivable path
N = Set of nodes
E = Set of edges

Decision variables

$$x_{ij} = \begin{cases} 1 & \text{if edge } (i, j) \text{ or } (j, i) \text{ is included in the solution,} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if node } i \text{ is a relay node,} \\ 0 & \text{otherwise} \end{cases}$$

$$c_{kijs} = \begin{cases} 1 & \text{if edge } (i, j) \text{ is used by commmodity } k \text{ in its split } s, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{ks} = \begin{cases} 1 & \text{if commodity } k \text{ uses split } s, \\ 0 & \text{otherwise.} \end{cases}$$

u_{kisp} = distance that commodity k goes before the relay node i in survivable path p of split s
v_{kisp} = distance that commodity k goes after the relay node i in survivable path p of split s
f_{ks} = flow of commodity k allocated to split s
f_{kijs} = flow of commodity k allocated to edge (i, j) in split s

Parameters

λ = maximum distance that a flow of a commodity can go on a given path without a relay
c_{ij} = cost of including edge (i, j) in the solution
d_{ij} = distance of edge (i, j)
l_{ij} = maximum flow allowed on edge (i, j)
F_k = total flow of commodity k
r_i = cost of locating a relay at node i
n_{spl} = number of splittable paths
n_{sur} = number of survivable paths
s_k = source node of commodity k
t_k = sink node of commodity k
M = a sufficiently big number

Objective function

$$\text{Min } z = \sum_{i \in N} r_i * y_i + \sum_{(i,j) \in E: i<j} c_{ij} * x_{ij} \tag{1}$$

Constraints

$$\sum_{(i,j) \in E} c_{kijs} - \sum_{(j,i) \in E} c_{kjis} = \begin{cases} n_{sur} * z_{ks} & \forall k, s, i = s_k, \\ -n_{sur} * z_{ks} & \forall k, s, i = t_k \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$u_{kisp} \geq v_{kjsp} + d_{ji} - \left[1 - c_{kjis}\right] * M * \lambda \quad \forall k, p, i, s, \text{ and } (i, j) \in E \tag{3a}$$

$$u_{kisp} \leq \lambda \quad \forall \quad k, p, i, s \tag{3b}$$

$$v_{kisp} \geq u_{kisp} - y_i * \lambda \quad \forall \quad k, p, i, s \tag{3c}$$

$$\sum_s c_{kijs} + \sum_s c_{kjis} \leq x_{ij} \quad \forall k, (i, j) \in E : i < j \tag{4}$$

$$c_{kijs} \leq z_{ks} \quad \forall k, s, (i, j) \in E \tag{5}$$

$$1 \leq \sum_s z_{ks} \leq n_{spl} \quad \forall k \tag{6}$$

$$\sum_s f_{ks} = F_k \quad \forall k \tag{7}$$

$$f_{ks} \leq F_k * z_{ks} \quad \forall k, s \tag{8}$$

$$f_{ks} - F_k * \left[1 - c_{kijs}\right] \leq f_{kijs} \leq f_{ks} \quad \forall k, s, (i, j) \in E \tag{9a}$$

$$f_{kijs} \leq F_k * c_{kijs} \quad \forall k, s, (i, j) \in E \tag{9b}$$

$$\sum_k \sum_s \left[f_{kijs} + f_{kjis}\right] \leq l_{ij} \quad \forall (i, j) \in E : i < j \tag{10}$$

$$z_{k(s+1)} \leq z_{ks} \; \forall k, \text{ and } s \in \{1, \ldots, (n_{spl} - 1)\} \tag{11}$$

$$u_{kisp}, \; v_{kisp}, \; f_{ks}, \; f_{kijs} \geq 0$$
$$x_{ij}, \; y_i, \; z_{ks}, \; c_{kijs} \in \{0, 1\} \tag{12}$$

The objective function minimizes the total cost, which consists of two parts: (1) relay costs and (2) edge costs. Constraint 2 is the standard node flow balance constraint. It ensures that a total of n_{sur} paths are established for each split flow of commodity k. Constraints 3a, 3b and 3c work together to determine the u and v variables, and the relay variable y is determined according to the u and v variables as also explained by [23] and [22]. Constraint 3a calculates the total distance traveled by commodity k to node i without visiting a relay on survivable path p of split s. Constraint 3a becomes

$u_i \geq v_j + d_{ji}$ if edge (j, i) is used by commodity k in split s. If node j is not a relay, Constraint 3c becomes $v_j \geq u_j$ and therefore we obtain $u_i \geq u_j + d_{ji}$ by substituting u_j for v_j in 3a. Thus, u_i is the cumulative distance from a relay, or the source node, to i without passing any relay. On the other hand, Constraint 3c becomes $v_j \geq u_j - \lambda$ in the case that j is a relay and therefore v_j can be zero since Constraint 3b is $u_j \leq \lambda$. And, if $v_j = 0$ then Constraint 3a becomes $u_i \geq d_{ji}$ and the distance is restarted after the relay at node j. In short, in the case that node j is a relay, the total distance after node j (v_j) becomes zero by Constraint 3c. Similarly, if node j is not a relay then v_j is forced to be equal to u_j since the objective function minimizes the relay costs (as well as the edge costs).

Constraints 2 and 4 ensure that there exist n_{sur} edge disjoint paths of each split of every commodity and all paths are edge disjoint. Constraint 5 forces z to be 1 if the c variable is 1 for a given split s. Constraint 6 limits the number of splits of each commodity to the specified upper limit of number of splittable paths. Constraint 7 states that for any commodity k, the sum of its split flows is equal to the total flow requirement of that commodity. Constraint 8 prohibits f_{ks} to be positive when z is not 1. For a given commodity, Constraints 9a and 9b set the upper and lower bounds of f_{kijs}, respectively. Constraint 9b also forces c_{kijs} to be 1 if f_{kijs} is positive. Constraint 10 is the edge capacity constraint. Constraint 11 prevents selection of a split s if split $(s - 1)$ is not included in the solution. Constraint 12 is simply the bounding constraint for the decision variables.

Two possibilities are presented to incorporate the edge capacity variable, l_{ij}, into above formulation: (1) continuous and (2) discrete. The model with continuous values of edge capacities uses the same formulation as above, Eqs. 1–12. However, the objective function is changed to include edge capacity costs. The new objective function is given in Eq. 13. Here, the capacity of edge (i, j) increases the total cost of the network by $l_{ij} * c_{ij}$ where c_{ij} is assumed to be the unit cost of capacity of edge (i, j).

$$\text{Min } z = \sum_{i \in N} r_i * y_i + \sum_{(i,j) \in E : i<j} c_{ij} * \left[x_{ij} + l_{ij}\right] \tag{13}$$

Note that, l_{ij} is a new decision variable and defined as:

l_{ij} = capacity of edge (i, j), $l_{ij} \geq 0$

This formulation optimizes the cost of relays, edge selections and edge capacities. The second model allows only discrete values for edge capacities. It has the same objective function as (13), however, the variable l_{ij} is defined as:

l_{ij} = capacity of edge (i, j), $l_{ij} = a * y_{(l_{ij})}$
where $y_{(l_{ij})} \in 0, 1, 2,$ and
a = parameter value of edge capacity value

The integer variable $y_{(l_{ij})}$ ensures that any edge capacity variable can take only discrete values: 0, "a", or "$2a$". In real applications, limited choices of capacity on each edge is a more reasonable assumption than a continuous capacity variable due to physical constraints on hardware. To illustrate, edge (i, j) is not included in the solution (and its capacity is zero) when $y_{(l_{ij})} = 0$, the capacity of edge (i, j) is "a" when $y_{(l_{ij})} = 1$, and capacity of edge (i, j) is "$2a$" when $y_{(l_{ij})} = 2$. As with the continuous edge capacity case, total cost of the network increases as the capacity of edge (i, j) increases (Eq. 13).

As will be shown in Sect. 5, due to the complexity of the problem, the MIP formulation does not yield satisfactory results for problem instances with more than 20 nodes. Therefore, a relaxation based heuristic method is presented in the next subsection.

4.2 The proposed heuristic

The proposed heuristic consists of three steps as shown in Fig. 3. Each step solves a modification of the original formulation given in Sect. 4.1. In the first step of the heuristic, relay placements are optimized without considering edge costs or capacities. The objective function only includes relay costs. In the second step, the uncapacitated problem is solved to assign edges using the relays located from the first step. Only edge costs are considered in this second step. In the third step, edge capacities are optimized with the specified relays obtained from the first step and edges from the second step. Thus, all cost elements (edge costs, edge capacity costs and relay costs) are included in the resulting cost function.

The main advantage of this heuristic is the reduced problem size. Although this method does not guarantee optimality, the gap is relatively low (see Sect. 5) and the solution time is less than the original formulation.

5 Computational experience

In this section, computational experience is presented. Approaches are considered for both specified edge capacities and variable edge capacities (continuous or discrete values). Exact results and those from the heuristic are shown.

The size of the problem is determined by the number of splits, the number of survivable paths, the number of commodities and the number of nodes (also determines the number of possible edges). Five problem instances of scenarios with different number of nodes were used to test the computational performance of the proposed methods. In one instance, the edges sets (available edges and their costs and distances) are the same as that of [23]. The remaining four

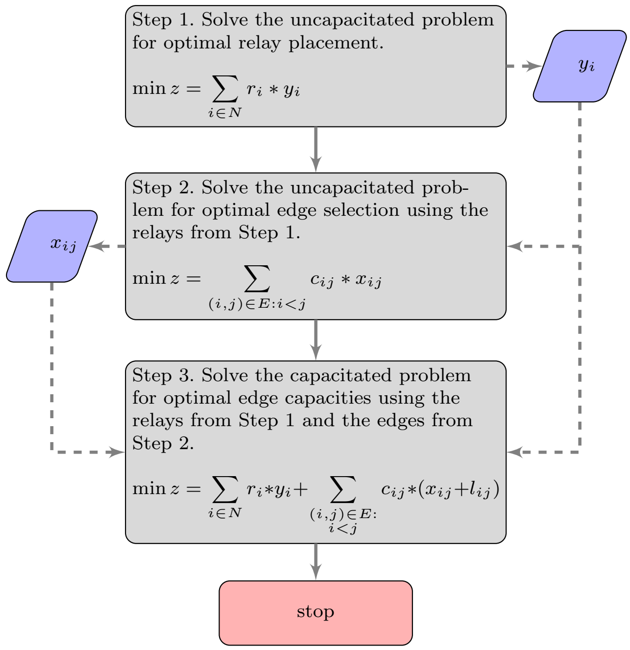


Fig. 3 The proposed heuristic

Table 1 Largest test problems that can be solved to optimality within 24 h with CPLEX

Edge capacity	n_{spl}*	Largest node size
Fixed	2	40
Discrete	2	8
Continuous	2	9
Fixed	1	55
Discrete	1	21
Continuous	1	22

* n_{spl} = Maximum number of splits

Table 2 The number of relays for an instance of the 35 node fixed edge problem with different values of λ, n_{spl} and n_{sur}

λ	$n_{spl} = 1$			2			3		
	$n_{sur} = 1$	2	3	1	2	3	1	2	3
20	6	9	10	6	7	10	6	7	10
25	5	5	8	5	5	7	5	5	6
30	4	4	6	4	4	6	4	4	6
40	3	3	4	3	3	4	3	3	4

instances were randomly generated based on [23]. Specifically, edge costs and distances are assumed to be identical and vary uniformly between 0 and 30. For all scenarios, the number of commodities is five, and flow of a commodity varies uniformly between 10 and 20. The source and the destination nodes of a commodity are randomly selected to be far apart from each other. The edge capacities are fixed at 30 for the problem instances with specified edge capacities. The maximum distance that a flow of any commodity can travel without a relay (λ) is 30, which is the same as that of [23]. The maximum number of splits is set to two in this paper, which is common in network design problems. The number of survivable paths is set to two for all problem instances. The problems were solved in CPLEX 11.2.1 (with a time limit of 24 h) on a Linux computer with 2.27 Ghz Intel Quad Core Xeon CPU and 4 GB memory.

To demonstrate the difficulty of the problem and validate the need for a heuristic, the largest test instances that can be solved to optimality within 24 h are reported in Table 1. As expected, the fixed edge capacity scenario with splittable flows can be solved for larger problem instances than the discrete and the continuous edge capacity scenarios with splittable flows. The continuous case is slightly easier to solve than the discrete one because the discrete case has more integer variables. Not surprisingly, unsplittable flow cases ($n_{spl} = 1$) are much easier to solve than splittable cases.

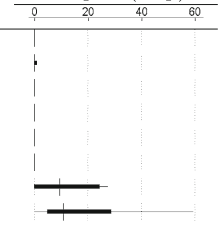
To demonstrate the effect of different parameter levels on the number of relays, Table 2 summarizes the results for one instance of the 35 node fixed edge problem with different λ, n_{spl} and n_{sur} values. Not surprisingly, as the value of λ increases the number of relays decreases since a greater optical reach allows signals to travel further without regeneration. The results show that allowing splits can reduce the number of relays because the capacity of edges can be better utilized. For example, for the $\lambda = 25$ and $n_{sur} = 3$ instance, the number of relays are 8, 7 and 6 when n_{spl} is 1, 2 and 3, respectively. Also, the number of relays increases as the number of survivable paths increases because these paths also require relays.

Results of the test sets for 6–35 nodes with five commodities are given in Tables 3, 4 and 5. Five different data sets are tested for each scenario. One data set was taken from [23] and four data sets were generated randomly by assigning edge costs, edge distances and relay costs. For MIP models, denoted as "exact", % gap values are directly taken from CPLEX. The gap of the heuristic is calculated according to its corresponding exact solution, i.e., 100*(Heuristic solution−Exact solution)/Exact solution. Therefore, a negative % gap is found when the heuristic finds a better solution than its corresponding CPLEX run.

Table 3 summarizes the results of the test instances with specified edge capacities. The optimal solution could not be found in 24 h for most problem instances with 50 nodes and larger. Discrete and continuous edge capacity problems are much harder to solve to optimality than fixed capacities (Tables 4 and 5).

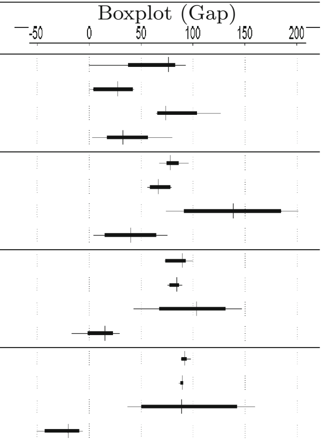
It is seen from Tables 4 and 5 that the heuristic method can find better results than the MIP model for larger sized problems in 24 h. For example, the heuristic found better results (negative % gap) for discrete problems than the MIP model for some problem instances with 25 nodes and larger. For some continuous case problem instances, the heuristic could

Table 3 Results of five runs of the fixed edge problems

Size	CPU Time (s) Average	CPU Time (s) Min	CPU Time (s) Max	Gap (%) Average	Gap (%) Best	Gap (%) SD	Boxplot (Gap) 
10N	3,312.53	0.40	16,560.00	0	0^5	0	
15N	17,288.52	3.20	L	0.76	0^4	1.7	
20N	223.03	4.68	1,070.96	0	0^5	0	
25N	7,187.52	1.85	30,396.00	0	0^5	0	
30N	249.13	3.12	946.25	0	0^5	0	
35N	991.71	16.86	4,455.82	0	0^5	0	
50N	52,052.46	409.65	L	12.05	0^2	12.85	
100N	69,240.95	604.77	L	19.04	0^1	23.41	

[a] L = 86,400 s
[1,2,4,5] Numbers indicate the number that of times the optimal solution is found within 24 h

Table 4 Results over five data sets of 10, 15, 20 and 25N problems

Size	Edge	CPU Time (s) Average	CPU Time (s) Min	CPU Time (s) Max	Gap (%) Average	Gap (%) Best	Gap (%) SD	Boxplot (Gap) 
10N	Continuous-exact	69,214.69	473.43	L[a]	59.61	0.01	36.73	
	Discrete-exact	70,617.19	7,485.93	L	23.47	0.00	19.86	
	Continuous-heuristic[1]	2.22	0.15	8.76	84.60	64.30	28.79	
	Discrete-heuristic[1]	2.22	0.16	8.78	36.90	3.04	31.60	
15N	Continuous-exact	L	L	L	80.08	67.36	10.33	
	Discrete-exact	L	L	L	67.79	56.18	10.39	
	Continuous-heuristic[1]	175.74	0.61	859.68	138.09	74.06	57.09	
	Discrete-heuristic[1]	175.72	0.60	859.57	39.89	4.28	30.92	
20N	Continuous-exact	L	L	L	85.19	72.91	11.50	
	Discrete-exact	L	L	L	82.50	75.51	5.59	
	Continuous-heuristic[1]	8,278.60	1.43	41,167.29	99.12	43.18	43.60	
	Discrete-heuristic[1]	8,276.98	1.95	41,169.47	10.72	-16.97	19.59	
25N	Continuous-exact	L	L	L	91.88	88.28	3.72	
	Discrete-exact	L	L	L	89.06	86.36	1.64	
	Continuous-heuristic[2]	9,968.02	2.83	49,828.20	95.11	36.92	61.44	
	Discrete-heuristic[2]	9,967.98	3.43	49,825.24	-25.56	-50.26	22.46	

[a] L = 86,400 s
[1,2] Numbers indicate the number of instances with no feasible solution

not find good solutions within 24 h. For instance, it found solutions with over 100 % gap for some test instances with 50 nodes and smaller. Note that a feasible solution could not be found for some problem instances within 24 h and Tables 4 and 5 were built to incorporate these missing values. Specifically, the heuristic method could not find a feasible solution for one instance each of 10, 15, 20, 30, 35 and 50N. Also, it could not find a feasible solution for two instances of 25, 35N (only the continuous case), 50 (only the discrete case) and 100N. According to statistical comparisons using paired *t* tests some definitive results were identified. Solution times of the heuristic (for both continuous and discrete cases) are significantly shorter than the corresponding MIP models (p values are 0.000). However, the continuous MIP formulation finds better solutions (p value = 0.000) than the heuristic for 10 through 35N problems, for the remaining scenarios (50 and 100N) the difference is not significant (p value = 0.145). For discrete case, the heuristic finds lower objective function values than the MIP formulation for 25 through 100N problems (p value = 0.026). However, the discrete MIP formulation finds better solutions for the smaller (10, 15 and 20N) problems (p value = 0.007).

In summary, for the most realistic case, that of discrete alternatives for edge capacities, the constructive heuristic is superior for networks of greater than 10 nodes and takes significantly less computational effort. For large networks (greater than 35 nodes) the heuristic also outperforms MIP solved by CPLEX and at a considerable computational savings.

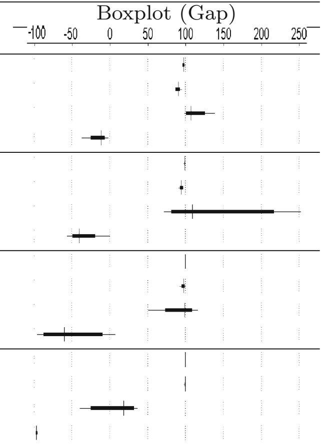
6 Discussion

In this paper, a realistic problem for telecommunications network design is considered mathematically for the first time. This problem formulation extends the work of [7] and [23] by adding capacitated edges (both fixed and variable) and k-splittable flow, both considerably complicating an already difficult problem. The edge capacity variable is considered as either discrete and continuous, however, discrete predetermined levels of capacity is a more realistic assumption due to hardware constraints. The formulation includes two-edge connectivity to ensure survivability and places relays as required for transmission integrity.

Table 5 Results over five data sets of 30, 35, 50 and 100N problems

Size	Edge	CPU Time (s)			Gap (%)		
		Average	Min	Max	Average	Best	SD
30N	Continuous-exact	L[a]	L	L	97.44	96.11	1.54
	Discrete-exact	L	L	L	90.17	86.86	3.45
	Continuous-heuristic[1]	18,378.26	4.30	L	113.01	98.75	17.86
	Discrete-heuristic[1]	17,849.97	4.26	L	-15.71	-36.94	14.79
35N	Continuous-exact	L	L	L	98.83	97.42	0.89
	Discrete-exact	L	L	L	94.44	90.33	2.62
	Continuous-heuristic[2]	8,045.44	8.43	40,144.10	144.39	71.77	95.41
	Discrete-heuristic[1]	8,057.64	8.36	40,144.78	-34.32	-56.55	23.98
50N	Continuous-exact	L	L	L	99.93	99.81	0.07
	Discrete-exact	L	L	L	96.74	92.51	2.67
	Continuous-heuristic[1]	8,531.83	33.81	40,075.59	90.91	50.55	28.20
	Discrete-heuristic[2]	8,551.97	33.90	40,169.09	-49.70	-95.88	52.11
100N	Continuous-exact	L	L	L	99.99	99.96	0.02
	Discrete-exact	L	L	L	99.38	98.17	0.69
	Continuous-heuristic[2]	34,882.34	682.02	L	5.23	-39.27	39.53
	Discrete-heuristic[2]	34,956.40	682.32	L	-96.73	-97.62	1.17

Boxplot (Gap)



[a] Maximum CPU time (86,000 s) was reached for all problem instances
[1,2] Numbers indicate the number of instances with no feasible solution

As expected, the difficulty of the problem increases dramatically as the number of nodes increases and the edge capacity variable dramatically increases the difficulty of the problem. The problem with discrete edge capacity is the hardest to solve in CPLEX due to the large number of integer variables in the formulation. Another complicating issue, split flows, increases the difficulty of the problem significantly.

The heuristic decomposes the problem into three steps and then sequentially solves each step optimally. The decomposed problems are: (1) Placing relays, (2) Finding edges and (3) Assigning edge capacities. The first two subproblems of the heuristic are solved for uncapacitated edges. The advantage of this heuristic is the reduced solution time in comparison to the original MIP formulation and improved network designs for medium to large sized problems. However, for a few problem instances, the heuristic could not find any feasible integer solution due to the imposed constraint that the three problems (placing relays, finding edges and assigning capacities) must be solved within 24 h total.

In addition to the exact method (MIP formulation) and heuristics of this paper, more effective heuristics might be developed using metaheuristic optimization methods, such as GA. Other integer programming methods, such as column generation or branch-and-price methods, might be also used to good effect.

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