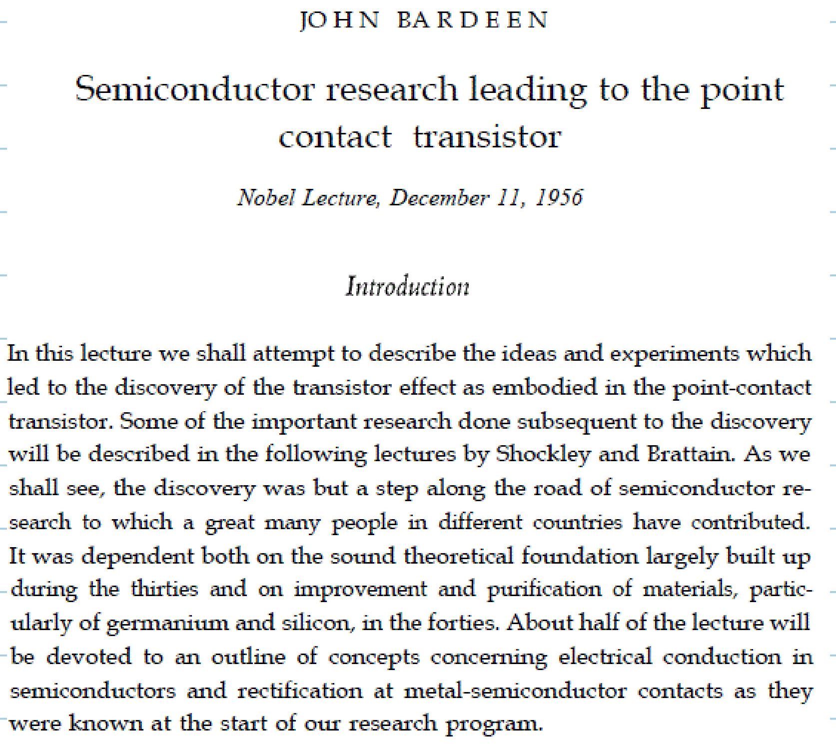
Electrons and holes in pure Silicon

Sunday, April 22, 2012
12:45 PM

Chapter goals:

1. Develop bond and band models for semiconductors.
2. Develop the concept of bonds, freed electron and hole
3. Develop the concept of intrinsic carrier concentration
4. Understand how holes move to carry current (to be a carrier)
5. Understand how doping affects both N and P



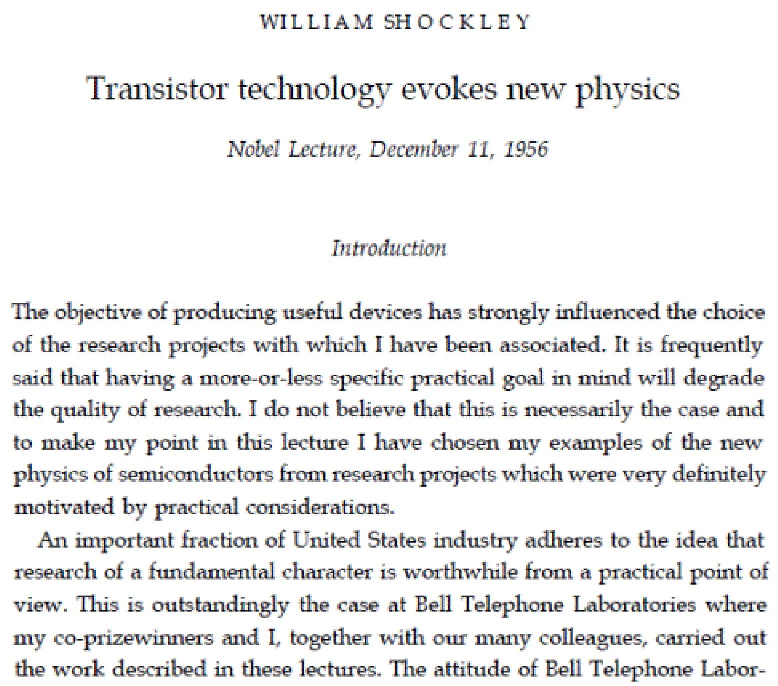
JOHN BARDEEN

Semiconductor research leading to the point contact transistor

Nobel Lecture, December 11, 1956

Introduction

In this lecture we shall attempt to describe the ideas and experiments which led to the discovery of the transistor effect as embodied in the point-contact transistor. Some of the important research done subsequent to the discovery will be described in the following lectures by Shockley and Brattain. As we shall see, the discovery was but a step along the road of semiconductor research to which a great many people in different countries have contributed. It was dependent both on the sound theoretical foundation largely built up during the thirties and on improvement and purification of materials, particularly of germanium and silicon, in the forties. About half of the lecture will be devoted to an outline of concepts concerning electrical conduction in semiconductors and rectification at metal-semiconductor contacts as they were known at the start of our research program.



WILLIAM SHOCKLEY

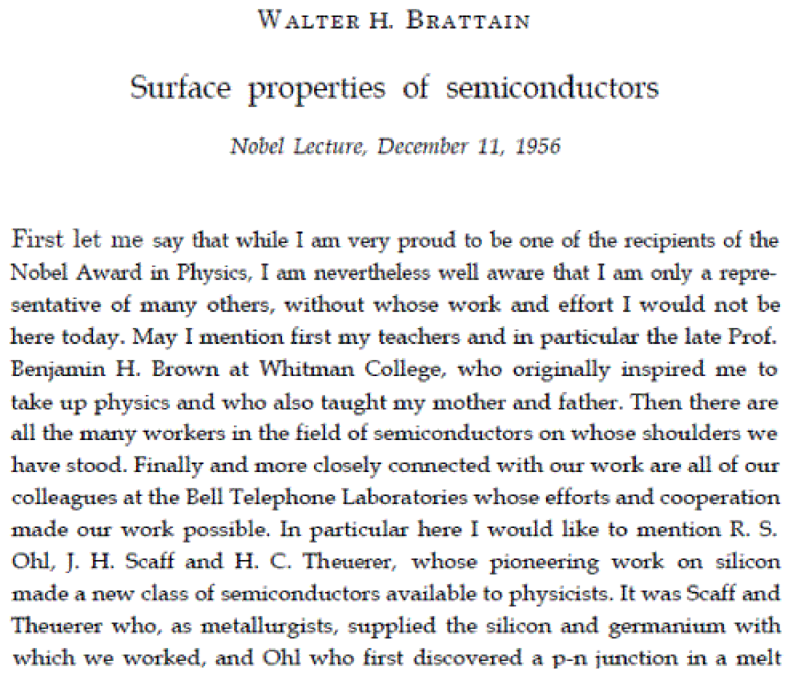
Transistor technology evokes new physics

Nobel Lecture, December 11, 1956

Introduction

The objective of producing useful devices has strongly influenced the choice of the research projects with which I have been associated. It is frequently said that having a more-or-less specific practical goal in mind will degrade the quality of research. I do not believe that this is necessarily the case and to make my point in this lecture I have chosen my examples of the new physics of semiconductors from research projects which were very definitely motivated by practical considerations.

An important fraction of United States industry adheres to the idea that research of a fundamental character is worthwhile from a practical point of view. This is outstandingly the case at Bell Telephone Laboratories where my co-prizewinners and I, together with our many colleagues, carried out the work described in these lectures. The attitude of Bell Telephone Labor-

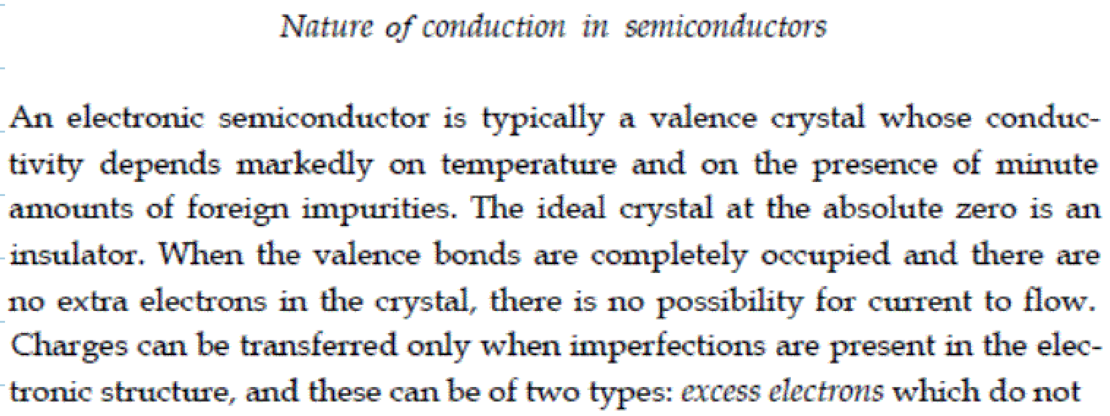


WALTER H. BRATTAIN

Surface properties of semiconductors

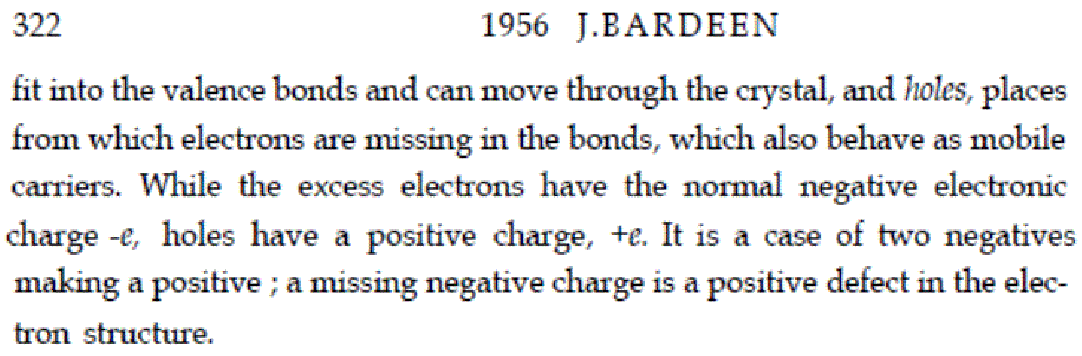
Nobel Lecture, December 11, 1956

First let me say that while I am very proud to be one of the recipients of the Nobel Award in Physics, I am nevertheless well aware that I am only a representative of many others, without whose work and effort I would not be here today. May I mention first my teachers and in particular the late Prof. Benjamin H. Brown at Whitman College, who originally inspired me to take up physics and who also taught my mother and father. Then there are all the many workers in the field of semiconductors on whose shoulders we have stood. Finally and more closely connected with our work are all of our colleagues at the Bell Telephone Laboratories whose efforts and cooperation made our work possible. In particular here I would like to mention R. S. Ohl, J. H. Scaff and H. C. Theuerer, whose pioneering work on silicon made a new class of semiconductors available to physicists. It was Scaff and Theuerer who, as metallurgists, supplied the silicon and germanium with which we worked, and Ohl who first discovered a p-n junction in a melt



Nature of conduction in semiconductors

An electronic semiconductor is typically a valence crystal whose conductivity depends markedly on temperature and on the presence of minute amounts of foreign impurities. The ideal crystal at the absolute zero is an insulator. When the valence bonds are completely occupied and there are no extra electrons in the crystal, there is no possibility for current to flow. Charges can be transferred only when imperfections are present in the electronic structure, and these can be of two types: *excess electrons* which do not



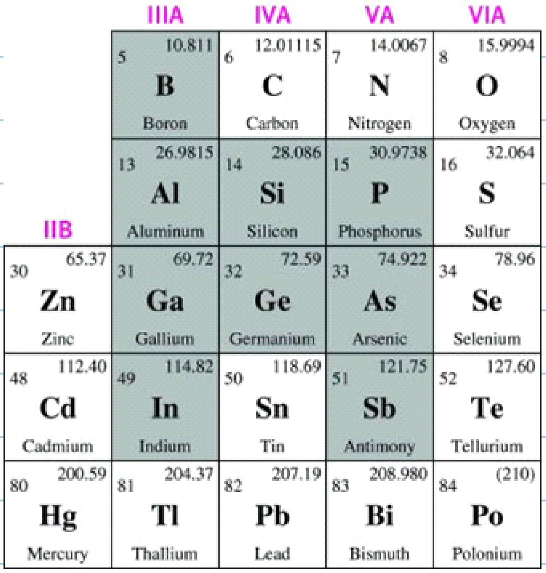
322 1956 J.BARDEEN

fit into the valence bonds and can move through the crystal, and *holes,* places from which electrons are missing in the bonds, which also behave as mobile carriers. While the excess electrons have the normal negative electronic charge $-e$, holes have a positive charge, $+e$. It is a case of two negatives making a positive ; a missing negative charge is a positive defect in the electron structure.

Solid-State Electronic Materials

- Electronic materials fall into three categories:
  - Insulators Resistivity (ρ) > 10^5 Ω-cm
  - Semiconductors $10^{-3} < \rho < 10^5$ Ω-cm
  - Conductors $\rho < 10^{-3}$ Ω-cm
- Elemental semiconductors are formed from a single type of atom, typically Silicon or Germanium (Si or Ge).
- Compound semiconductors are formed from combinations of column III and V elements or columns II and VI, e.g. GaAs.
- Germanium was used in many early devices.
- Silicon quickly replaced silicon due to its higher band gap energy, lower cost, and is easily oxidized to form silicon-dioxide insulating layers.

- GaAs has excellent optical properties
- Increase of temperature, light or addition of impurities can greatly increase conductivity of semiconductors through increase of electrons and holes



IIB	IIIA	IVA	VA	VIA
	5 10.811 **B** Boron	6 12.01115 **C** Carbon	7 14.0067 **N** Nitrogen	8 15.9994 **O** Oxygen
	13 26.9815 **Al** Aluminum	14 28.086 **Si** Silicon	15 30.9738 **P** Phosphorus	16 32.064 **S** Sulfur
30 65.37 **Zn** Zinc	31 69.72 **Ga** Gallium	32 72.59 **Ge** Germanium	33 74.922 **As** Arsenic	34 78.96 **Se** Selenium
48 112.40 **Cd** Cadmium	49 114.82 **In** Indium	50 118.69 **Sn** Tin	51 121.75 **Sb** Antimony	52 127.60 **Te** Tellurium
80 200.59 **Hg** Mercury	81 204.37 **Tl** Thallium	82 207.19 **Pb** Lead	83 208.980 **Bi** Bismuth	84 (210) **Po** Polonium

Let us start with pure Silicon:

Pure (Intrinsic) Silicon

Isolated Si atoms

Tuesday, May 22, 2012
10:26 PM

Valence electrons in isolated Si Atom

The energy of electrons in atomic systems is restricted to a limited set of values – the wonder of quantum mechanics.
Electrons fill allowed energy levels, starting from lowest one

- Ten of the 14 Si-atom electrons occupy very deep lying energy levels and are tightly bound to the nucleus

- The remaining **4** electrons, called **valence electrons** are not very strongly bound and occupy 4 of the 8 allowed slots.

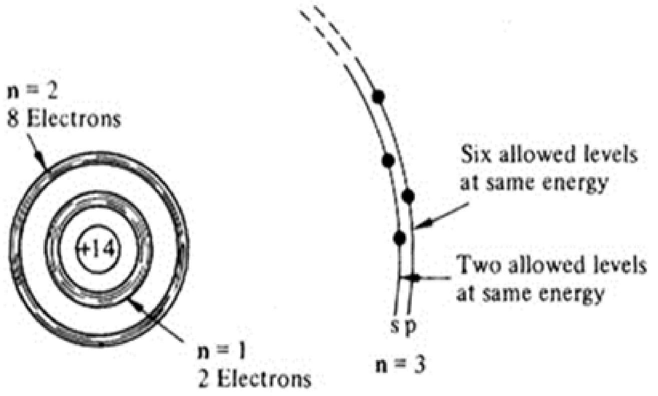
- 

Figure 2.2 Schematic representation of an isolated Si atom.

Si crystal and covalent bonds

Tuesday, May 22, 2012
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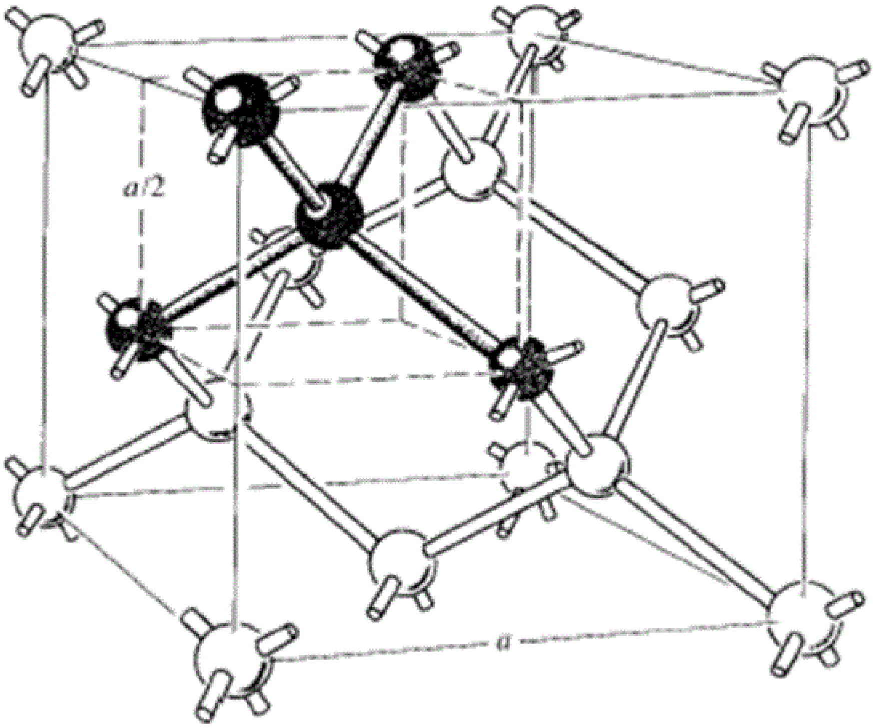
Covalent bonds in 3-D Si Crystal Lattice

In Si crystal, each Si atom has 4 closest neighbors, and shares its valence electrons with its 4 nearest neighbor.

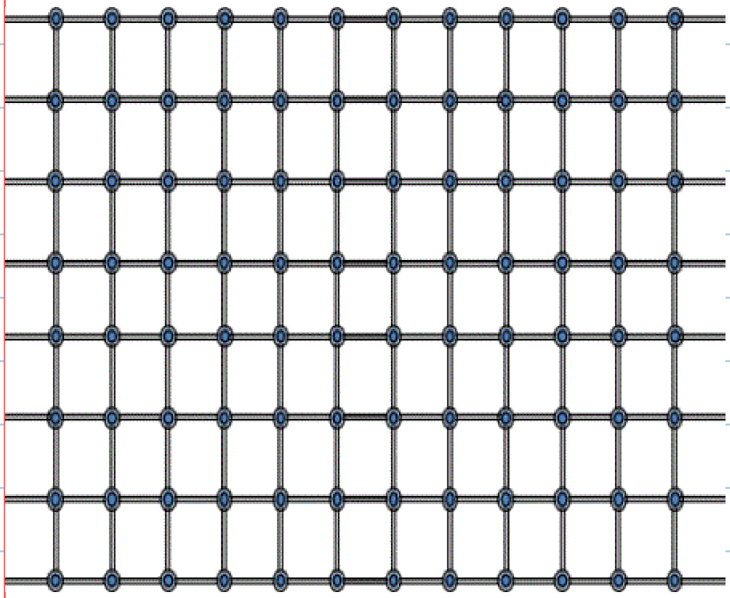
This is called a **covalent bond**. These bonds glue the atoms together.



2-D Representation of Bonds in

Silicon lattice

- 3-d silicon lattice and bonding between atoms are very hard to visualize
- It can also be visually described as 2-D square lattice, with "two lines" representing a **covalent bond**
- Like in 3d, each atom forms covalent bond with 4 closest neighbors
- **Each bond contains two valence electrons**

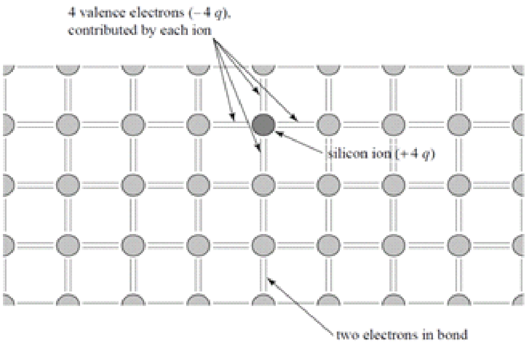


each Si atom has four nearest neighbors,

each isolated Si atom has 4 electrons in outer shell
Each atom shares its electrons with its nearest neighbor in a Si crystal. This is called a covalent bonding
At 0 K , no electrons are available for conduction in this covalent structure, so the material is and should be an insulator at 0 K

Perfect Bonds at 0K

At 0K, all bonds are perfect, Si crystal is an insulator.



Bonds Break Above 0K due to lattice vibration

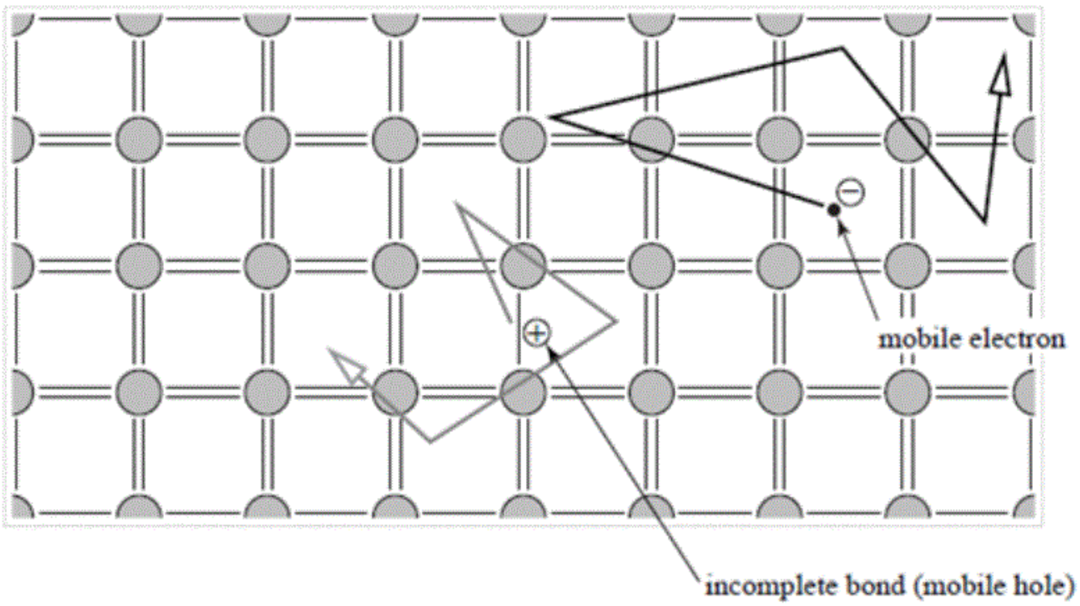
- Above 0K, atoms are vibrating around their lattice positions due to thermal energy.

- Atoms are closely packed and interact with one another, some atoms oscillate more violently than others.
- At any given instant of time, oscillations are large in some locations, causing some **bonds to break**, and producing
  - "**excess electrons**" that do not fit into valence bonds and can move around crystal. These "electrons" are mobile, and have the normal -q charge
  - **holes**, places from which electrons are missing in the bonds, which also behave as mobile carriers. These holes have a "+q" charge.

- The atomic agitation increases with absolute temperature – this is why it is also called thermal agitation

Mobile Electron and Hole

We now have broken bond due to thermal energy
“free” electrons (mobile negative charge –q)
“free” holes (mobile positive charge, +q)



Some valence electrons pick up enough energy to strike out, and become “**free electron**”, leaving behind vacancies (**holes**) or broken bond

How many free electron-hole pairs do we have at room

temperature in pure Si?

- At 300K, very few electrons become free as thermal energy kT is only 25.8 meV, while the thermal ionization energy required to break a bond is 1.12 eV This is also called band gap energy - Eg, as we will learn soon.
  - k is Boltzmann constant 8.62e-5 eV/K
  - kT=25.8meV at 300K is worth remembering as you will use it repeatedly when working with semiconductor devices
- As an example, consider that there are about 10^{22} atoms in a cubic centimeter of crystal, but there are only about 10^{10} /cm^3 free electrons in silicon – **only 1 in one trillion atoms.**

Energy Unit for Electronics: eV

We often use eV as energy unit in electronics.

1 eV is the amount of energy a 1V battery provides when moving an electron across it.

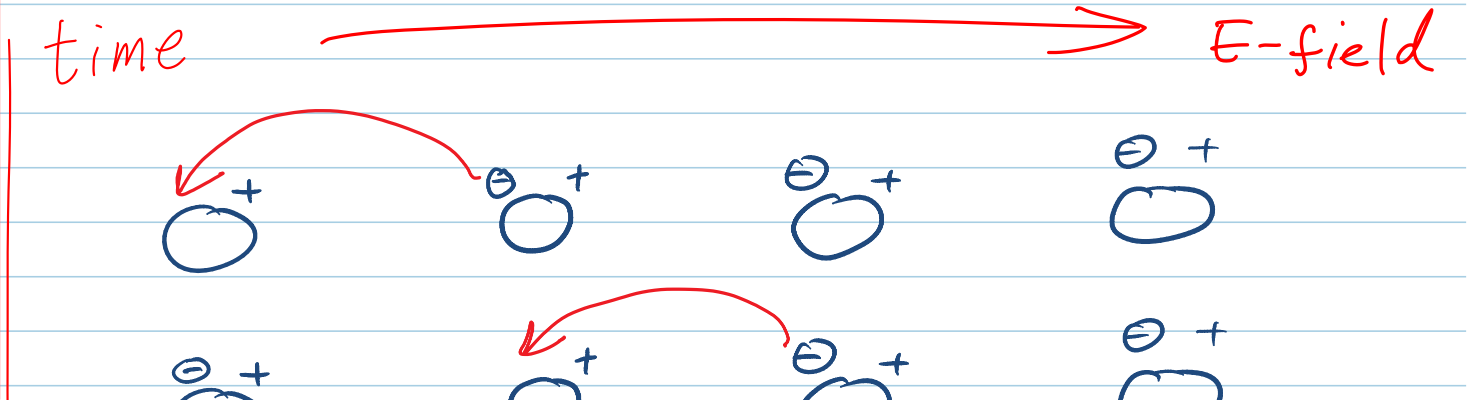
So 1 eV = 1 * 1.6e-19 C * V = 1.6e-19 J.

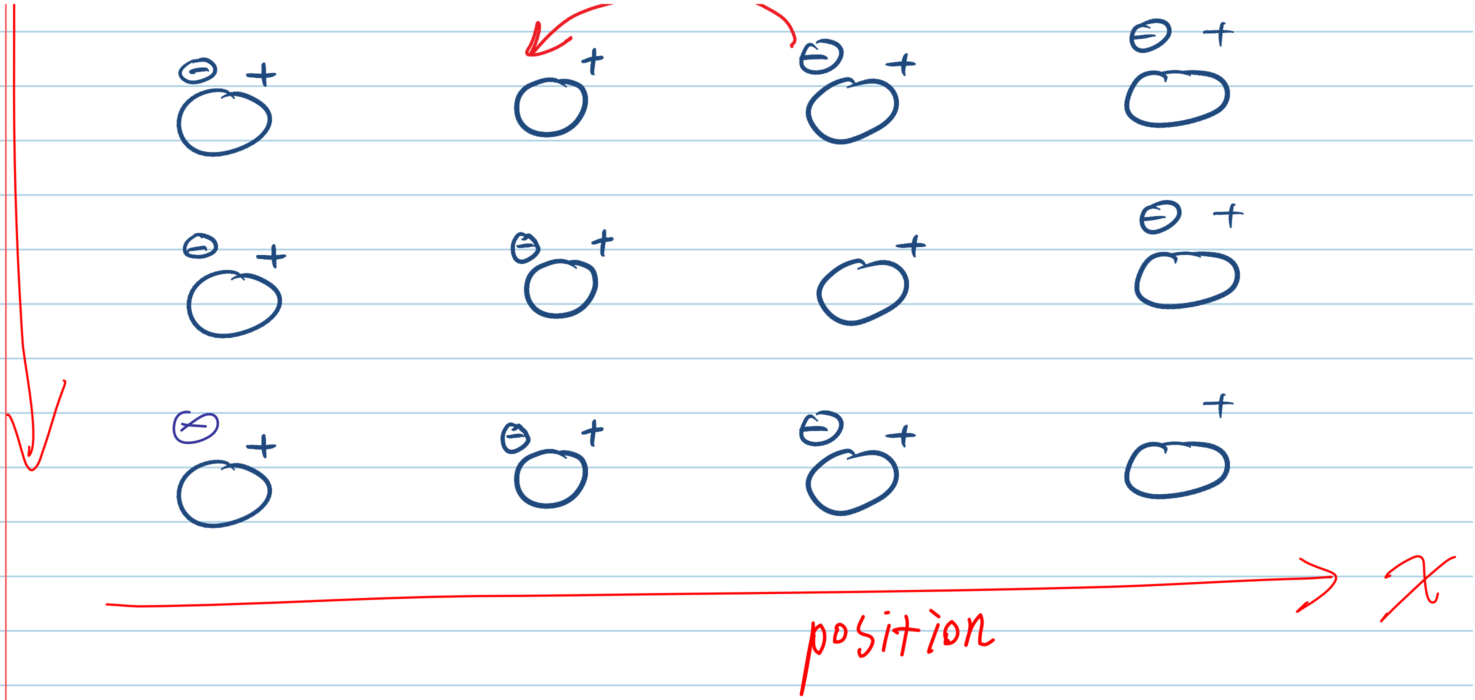
How do holes carry current?

Sunday, May 20, 2012
8:57 AM

- Freed electrons have left positively ionized Si atoms behind.
- Normally, we assume that the silicon ions do not move; however, their charges can.
- Valence electrons are traded among atoms,
- a positive ion will neutralize itself by capturing another valence electron, then some other atom is left ionized.
- **While the ion did not move, the +q charge has moved.**
- **We could just call this +q charge a mobile hole.**
- **The motion of holes becomes directed when an electric field is applied, producing a current.**
- **Free electrons and holes will move in opposite directions in an electric field, but the resulting currents will add**

“Hole” movement movie




position
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Energy bands

Sunday, April 22, 2012
12:54 PM

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Electrons fill allowed energy levels, starting from lowest one

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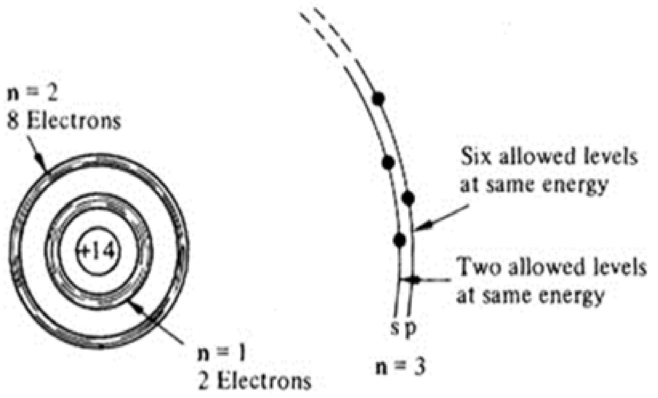
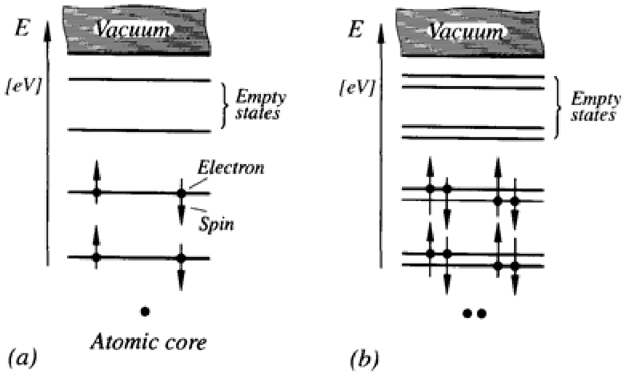
- 

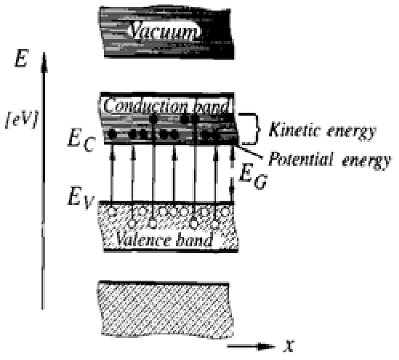
Figure 2.2 Schematic representation of an isolated Si atom.

Band Formation

- In an isolated atom, electrons can only have certain distinct energy states.
- As atoms are brought together to form Si crystal, the allowable energy states will split into closely spaced states because of the Pauli exclusion principle.
- When N atoms are put together, the original energy level will split into N different allowed states forming an energy band.
- Number of atoms is VERY Large in a crystal (on the order of $10^{22}/cm^{3}$), the separation between states is extremely small, and electrons can easily move between them.
- We can speak of a “continuous” band of allowable states.



Schematic presentation of energy levels: (a) single atom and (b) two closely spaced



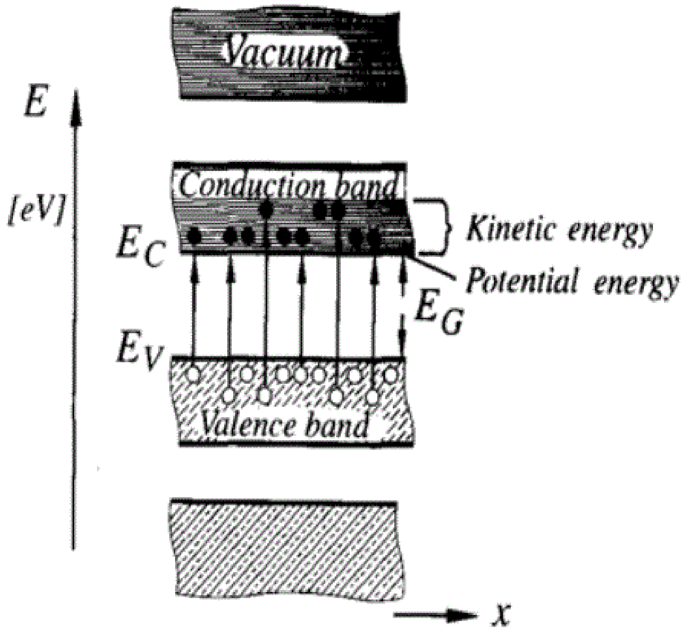
gure 1.2 Schematic energy band diagram of a crystal

Picture from Kaufman's book.

Conduction and valence band

- Of interest are only those energy bands with the highest energy, the conduction band and valence band
- All other energy bands are completely filled by electrons.
- **Electrons in the conduction band are called conduction electrons**
- **Valence band electrons are called valence electrons.**
- **The conduction band bottom is called Ec**
- **The valence band top is called Ev**
- **Eg = Ec - Ev**
- The conduction and valence bands are separated by the

band-gap **Eg**, where no energy state exists – forbidden gap.



gure 1.2 Schematic energy band diagram of a crystal

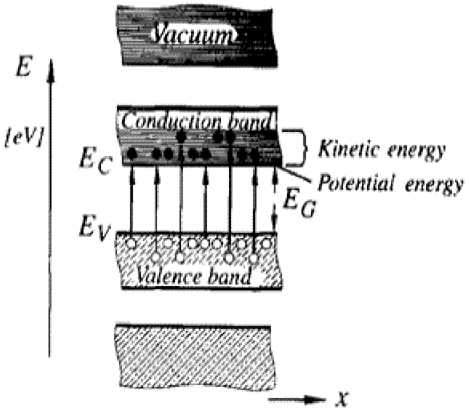
The band-gap Eg corresponds to the energy required to break a valence bond.

Eg is a fundamental parameter of a semiconductor.

The parameter characterizing the semiconductor is the energy *Eg* necessary to dissociate a hole electron pair. **It is approximately three quarters of an electron volt for germanium, one electron volt for silicon and ranges in other semiconductors from a few tenths to several electron volts.**

- from the Nobel Lectures.

Band model description - Valence electrons pick up enough energy to overcome the band-gap and jump into the conduction band, leaving behind empty states (holes) in the valence band.
Bond model - some valence electrons pick up enough energy to strike out, and become "free", leaving behind vacancies (holes) or broken bond



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semiconductor devices

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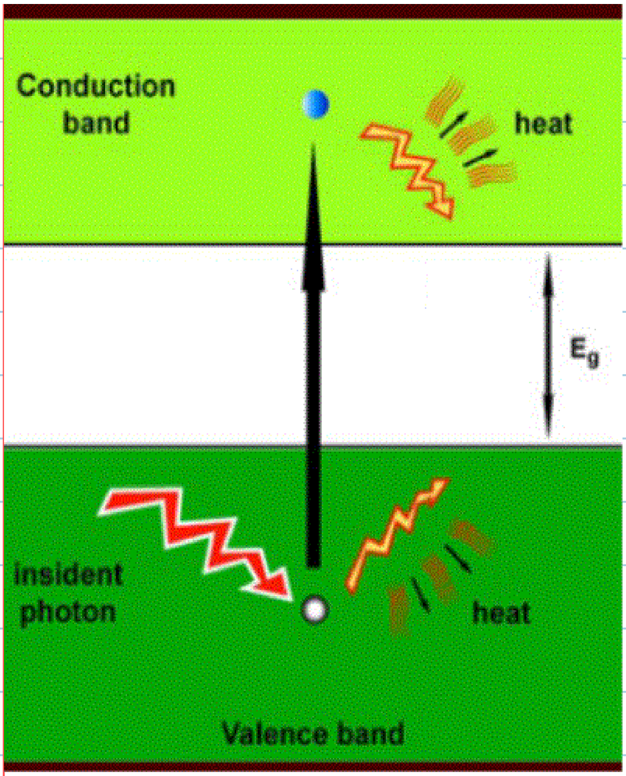
Carriers

Electrons and holes are called carriers – because they can carry current, i.e. when they move around the crystal, a current is produced

The number of carriers per volume is called carrier concentration

Light as Energy Sources to Produce Electron-Hole Pairs



Light has wave-particle duality.

Light is a stream of "particles" called "photons", which carry energy

On a clear day, $4.4\text{x}10^7$ photons hit 1 square meter of Earth's surface every second.

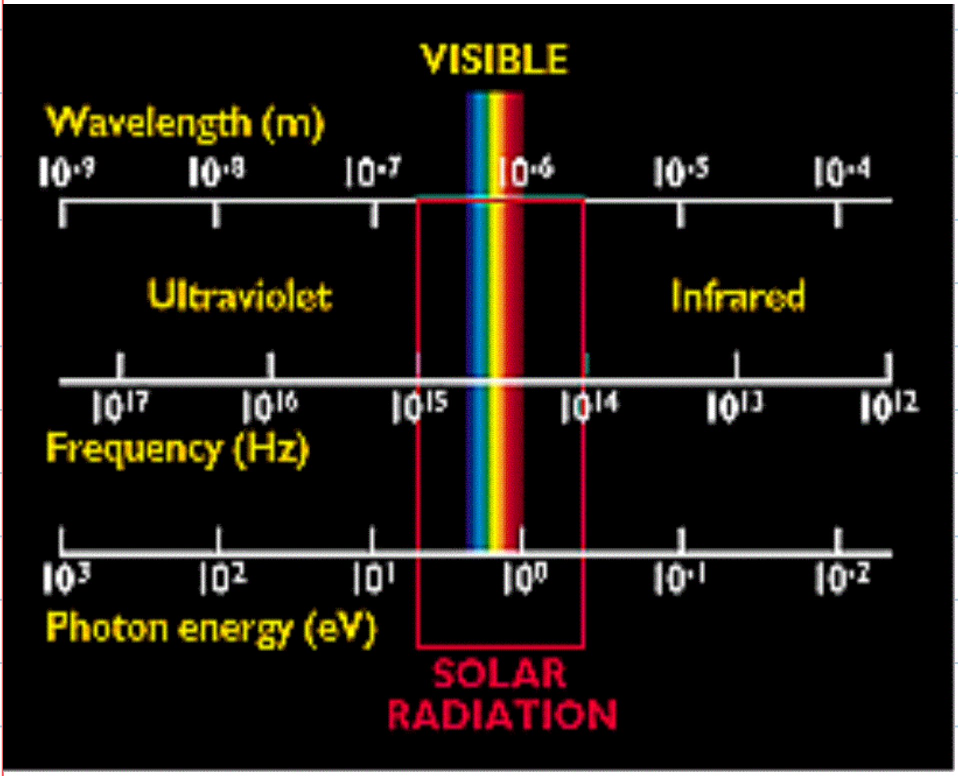
Photon energy E = h * f = h*c/wave_length

h: Planck's constant, 6.625e-34 J-s
f: frequency of light

c (light speed) = wave_length / T = wave_length * f, c = 3e8 cm/s

example: solar cells, photo conductors (light sensitive resistors), photo-transistors

Photon energy has to be higher than the bandgap Eg for optical e-h pair generation to occur



Visible light wavelengths are usually expressed in nanometers (nm), and give photon energies that are given in electron volts (eV).

P and n at equilibrium

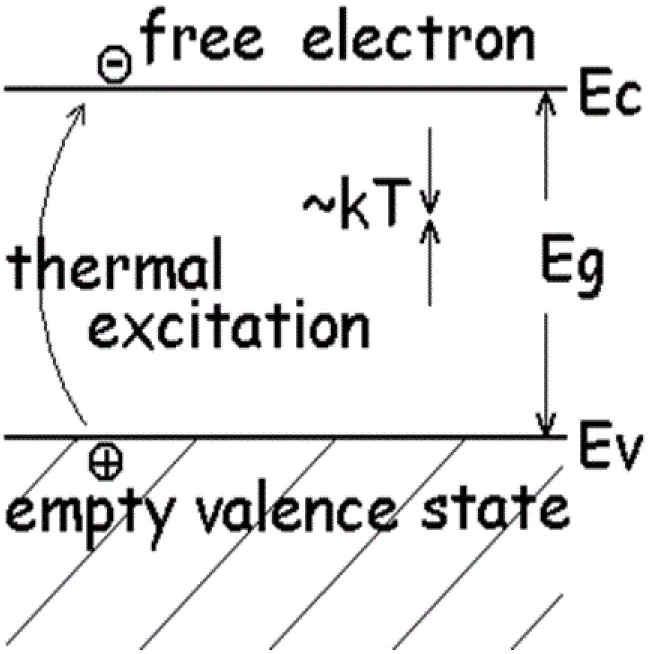
Tuesday, August 21, 2012
9:41 AM

Intrinsic concentration ni

Sunday, May 20, 2012
9:01 AM

Intrinsic Carrier Concentration (ni)

- The process of freeing electrons in pure silicon is an ionization of the silicon atoms,
- free-electron concentration is equal to hole concentration (whole pure Si crystal is charge neutral), and either one is represented by the symbol n_i, which is called the intrinsic carrier concentration.
- n_i is produced by a dynamic process; free electron hole pairs are continually being generated, while other electron hole pairs are recombining



- E_G = semiconductor **bandgap energy in eV** (electron volts)
- k = **Boltzmann's constant**, 8.62 x 10^{-5} **eV/K**
- T = **absolute temperature**, K
- B = **material-dependent parameter**, 1.08 x 10^{31} K^{-3} cm^{-6} for Si
- Band gap energy is the minimum energy needed to free an electron by breaking a covalent bond in the semiconductor crystal.

Numerical Example of n_i for Si:

$$\exp\left(-\frac{E_G}{kT}\right) \text{cm}^{-6}$$

For silicon, B = 1.08 x 10^{31} and EG = 1.12 eV, find ni at 300K using the equation given above:

Solution:

$$n_i = 6.73 \times 10^9/\text{cm}^3.$$

Python code for step by step calculation:

```
from __future__ import print_function
from math import exp, sqrt


b = 1.08e31 # K^{-3} cm^{-6}
k = 8.62e-5 # eV/K

T = 300

eg = 1.12

kt = k * T
ni2 = b * T**3 * exp(-eg / kt)
ni = sqrt(ni2)

print('Si: Eg = {eg} eV, T = {T} K, B = {b}, ni = {ni:6.2e} /cm^3'.format(**locals()))
```

Python Output:

```
Si: Eg = 1.12 eV, T = 300 K, B = 1.08e+31, ni =
6.73e+09 /cm^3
```

Numerical Example Ge:

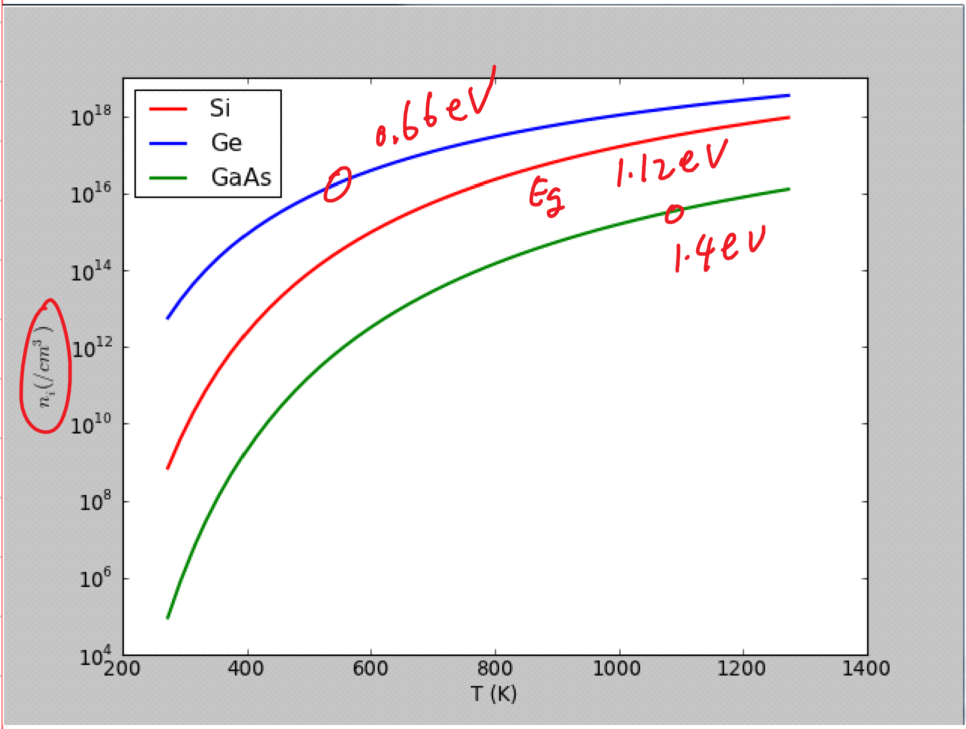
For germanium, B = 2.31 x 10^{30} and EG = 0.66 eV, at 300K:

ni = 2.27 x10^{13}/cm^3 .

n_i of popular semiconductors

One expects that the intrinsic concentration is a function of temperature T and of the amount of energy required to break crystal bonds (ionization energy), which is also called Eg as we will introduce soon.

As expected, the largest Intrinsic density results at higher temperature and in materials with a small band-gap.



I generated a graph for ni - T for three most popular semiconductors.

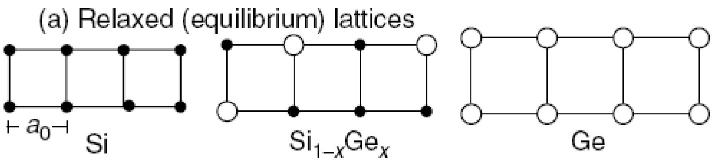
The main difference between materials is from Eg.

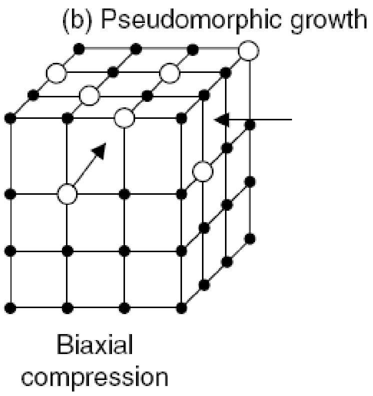
Eg = 1.12 eV for Si, 0.66 eV for Ge and 1.4 eV for GaAs.

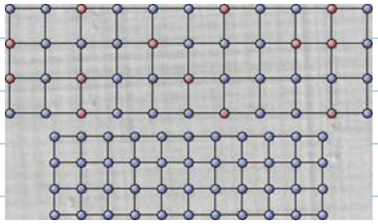
$n_i = 10^{10} / cm^3$ at 300K in Si

Bandgap engineering

Wednesday, August 15, 2012
5:36 PM

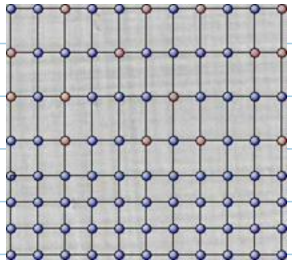




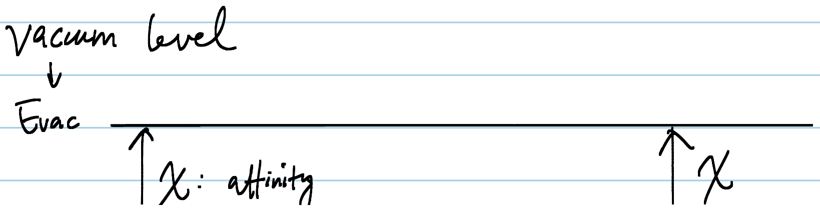


Bulk SiGe

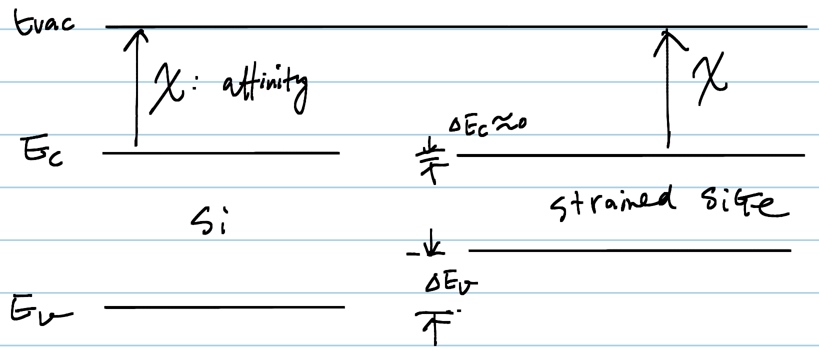
Si



Strained SiGe



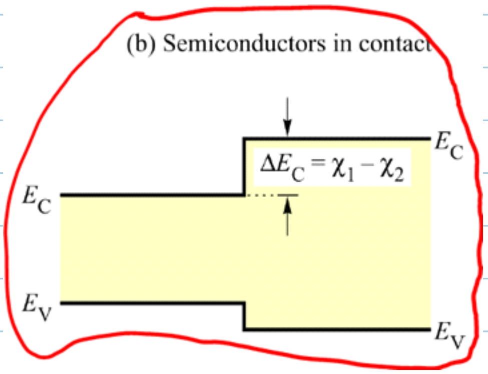
$\chi_{Si} \approx \chi_{SiGe}$

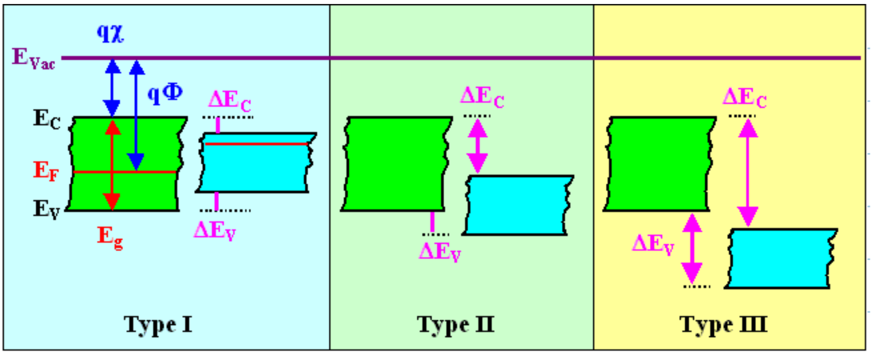


$\chi_{Si} \approx \chi_{SiGe}$

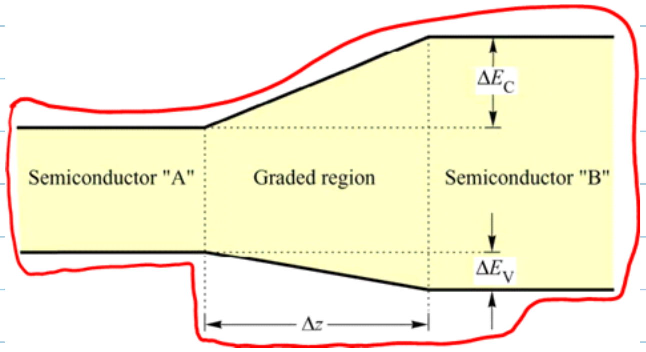
* In general, however, χ is a function of Ge mole fraction x

* So χ_{SiGe} can vary with position





We can also create graded structures:



Doping engineering

Sunday, April 22, 2012
1:17 PM

Donors
Acceptors
Charge Neutrality
Mass Action Law (pn product)
Carrier Concentration Engineering at Equilibrium
Majority vs Minority Carrier

- The discussion of semiconductor properties has so far been restricted to pure crystals. In semiconductor terminology an absolutely pure crystal is called an intrinsic semiconductor.
- Intrinsic semiconductor is not very useful as we will learn later.
- To make useful devices, we need to add small amounts of other elements having a different atomic structure, e.g. group III element Boron, or group V elements arsenic, phosphorous or antimony – also known as

"impurities" – these are called extrinsic semiconductors

impurities

dopants

Acceptors and donors (of electrons)

Sunday, April 22, 2012
1:18 PM

Quiz: molar concentration $\underline{\underline{10^{-7}}}$ $[H^+]$ $[OH^-]$

1) what is the pH of pure water? What is the molar concentration of positive hydrogen ions and negative hydroxide ions in deionized (pure) water?

pure Si

$[H^+]$ 10^{-7} $\quad$ p $10^{10}/cm^3$

$[OH^-]$ 10^{-7} $\quad$ n $10^{10}/cm^3$

2) What can we add to water to increase hydrogen ion concentration?

$\underline{\underline{\text{acid}}}$ $\quad [H^+]\uparrow$ $\quad 10^{-7} \rightarrow 10^{-4}$

$[OH^-]\downarrow$ $\quad 10^{-7} \rightarrow 10^{-10}$

3) if we increase hydrogen ion concentration by 1000 times, what will the concentration of hydroxide ions be?

Water

$H_2O \rightleftharpoons H^+ + OH^-$

In pure water

$[H^+] = [OH^-] = 10^{-7} M$

$[H_2O] = 55.5 M$

$[H^+]$ or $[OH^-] \ll [H_2O]$

$[H^+][OH^-] \quad 10^{-14}$

Semiconductor

valence bond $\rightleftharpoons h^+ + e^-$

In pure semiconductor (intrinsic)

$p = n = n_i = 10^{10} cm^{-3}$ Si at 300K

$p \cdot n \ll$ valence bond concentration

$4 \times 5 \times 10^{22}/cm^3$

$pn = 10^{20} cm^{-6}$

$[H^+][OH^-] = 10^{-14}$

ion product constant

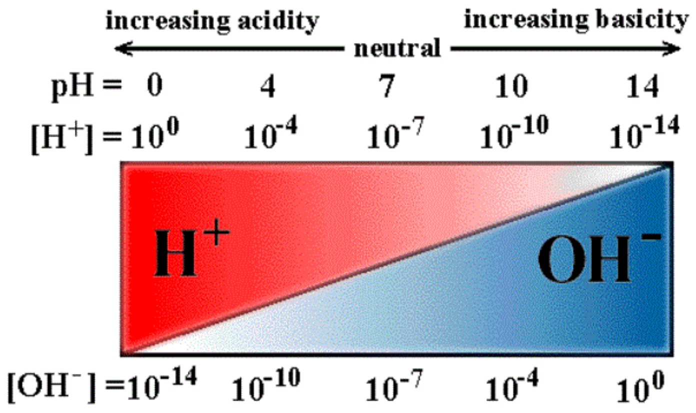
$pn = 10^{20}\,cm^{-6}$

in Si at 300 K

pn product

$$HCL \xrightarrow{H_2O} H^+ + Cl^-$$

$$NaOH \xrightarrow{H_2O} OH^- + Na^+$$



To increase $[H^+]$, we add acid

$$HCL \xrightarrow{H_2O} H^+ + Cl^-$$

total charge needs to stay zero, positive = negative

$$[H^+] = [OH^-] + [Cl^-]$$

$$[H^+][OH^-] = 10^{-14}$$

In semiconductors, we add acceptors to increase "p" - concentration of positive charged holes

atoms which become negatively ionized when placed at Si lattice locations are called acceptors. We denoted acceptor concentration Na. Ionized concentration Na-. Often we can assume Na- = Na, or complete ionization.

Semiconductor

To increase $\overset{p}{[H^+]}$, we add $\overset{\text{acceptor}}{\text{acid}}$

$$\text{Acceptor} \rightarrow \underset{\text{Ionized acceptor}}{\text{Acceptor}^-} + \underset{\text{hole}}{h^+}$$

$$HCl \xrightarrow{H_2O} H^+ + Cl^-$$

total charge needs to stay zero, positive = negative

$$[H^+] = [OH^-] + [Cl^-]$$

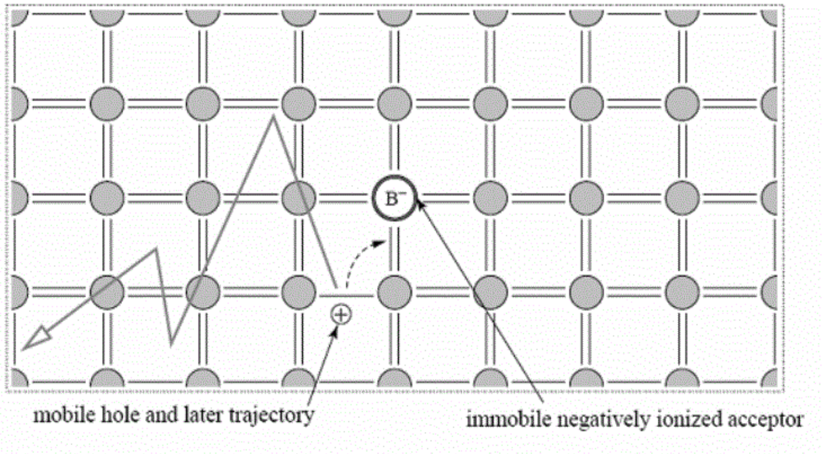
$$p = n + N_a \qquad pn = n_i^2$$

$$N_a^- \approx N_a \text{ at } 300\text{ K}$$

(complete ionization)

Acceptor in Si (Boron)

- Boron atom has 3 valence electrons and 3^+ ion core
- 4th bond site grabs (accepts) an electron from a neighboring Si atom, leaving behind a "hole"
- Each ionized boron atom is thus negatively charged
- Each hole is positively charged



To increase $[OH^-]$ n

$NaOH \xrightarrow{H_2O} Na^+ + OH^-$ donor → $\text{donor}^+ + e^-$

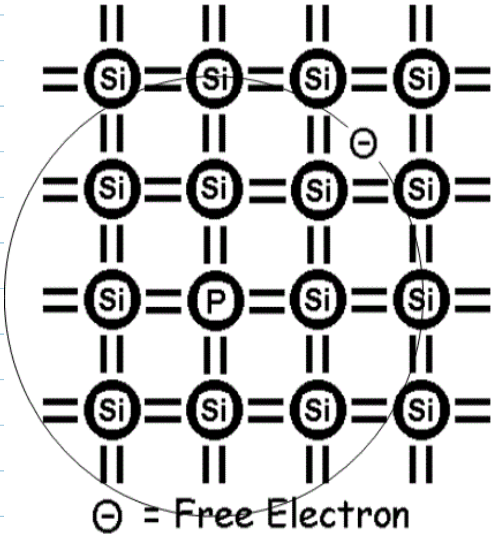
$[H^+] + [Na^+] = [OH^-]$

$p + N_d = n$ $pn = n_i^2$

$N_d^+ \approx N_d$ Complete Ionization

Donor in Si (phosphorous / arsenic)

- P atom: 5 valence electrons and a 5+ ion core
- The 5th valence electron has no available bond – so there is an extra electron
- Each extra free electron, -q, has one +q ion
- # P atoms = # free elect, so material is charge neutral at RT (300K), approximately



Popular misunderstanding of doping impact

Sunday, April 22, 2012
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How many electrons and holes are there in a cubic cm pure silicon at 300K? (assuming ni=10^{10}/cm3)

N=? 10^{10}

P=? 10^{10}

What if we add Phosphorous, at a concentration of 10^{16}/cm3

N=? $\underline{\underline{10^{10}}} + \underline{\underline{10^{16}}}$

P=? 10^{10}

Mass action law:

In thermal equilibrium and for a given semiconductor, np product is a constant

$$n_0 * p_0 = n_i^2$$

Typical doping level ranges from 10^{14}/cm^3 to 10^{21}/cm^3

Doping Design of Equilibrium Uniform Semiconductor

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Problem statement:

Design:

To achieve a given n or p (product is ni^2), determine doping type and level.

Charge Types and Charge Neutrality

Electrons (n) - negative
Holes (p) - positive
Ionized Donors (N_d^+)- positive
Ionized Acceptors (N_a^-) - negative

For a uniform sample, at equilibrium, net charge is zero (charge neutrality).

At room temperature, dopants are nearly 100% ionized. That is, Nd+ = Nd, Na- = Na approximately.

$N_d^+ = N_d$

$N_a^- = N_a$

$p + N_d = n + N_a$

$$p + N_d = n + N_a$$

or

$$p - n + N_d - N_a = 0$$

This is the first weapon we have in solving equilibrium uniform semiconductor problem.

The second weapon we have is the $pn = constant = n_i^2$ law, also known as mass action law.

Mass Action Law at Equilibrium:

$pn = n_i^2$ at equilibrium

We have therefore two equations, charge neutrality and pn product, which can be solved for p and n.

$$p + N_d = n + N_a$$

$$pn = n_i^2$$

Before we mathematically solve these equations, let us make some simple observations in cases of donor and acceptor doping only.

Donor only case: n-type

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Consider donor doping only, that is, $N_d > 0, N_a = 0$.

$p + N_d = n + N_a$ becomes:
$p + N_d = n$ which means $n > p$ as consequence of donor doping. The semiconductor is said to be n-type.

The $n - p$ difference is set by donor concentration N_d.

In the mean time, $pn = n_i^2$

Therefore $n > n_i > p$

$N_d > 0 \qquad N_a = 0$

$$\left.\begin{array}{l} p + N_d = n \\ pn = n_i^2 \end{array}\right\} \Rightarrow$$

Q:
is n > p or p > n?

Is n > ni?

Is p > ni?

N-type Doping Design Example:

Consider Si, 300K, ni = 1e10cm-3. Determine the doping type and level required to achieve n=1e15cm-3.

Solution:

1. n=1e15 > ni, therefore we need n-type doping.
2. The level of n-type doping, Nd, can be found from pn product and charge neutrality
   a. p=ni^2/n
   b. Nd = n-p

```
>>> ni=1e10

>>> n=1e15

>>> p=ni**2/n

>>> p

100000.0
>>> nd = n-p

>>> nd

999999999900000.0
>>> print "%5.2e" % nd
```

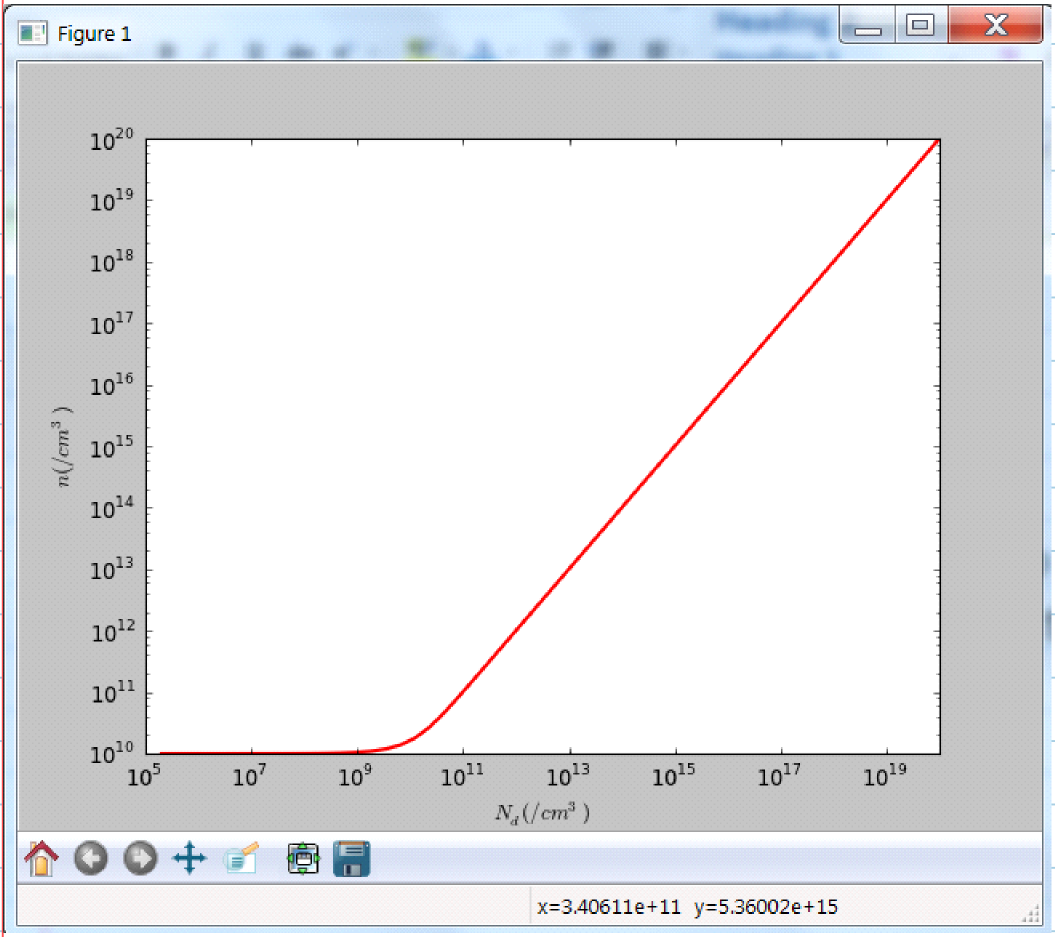
```
1.00e+15
>>> print ″%5.2e″ % n

1.00e+15
>>> print ″%5.2e″ % p

1.00e+05
```

Nd is smaller than n, by p. However, value wise, Nd is practically the same as "n" in our computer.

As shown above, Nd = 999999999900000.0 while n = 1e15.

Graphing n - Nd curve:

An easy way to visualize how Nd affects n is to assume an array of n values, use p=ni^2/n to calculate p, then find out the corresponding Nd. We can then plot out n as a function of Nd:



x=3.40611e+11 y=5.36002e+15

Observe from the above graph that

1. only when Nd is much greater than ni, n increases with Nd considerably. For Nd > 10ni (1e11), practically n = Nd.
2. When Nd << ni (1e10), n is approximately ni.
3. Thus for doping to be effective in controlling n, Nd needs to be much greater than ni
4. We will prove this analytically soon

Python codes:

```
def n_vs_nd():
    ni = 1e10

    n = ni + logspace(-5, 10, 50)*ni
    ni2 = ni**2
    p = ni2/n
    nd = n-p
    myplot(x=nd, y=n, xlog=True, ylog=True, xlabel='$N_d(/cm^3)$'
```

```
myplot(x=nd, y=n, xlog=True, ylog=True, xlabel='$N_d(/cm^3)$',
          ylabel='$n(/cm^3)$')
```

Acceptor Only: p-type

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$N_d = 0 \qquad N_a > 0$

Q:
is n > p or p > n?

Is n > ni?

Is p > ni?

$p = n + N_a$

$pn = n_i^2$

$p = n + N_a$
Therefore as consequence of acceptor doping
$p > n$
We therefore also call acceptor doping p-type doping.
The semiconductor is said to be p-type.
The difference $p - n$ is set by acceptor doping level or acceptor concentration N_a.

As $pn = n_i^2$, $p > n_i > n$.

p-type doping design example:

Consider Si, 300K, ni = 1e10cm-3. Determine the doping type and level required to achieve p=1e15cm-3.

Solution:

1. p=1e15 > ni, therefore we need p-type doping.
2. The level of p-type doping, Na, can be found from pn product and charge neutrality
   a. n=ni^2/p
   b. Na = p-n

```
>>> ni=1e10

>>> p=1e15

>>> n=ni**2/p

>>> na=p-n

>>> print "n=%g p=%g na=%g" % (n, p, na)

n=100000 p=1e+15 na=1e+15
```

You may wonder why na and p are the same, they are not, they look the same just because of the way they are printed, to show the difference:

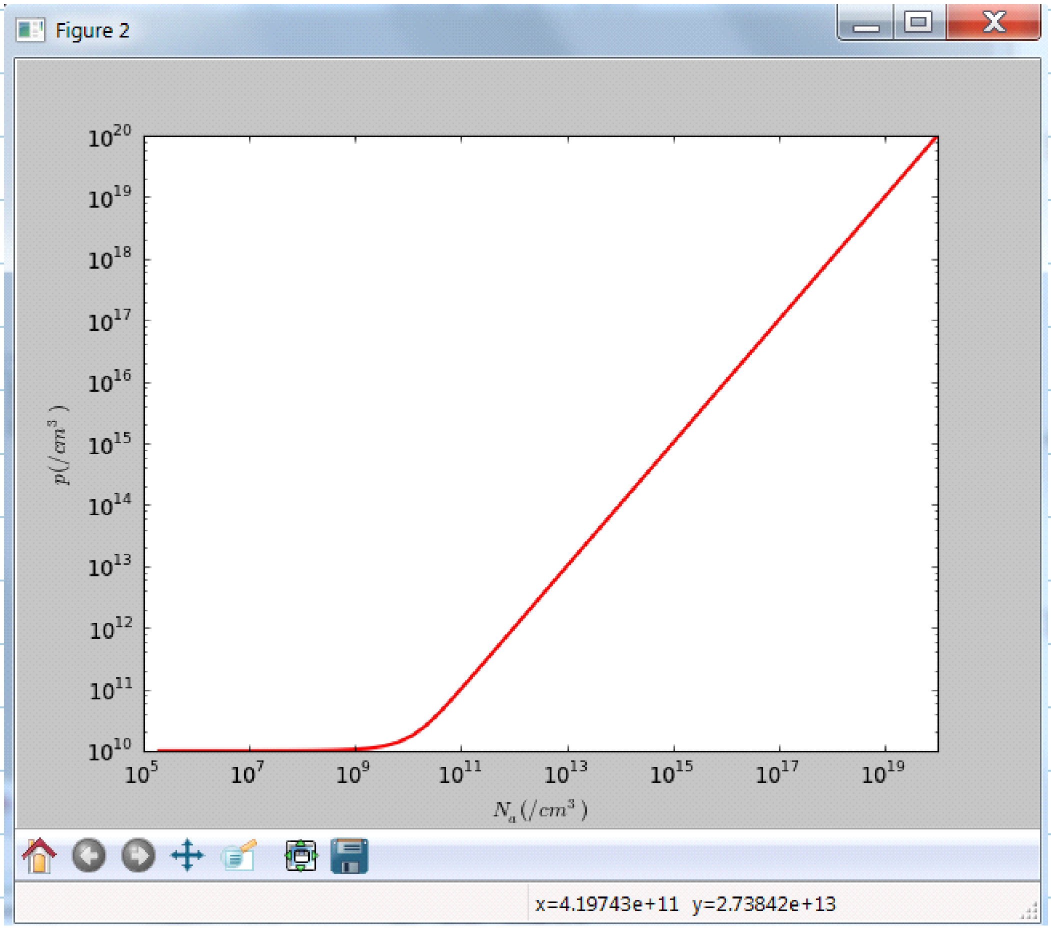
```
>>> print na
```

```
9.999999999e+14
>>> print p

1e+15
>>> print p-na

100000.0
```

Graph of p versus Na:



See if you can make observations similar to those in the n versus Nd graph.

Python codes:

```
def p_vs_na():
```

```
ni = 1e10

p = ni + logspace(-5, 10, 50)*ni
ni2 = ni**2
n = ni2/p
na = p-n
myplot(x=na, y=p, xlog=True, ylog=True, xlabel='$N_a(/cm^3)$',
       ylabel='$p(/cm^3)$')
```

Donor and acceptor co-exist - compensation

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Often we have a p-type doping to begin with, we overwrite it by adding a lot more n-type dopant, this is called compensation, the net doping Ndop is Nd – Na,
If Net doping Ndop >0, the final material is n-type, or n > p.
If net doping Ndop < 0, final material is p-type, or p > n.

$$p + N_d = n + N_a$$

$$p - n = N_a - N_d$$

$$n - p = N_d - N_a$$

That is, $N_d - N_a$ sets $n - p$, while $N_a - N_d$ sets $p - n$.

⭐ **Design Example:**

A p-type Si wafer is doped with Na=1e15/cm^3, find the level of donor doping we need to convert the wafer from p-type to n-type, with a target electron density n = 1e16/cm^3.

In fact, this is how real devices are made, we start from a substrate, and then use compensation to create regions of different type and doping level to make integrated circuits.

Solution:

We know n, so p=ni^2/n.

$$n - p = N_d - N_a$$

Thus

$$N_d = (n - p) + N_a$$

```
>>> na=1e15
>>> n=1e16
>>> p=ni**2/n
>>> nd=(n-p)+na
>>> nd
1.099999999999e+16
```

Pn product - direct derivation

Sunday, April 22, 2012
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We have used the water analogy to show that the pn product at equilibrium is constant, and therefore, is equal to ni^2.

We will directly derive the pn product here.

The key is to understand that at any point of time, e-h pairs are generated, and in the same time, e-h pairs recombine. In equilibrium, the number of e-h pairs generated exactly equals the number of e-h pairs recombined.

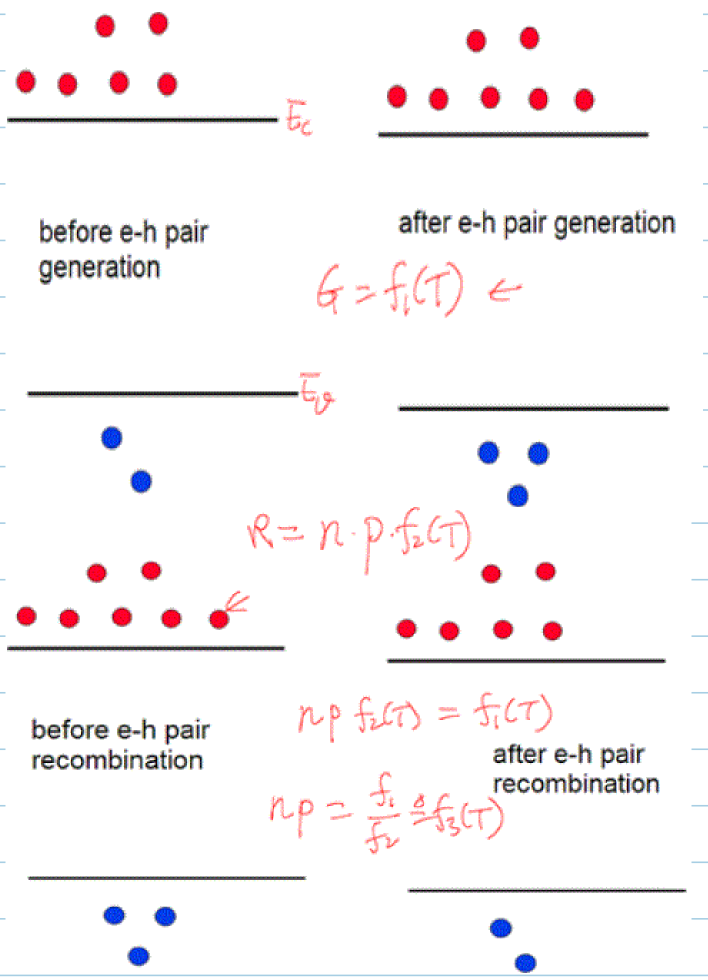
a dynamic equilibrium exists

some valence electrons are constantly being excited from VB to CB - a process called generation of e-h pairs

some CB electrons are losing energy and falling back to VB - called recombination of e-h pairs, annilating both carriers

To first order, generation rate G depends on T, but independent of number of carriers already present

Recombination rate R, depends on n and p, because both species must interact for recombination to occur



$G = f_1(T) \leftarrow$

$R = n \cdot p \cdot f_2(T)$

$np\, f_2(T) = f_1(T)$

$np = \frac{f_1}{f_2} \triangleq f_3(T)$

$$G = f_1(T)$$

$$R = p \cdot n \cdot f_2(T)$$

$$f_1(T) = pn\, f_2(T) \qquad pn = \frac{f_1(T)}{f_2(T)} \triangleq f_3(\overset{\uparrow}{T})$$

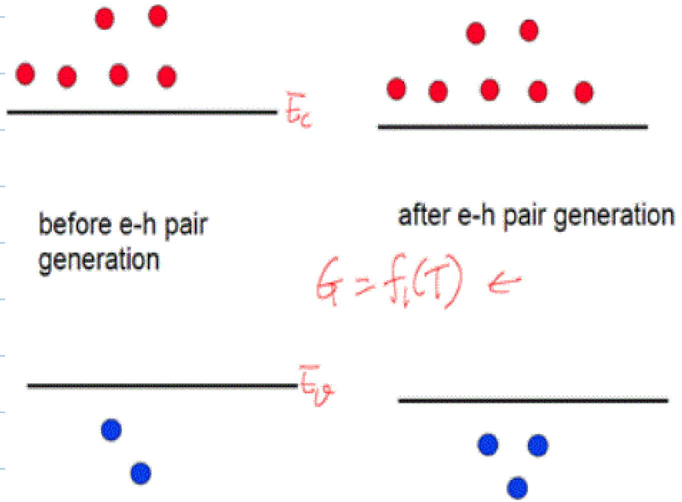
Generation rate (the number of e-h pairs per volume generated per second):

Generation of e-h pairs requires energy from thermal or optical sources (or other external sources)

G is independent of the density of bonds - which are virtually infinite. So supply of breakable bonds is inexhustible. In band model, the density of valence electrons is practically infinite.

So the generation rate depends on temperature (thermal energy), but to first order, is independent of the number of existing electrons or holes.

It makes intuitive sense, as most of CB states are empty. All of them can take an excited electron.

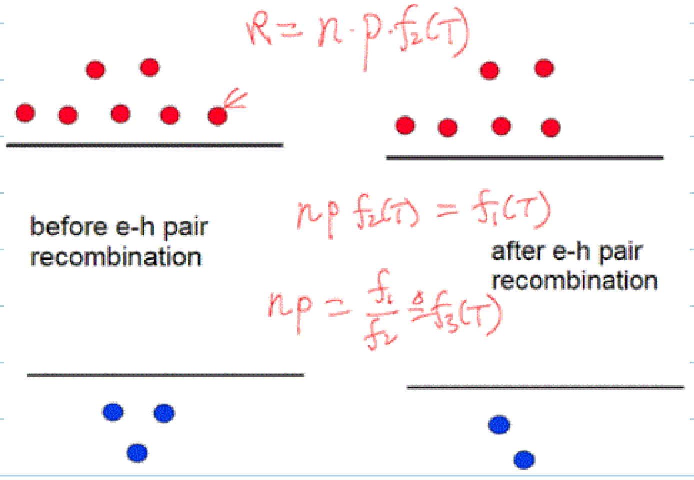


$G = f_1(T)$ ←

$$G = f_1(T)$$

Recombination,

Recombination rate R:



$R = n \cdot p \cdot f_2(T)$

$np\, f_2(T) = f_1(T)$

$np = \frac{f_1}{f_2} \triangleq f_3(T)$

number of e-h pairs recombined per second.

To first order, generation rate G depends on T, but independent of number of carriers already present

$$R = n \cdot p \cdot f_2(T)$$

$$G = f_1(T)$$

$$f_1(T) = n \cdot p \cdot f_2(T)$$

$$np = f(T)$$

a dynamic equilibrium exists

some valence electrons are constantly being excited from VB to CB - a process called generation of e-h pairs

some CB electrons are losing energy and falling back to VB - called recombination of e-h pairs, annilating both carriers

To first order, generation rate G depends on T, but independent of number of carriers already present

Recombination rate R, depends on n and p, because both species must interact for recombination to occur

Important consequence:

In thermal equilibrium and for a given semiconductor, np product is a constant that depends only on temperature!

$$p_0 n_0 = n_i^2$$

$$\underset{\text{equilibrium}}{\overset{\downarrow}{0}}$$

What if this is intrinsic semiconductor?

$$p_0 = n_i \qquad n_0 = n_i$$

$$pn = n_i^2$$

Summary of PN Product

- At equilibrium, the generation rate must equal the recombination rate, that is G=R
  - n0 * p0 * f2(T) = f1(T) or
  - n0 * p0 = f1(T) / f2(T) = f3(T)
- The equilibrium **np** product is a function of only temperature and the bandgap **Eg**
- **In an intrinsic semiconductor (undoped), all n and p are from thermal ionization or excitation across the bandgap. Consequently n0 = p0 = ni, "i" indicates intrinsic**
- **$n_0 * p_0 = n_i^2$**

Analysis of Equilibrium Uniform Semiconductor

Thursday, May 24, 2012
6:22 PM

Analysis Problem Statement:
At manufacturing, Nd and Na are established, we need to find the resulting carrier concentrations, i.e. n and p values.

Let us solve the neutrality and pn product equations for p and n.

We will start with a n-type example, then look at a p-type example where a numerical issue is revealed.

We can then develop a generic solution that applies to any doping level.

n-type example

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Example: find n and p in a uniformly doped sample with Nd-Na >0

- Knowns: doping Nd, Na
- Unknowns: n and p
- Weapons we have: charge neutrality, mass action law

$$n = \frac{(N_D - N_A) \pm \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2}$$

$$= \frac{(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2}$$ to be meaningful

and $p = \frac{n_i^2}{n}$

$$\left.\begin{array}{l} pn = n_i^2 \\ p - n + N_d - N_A = 0 \end{array}\right\}$$

$$N_{dop} \overset{\Delta}{=} N_d - N_a$$

$$\frac{n_i^2}{n} - n + N_{dop} = 0$$

$$n^2 - n_i^2 - n N_{dop} = 0$$

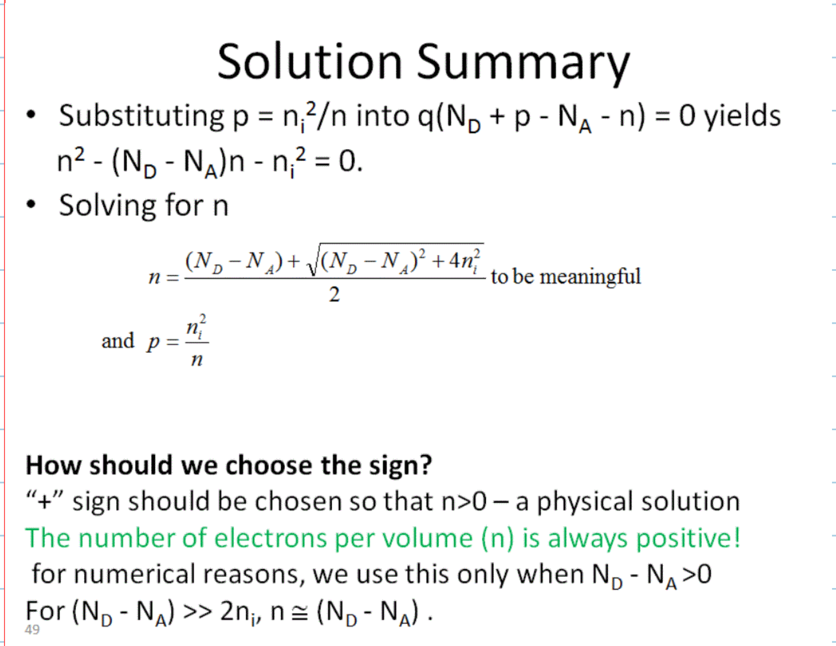
$$n = \frac{N_{dop} \pm \sqrt{N_{dop}^2 + 4n_i^2}}{2}$$

the "−" solution
is never physical
* should be dropped!

If $N_D - N_A > 0$, $n > n_i$. $p < n_i$, $n > p$. n-type

If $N_D - N_A = 0$, no net doping, $n = n_i$, $p = n_i$. intrinsic

If $N_D - N_A < 0$, $n < n_i$. $p > n_i$. $p > n$. p-type



Solution Summary

- Substituting $p = n_i^2/n$ into $q(N_D + p - N_A - n) = 0$ yields $n^2 - (N_D - N_A)n - n_i^2 = 0$.
- Solving for n

$$n = \frac{(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2} \text{ to be meaningful}$$

$$\text{and } p = \frac{n_i^2}{n}$$

How should we choose the sign?

"+" sign should be chosen so that n>0 – a physical solution

The number of electrons per volume (n) is always positive!

for numerical reasons, we use this only when $N_D - N_A > 0$

For $(N_D - N_A) >> 2n_i$, $n \cong (N_D - N_A)$.

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How many electrons and holes are there in a cubic cm pure silicon at 300K? (assuming ni=1e10/cm3)

N=? 10^{10}

P=? 10^{10}

What if we add Phosphorous, at a concentration of 10^{16}/cm3 $\gg 2\times10^{10}$

N=?

P=?

$$n = \frac{10^{16} + \sqrt{(10^{16})^2 + (2\times10^{10})^2}}{2}$$

$$= 10^{16}/cm^3$$

$$p = \frac{n_i^2}{n} = 10^4/cm^3$$

p-type example

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P-type material example (Nd – Na <0)

What is n and p if Nd = 0, Na=1e17?

$$N_{dop} = N_d - N_a = -10^{17}/cm^3$$

$$n = \frac{N_{dop} + \sqrt{N_{dop}^2 + 4n_i^2}}{2}$$

$$= \frac{-10^{17} + \sqrt{(10^{17})^2 + 4\cdot 10^{20}}}{2}$$

$$p = \frac{n_i^2}{n}$$

We now need to take difference of two very large numbers, that difference gets lost easily in computers due to truncation errors! Try to do this calculation yourself with calculator and see what you get.

Solution: solve for the larger number "p" first or try to do something to your equation to avoid taking such difference, can you think of how?

$$n = \frac{-10^{17} + \sqrt{(10^{17})^2 + 4\cdot 10^{20}}}{2}$$

$$= \frac{\left(-10^{17} + \sqrt{(10^{17})^2 + 4\cdot 10^{20}}\right)\left(-10^{17} - \sqrt{(10^{17})^2 + 4\cdot 10^{20}}\right)}{2\cdot\left(-10^{17} - \sqrt{(10^{17})^2 + 4\cdot 10^{20}}\right)}$$

$$\left(-10^{17} - \sqrt{(10^{17})^2 + 4\cdot 10^{20}}\right)$$

$$= \frac{(10^{17})^2 - \left[(10^{17})^2 + 4\cdot 10^{20}\right]}{2\left(-10^{17} - \sqrt{(10^{17})^2 + 4\cdot 10^{20}}\right)}$$

$$= \frac{4\cdot 10^{20}}{2\left(10^{17} + \sqrt{(10^{17})^2 + 4\cdot 10^{20}}\right)}$$

$$\approx \frac{10^{20}}{10^{17}} = 10^{3}\ /cm^{3} \quad \text{as} \quad 10^{17} \gg 2n_i$$

$$(n_i \approx 10^{10})$$

A better way is to always solve for the larger number of "n" and "p" first, as shown below, to avoid the numerical issue above. We already know from previous analysis that if the net doping is p-type, p > n, so we can solve for "p" first as shown below, and then use pn = ni^2 to solve for "n"

Example: solve for p and n in p-type material where N_d-N_a<0.

- Weapons: **charge neutrality** and **mass action law**.

$$p - n + N_D - N_A = 0$$

$$p - \underline{n_i^2} + N_D - N_A = 0 \qquad pn = n_i^2$$

$$p^2 - n_i^2 + (N_D - N_A)\cdot p = 0$$

$$p = \frac{(N_A - N_D) + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2}$$

$$\frac{10^{17} + \sqrt{(10^{17})^2 + 4\cdot 10^{20}}}{2}$$

$$N_A - N_D > 0$$

$$= 10^{17}$$

$$n = 10^3$$

General solution strategy

Sunday, April 22, 2012

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$\underline{\text{for given } N_d,\ N_a}$

If $N_d > N_a$

$$n = \frac{N_d - N_a + \sqrt{(N_d - N_a)^2 + 4n_i^2}}{2}$$

$$p = n_i^2/n$$

If $N_d - N_a \gg 2n_i$, $n \approx N_d - N_a$

$p = n_i^2/n$

If $N_a > N_d$

$$p = \frac{N_a - N_d + \sqrt{(N_a - N_d)^2 + 4n_i^2}}{2}$$

$$n = n_i^2/p$$

If $N_a - N_d \gg 2n_i$

$$(N_a - N_d)^2 \gg 4n_i^2$$

$$p \approx N_a - N_d$$

$$n = n_i^2/p = n_i^2/(N_a - N_d)$$

Practical doping level

Sunday, May 20, 2012
9:35 AM

- **Typical doping level ranges from $10^{14}/cm^3$ to $10^{21}/cm^3$.**

The other effect of doping

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The other effect of Doping -

- if we add donor Phosphorous to Si, at a concentration of $10^{16}/cm^3$
  - n0=1e16 $/cm^3$
  - Compared to n0 = ni = 1e10/cm^3 in an intrinsic crystal, electron concentration is increased by 6 orders of magnitude
  - p0 = ni^2 / n0 = (1e10)^2 / 1e16 = 1e4 $/cm^3$
  - Compared to p0 = ni = 1e10/cm^3 in an intrinsic crystal, hole concentration is decreased by 6 orders of magnitude
- It is thus logical to say that , in a n-type crystal, electrons are the majority carriers, while the holes are the minority carriers!

Doped Silicon Carrier Concentrations Summary

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- If **n > p**, the material is **n-type**.
  If **p > n**, the material is **p-type**.
- N_d = donor impurity concentration
  atoms/cm3 –
  N_a = acceptor impurity concentration
  atoms/cm^3
- Ionized donors are positvely charged
- Ionized acceptors are negatively charged
- Ndop = N_d - N_a is called Net Doping
- $n - p = N_d - N_a, p - n = N_a - N_d$
- typically both N_D and N_A exist, this is called "**compensation**“
- **there is no net charge anywhere in sample – charge neutrality,**
- **Charge neutrality requires q(N_d - N_a + p - n) = 0**
- **Mass action law requires pn = n_i^2**

Band diagram essentials for uniform samples

Tuesday, August 21, 2012
9:52 AM

Band diagram - uniform sample at equilibrium

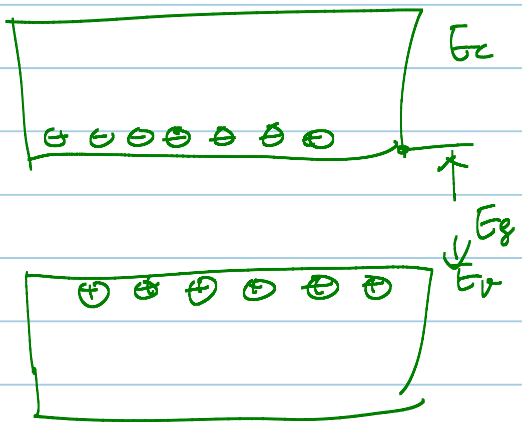
Wednesday, August 15, 2012
5:43 PM

Ec: bottom of conduction band
Ev: top of valence band
Ef: Fermi-level - it **only exists for equilibrium,**

Eg = Ec - Ev is bandgap, measured typically in eV.

1eV = 1.6e-19C * V = 1.6e-19 J

Band edges E_c E_v ,



$E_g = E_c - E_v$

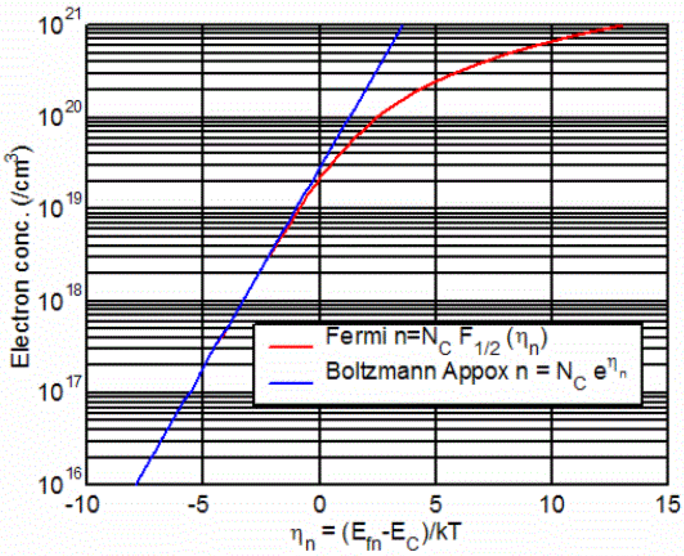
Electrons and "Holes"

Ef represents the state of an equilibrium uniform sample.

From Ef, we know both p and n.

For low doping levels (<1e17 barely useful for real devices), we can use Boltzmann approximation of "p" and "n".

For n-type case:



n is overestimated by Boltzmann Approximation when fermi level is within 3kT of Ec

$$n = N_c e^{\eta} \qquad \eta = \frac{E_f - E_c}{kT}$$

Fermi-dirac integral is defined by

$$F_{\frac{1}{2}}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} \frac{x^{\frac{1}{2}}}{1+e^{x-\eta}} dx$$

"n"

can be derived as

$$n = N_c F_{\frac{1}{2}}(\eta_n) \qquad \eta = \frac{E_f - E_c}{kT}$$

A similar derivation for holes yields

$$p = N_v F_{\frac{1}{2}}(\eta_p) \qquad \eta_p = \frac{E_v - E_f}{kT}$$

The Fermi integral of order $\frac{1}{2}$

Cannot be evaluated explicitly.

Intrinsic "p" "n" and "Ef"

Tuesday, August 21, 2012
10:03 AM

intrinsic carrier concentration ni

without any doping, n=p this is called n_i, the intrinsic carrier concentration

$$p = N_v \exp\left(-\frac{E_f - E_v}{kT}\right) \quad ①$$

$$n = N_c \exp\left(-\frac{E_c - E_f}{kT}\right) \quad ②$$

$E_c - E_v =$ E_g (fundamental material properties)

Observe that the np product

① × ② ⇒

$$np = N_c N_v \exp\left(-\frac{E_c - E_v}{kT}\right) = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$

N and P in Pure semi

Pure. n and p are equal (thermal generation)

$n = p \Rightarrow$ define $n_i \triangleq n = p$

$$n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right).$$

$E_g = 1.12\,eV$

$kT = 0.0256\,eV$

$n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$.

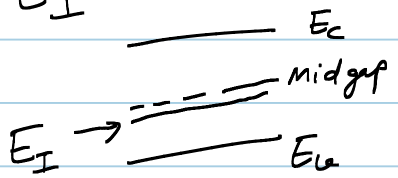
$kT = 0.0256\,eV$

$N_c =$

$N_v =$

$n_i \approx 10^{10}/cm^3$, Si at RT (300K)

E_f in Pure semi $\rightarrow E_I$



$E_I \hat{=} E_f$ when $n=p=n_i$

$=$

$$\rightarrow \left.\begin{aligned} n_i &= N_c e^{-(E_c - E_I)/kT} \\ n_i &= N_v e^{-(E_I - E_v)/kT} \end{aligned}\right\} \Rightarrow E_I = \frac{1}{2}\left[E_c + E_v + kT \ln\frac{N_v}{N_c}\right]$$

take log, then subtract

this is referred to as Ei, the intrinsic Fermi level corresponding to n=p=ni in equilibrium

Ei is frequently used as a reference in energy level for extrinsic (doped) semiconductors - particularly in traditional Si textbook

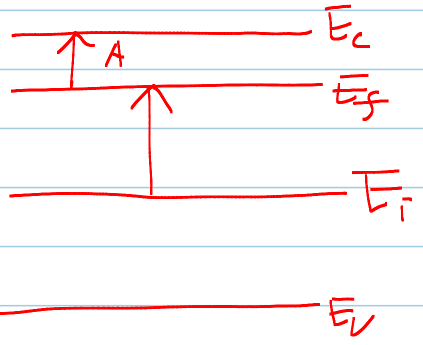
Ei, however, is much less useful for heterojunction devices, like in a graded base

Band diagrams for n-type and p-type

Wednesday, August 15, 2012
5:45 PM

n_i, E_i Referenced n & p Expressions

① $n = N_c e^{-\frac{E_c - E_f}{kT}}$



② $n_i = N_c e^{-\frac{E_c - E_i}{kT}}$

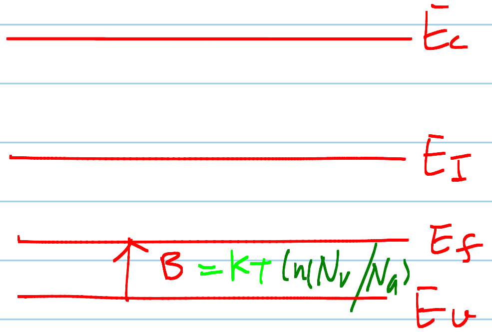
$\frac{①}{②} \Rightarrow \frac{n}{n_i} = e^{\frac{E_f - E_i}{kT}}$

$n = n_i e^{\frac{E_f - E_i}{kT}}$

n_i, E_f referenced

Similarly $p = n_i e^{\frac{E_i - E_f}{kT}}$

Recall that $p = N_v e^{-\frac{E_f - E_v}{kT}}$



Since $p \approx N_a$

$B \triangleq (E_f - E_v) = kT \ln \frac{N_v}{N_a}$

B is a log function of N_v/N_a

$$E_f = E_v + B$$
$$= E_v + kT \ln \frac{N_v}{N_a}$$
$$= E_v - kT \ln \frac{N_a}{N_v}$$

$N_a\downarrow$, $B\uparrow$ $E_f\uparrow$

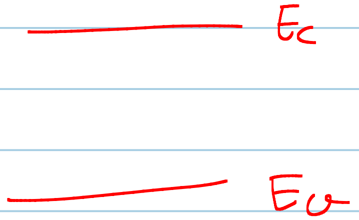
$N_a\uparrow$, $B\downarrow$, $E_f\downarrow$

p-type band diagram drawing - Ev referenced

Wednesday, August 15, 2012
5:49 PM

p-type Band Diagram Drawing method ①

① draw E_c, E_v, ⇒

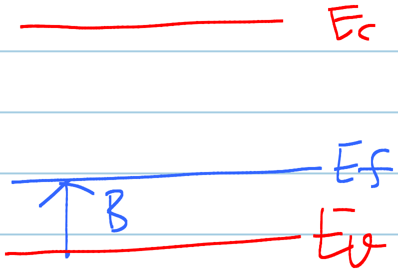


①

② determine

$B \triangleq E_f - E_v$

$= kT \ln \frac{N_v}{N_a}$



p-type band diagram drawing - Ei referenced

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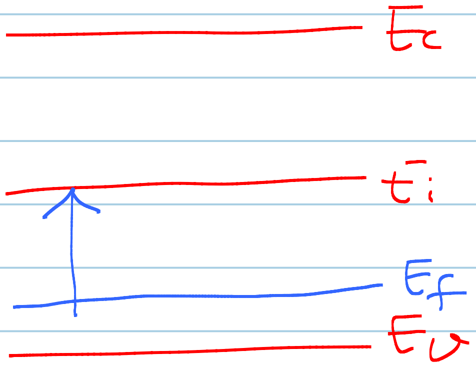
p-type B.D. drawing (method 2)

① E_c, E_v, E_T

determine

② $(E_i - E_f) = k_T \ln \frac{p}{n_i}$



* method ① and ② are completely equivalent

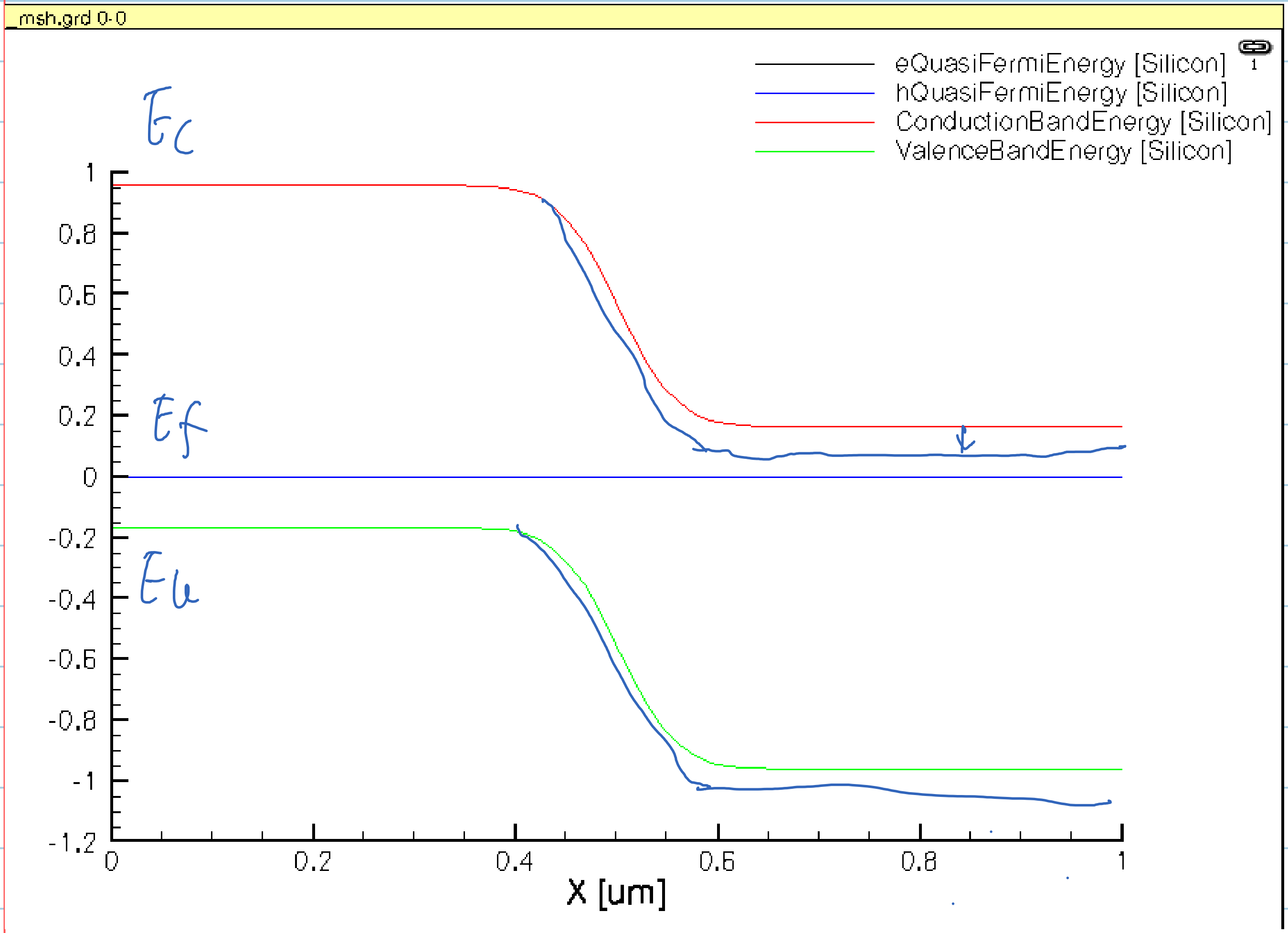
* both are useful in semiconductor analysis

* For heterojunction, method ① is more useful

N-type / P-type Band Diagram Quiz, Aug 23 lecture

Thursday, August 23, 2012
9:12 AM

The band diagram of a Pn junction under zero bias is shown below.



Note the blue is Fermi level.

In-class Quiz:

For the same equilibrium PN junction problem given above, let us fix the Ef (shown in blue line) at 0 eV, on top of the band diagram, draw a new band diagram for

- Another pn junction with higher Nd than this one
- Another pn junction with higher Na than this one

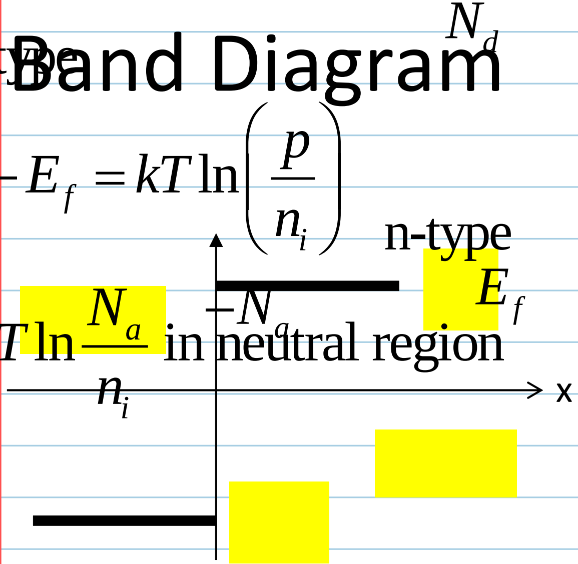
Verify your answers by modifying the TCAD example provided.

Sunday, August 26, 2012
12:16 PM

Abrupt PN Junction Band Diagram

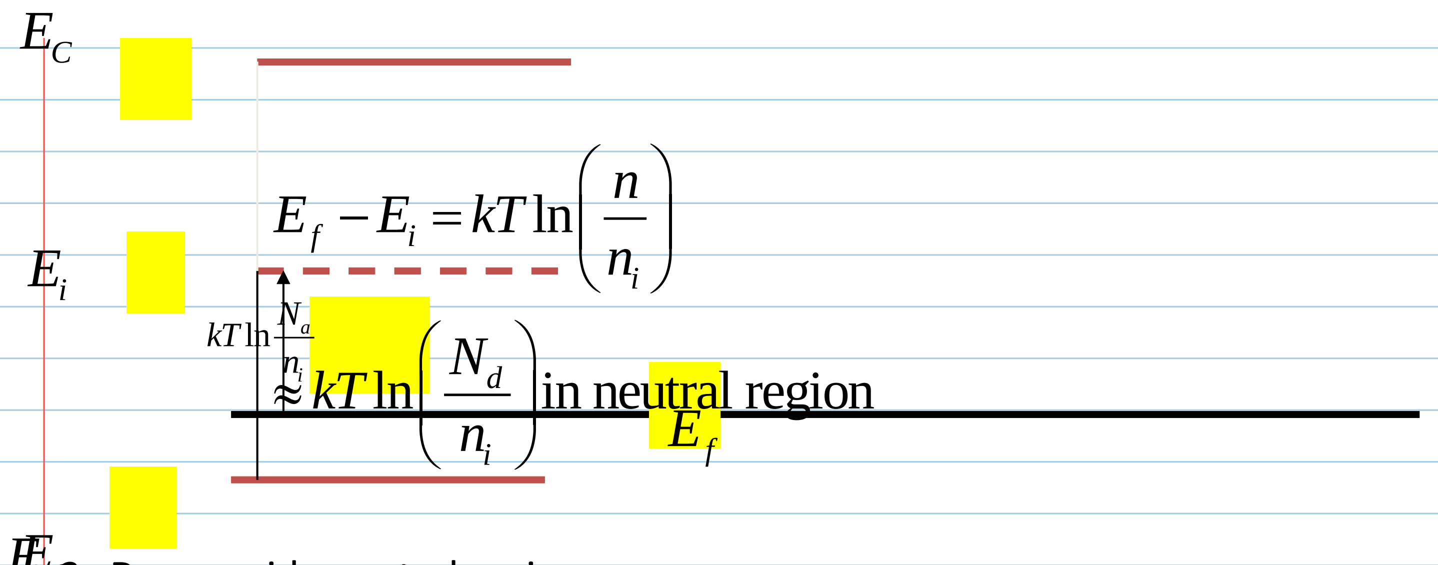
$$E_i - E_f = kT\ln\left(\frac{p}{n_i}\right)$$

$$\approx kT\ln\frac{N_a}{n_i} \text{ in neutral region}$$



1. Lay down Ef

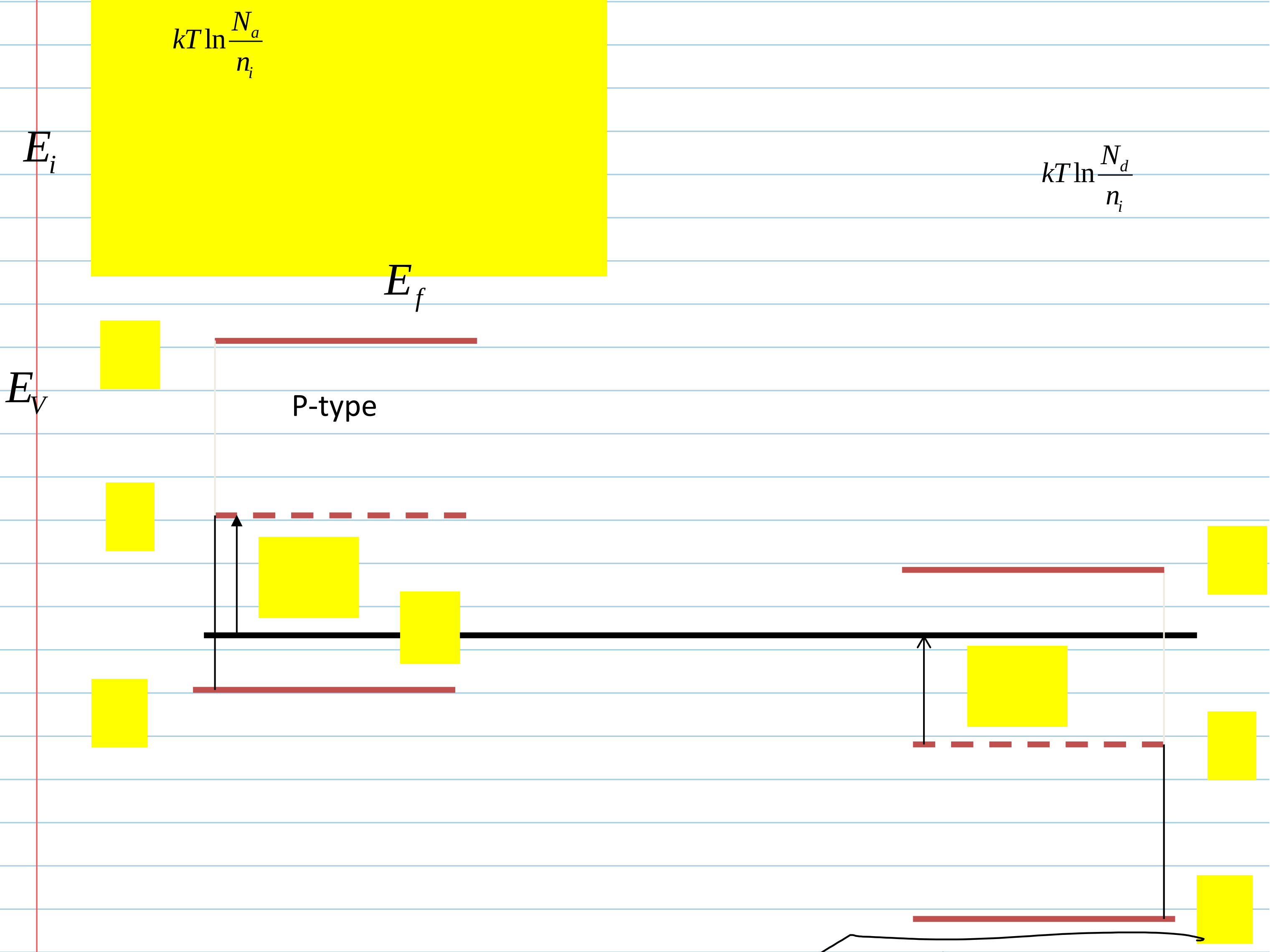
2. Draw p-side neutral region band diagram - just regular p-type band diagram!



$$E_f - E_i = kT\ln\left(\frac{n}{n_i}\right)$$

$$\approx kT\ln\left(\frac{N_d}{n_i}\right) \text{ in neutral region}$$

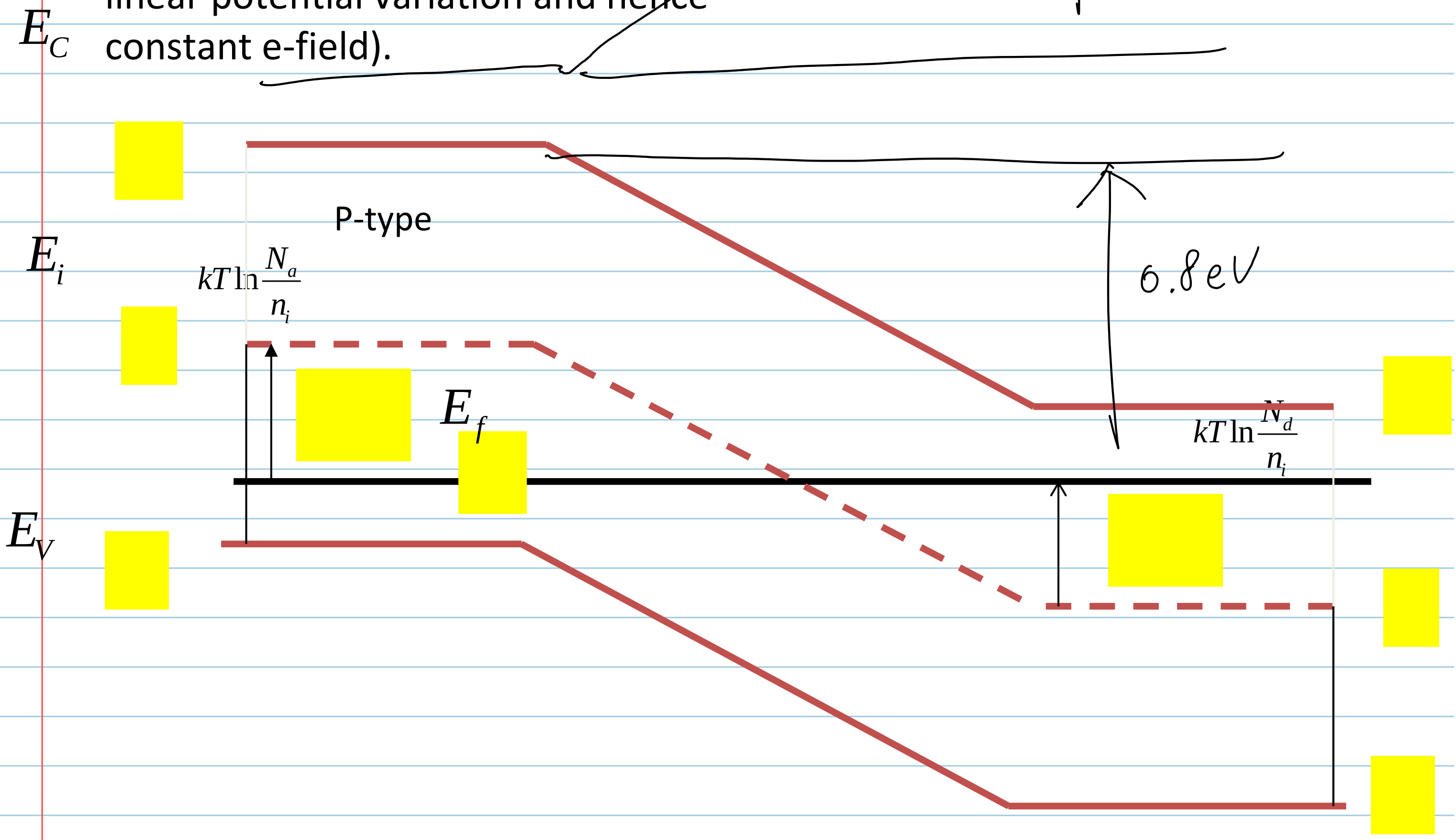
3. Draw n-side neutral region



4. Connect the two neutral regions with transition region. For simplicity, just use straight lines (but do not interpret it as linear potential variation and hence

0.8V

straight lines (but do not interpret it as linear potential variation and hence constant e-field).

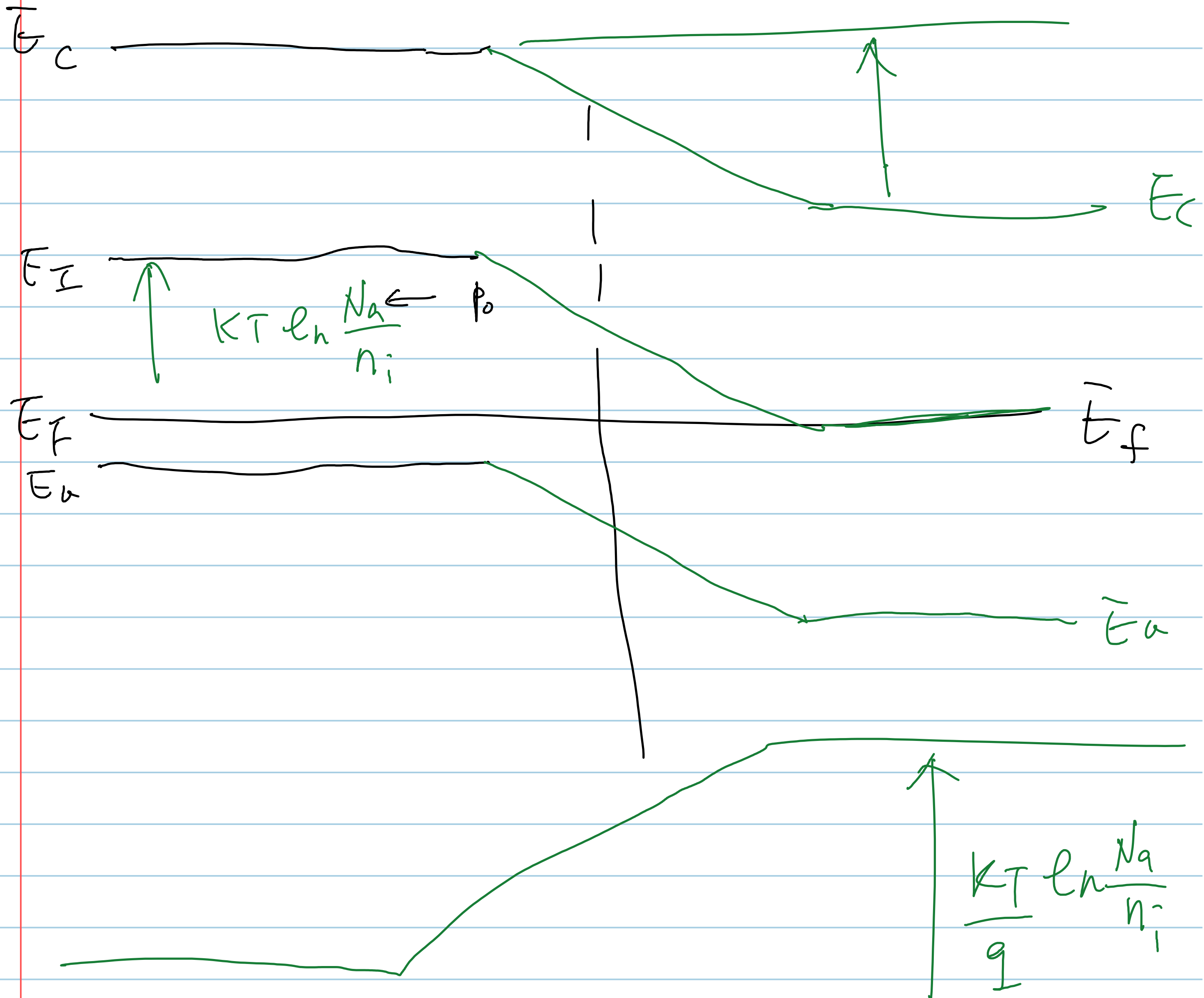


PI Junction exercise

Sunday, August 26, 2012
12:23 PM

Draw p-type to intrinsic junction band diagram on your own.

What is the built-in potential drop from p-type to intrinsic?



$$-\frac{kT}{q} \ln \frac{p_o}{n_i}$$

ϕ_{bi} (from p to intrinsic)

$$= \frac{E_F - E_I}{q}$$

$$\frac{E_I - E_F}{q} \triangleq \phi_f$$

NI Junction exercise

Sunday, August 26, 2012
12:23 PM

Draw n-type to intrinsic junction band diagram on your own.

What is the built-in potential drop from n-type to intrinsic?

Built-in potential to intrinsic reference

Tuesday, August 28, 2012
1:05 PM

So the built-in potential to an intrinsic reference is

$$\phi_{bi}(\text{intrinsic reference}) = \frac{E_F - E_I}{q}$$

Drift

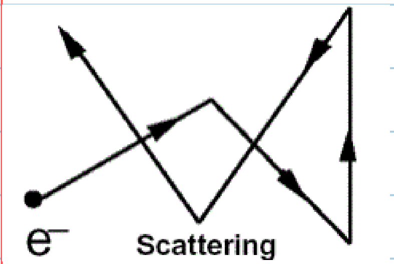
Sunday, April 22, 2012
1:44 PM

Drift – movement of charged particle in electric field

- In presence of electric field, electrons experience a force –qE, and thus drift against electric field
- In ELEC5700, we will show that such velocity is linearly proportional to electric field
- The proportionality constant is called mobility
- If we use cm/s for velocity, V/cm for e-field, what is the unit of mobility?

Random thermal motion

Thursday, May 24, 2012
7:49 PM



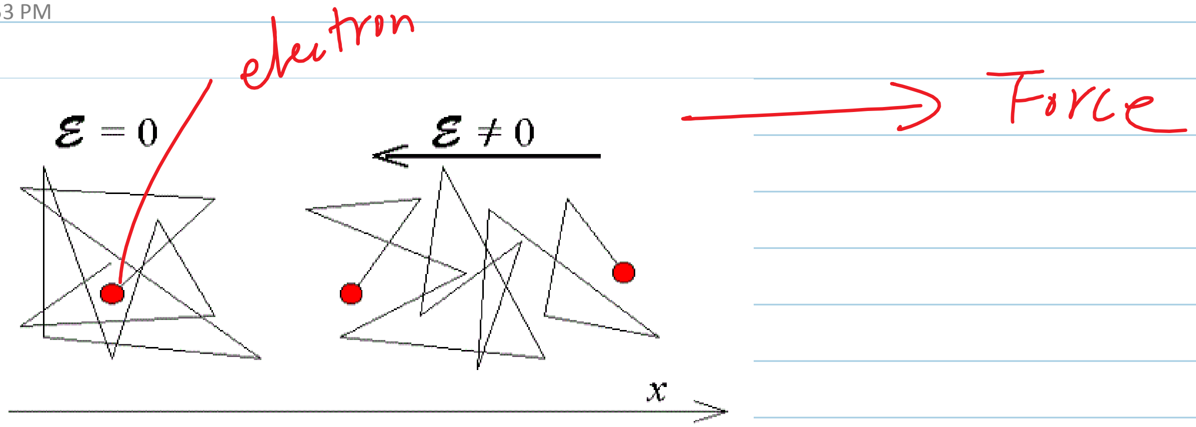
In thermal equilibrium, mobile electrons in the conduction band or holes in the valence band will be in random thermal motion, through scattering from lattice vibrations, impurities, other electrons, or defects. From statistical mechanics,
thermal energy = kT/2 per degree of freedom
where k = Boltzmann's constant

At 300K, in Si, thermal velocity is on the order of 10^7cm/s.

This is pretty fast! About 223693 miles per hour (mph).

Directed motion on top of random thermal motion

Thursday, May 24, 2012
7:53 PM



Drift by definition means the motion of charged particles driven by electric field.

From our earlier discussion on carrier motion, the force acting on -q charged electron tends to accelerate electron in the direction of the force - opposite to direction of field.

The acceleration, however, is frequently interrupted by collisions with vibrating atoms (or phonons in quantum terms), ionized dopants (Columb scattering), other electrons or holes etc.

It is important to realize that the drift velocity is often much less than the thermal velocity - the random Brownian motion, at least at relatively low electric field.

The average thermal motion at a macroscopic level, however, is zero, due to the random nature of thermal motion.

Drift, however, is directional!

For current, we are interested in the directional "net" velocity.

Drift-current and In-class questions

Sunday, April 22, 2012
1:44 PM

Drift – motion of charged particle in electric field

- In presence of electric field, electrons experience a force –qE, and thus drift against electric field, similarly holes drift along electric field

- Unlike in vacuum, electron or hole velocity keeps on increasing over time in presence of electric field. Instead, acceleration occurs over a short time period only, as directed motions are frequently interrupted by vibrating Si atoms or charged dopants.

Mobility

- In ELEC5700 - semiconductor fundamentals, we will show that average drift velocity of carriers is linearly proportional to electric field - this is the physics behind Ohm's law, as we will show below on next page.
- The proportionality constant is called mobility

☆ If we use cm/s for velocity, V/cm for e-field, what is the unit of mobility?

$$\underset{\substack{\uparrow \\ \text{cm/s}}}{v_n} = -\mu_n \underset{\substack{\uparrow \\ \text{V/cm}}}{E} \qquad v_p = \underset{\text{cm}^2/\text{V}\cdot\text{s}}{\mu_p} \cdot E$$

Drift velocity of holes

- Holes are +q charged, and thus drift along electric field

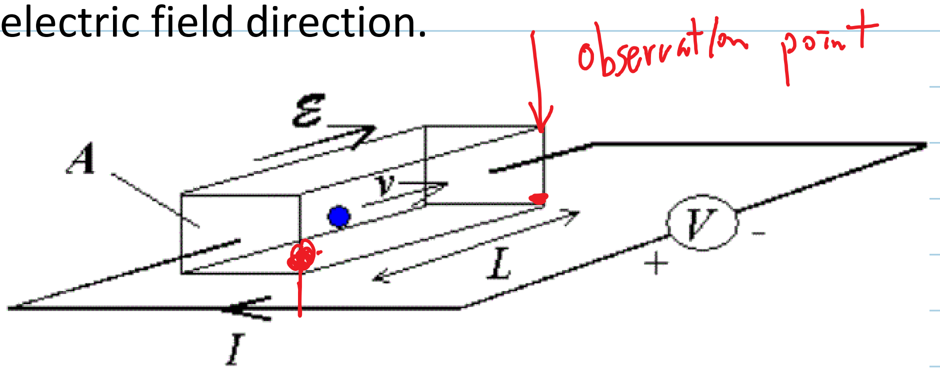
$\oplus \rightarrow v_p$

$\longrightarrow E$

- Thus hole drift velocity is given by

$v_{p,drift}=\mu E$

Consider a uniform bar, hole density is p, a voltage is applied as shown. Holes move at velocity v. Cross sectional area is A, length is L in the electric field direction.



$$\frac{q\,p\cdot A\cdot L}{\frac{L}{v_p}} = I$$

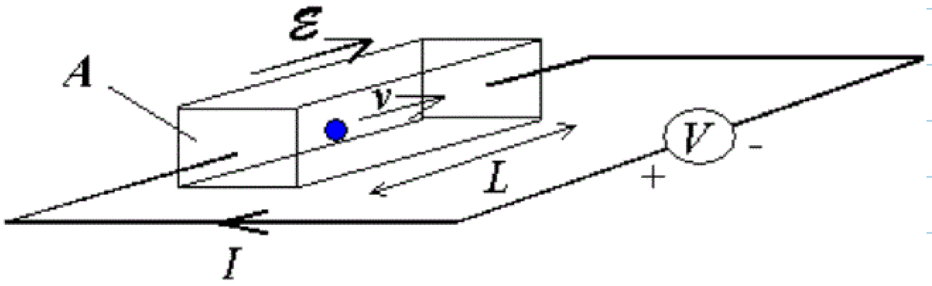
What is the total amount of hole charge in this bar? hole concentration is p.

How long does it take a hole to travel across the length of L? $\frac{L}{v_p}$

If you stand at the end of the bar, how much current do you observe?

What is the current per cross sectional area (current density)?

Drift Current and Drift Current Density Derivation

- Current = charge flowing through per unit time
- Current = The amount of charge / the amount of time taken for all the charges to move out
- 

$$I = \frac{Q}{t_r} = \frac{qpAL}{\frac{L}{v}} = Aqpv$$

$p: positive\ partical\ volume\ density$

$$Current = \frac{total\ charge\ inside\ the\ bar}{time\ of\ travel}$$

- Current density = current / cross section area

$J = \frac{I}{A} = qpv$ for positive charge
$J_n = -qnv$ for electrons
$J_p = qpv$ for holes

Drift Current Densities and Conductivity

$$v_{n,drift} = -\mu_n E$$
$$v_{p,drift} = \mu_p E$$
$$J_{n,drift} = -qnv = qn\mu_n E$$

$J_{p,drift} = qpv = qp\mu_p E$

$J_{total,drift} = (qn\mu_n + qp\mu_p)E = \sigma E$

$\sigma = qn\mu_n + qp\mu_p$ *is called conductivity*

Derivation of Ohm's law

Sunday, April 22, 2012
1:49 PM

$$J = q n \mu_n E + q p \mu_p E$$

$$= \sigma \cdot E \qquad \sigma = q n \mu_n + q p \mu_p$$

$$\frac{I}{A} = \sigma \cdot \frac{V}{L} \qquad \rho = \frac{1}{\sigma} \qquad \Rightarrow$$

$$\frac{I}{V} = \sigma \cdot \frac{A}{L} \qquad \Rightarrow$$

$$\frac{V}{I} = \frac{1}{\sigma} \cdot \frac{L}{A} = \rho \cdot \frac{L}{A}$$

Deriving Ohm’s Law

- Now we have established the relation between current density J and electric field E

$$\sigma = qn\mu_n + qp\mu_p$$

$$J = \frac{I}{A} = \sigma E = \frac{\sigma V}{L}$$

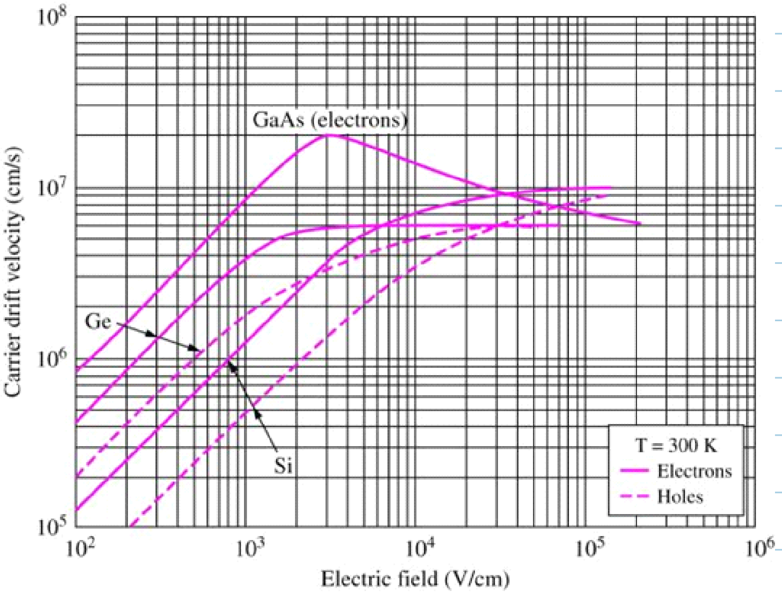
$$R = \frac{V}{I} = \frac{1}{\sigma}\frac{L}{A} = \rho\frac{L}{A}$$

$resistivity\ is\ thus\ the\ reciprocal\ of\ conductivity$

$$\rho = \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$$

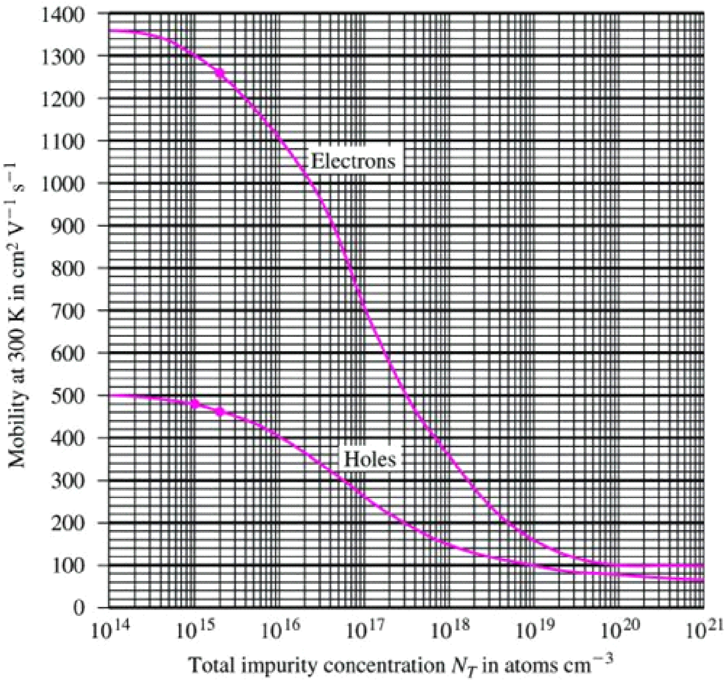
Velocity Saturation

At high fields, carrier velocity saturates and places upper limits on the speed of solid-state devices.



Mobility and Resistivity in Doped Semiconductors (well - this is for typical textbooks, not

too useful for active regions of transistors - you should use this only for resistance calculation, not for transit time etc)



Mobility approximations

$$\mu_n = 92 + \frac{1270}{1 + \left(\frac{N_T}{1.3 \times 10^{17}}\right)^{0.91}}$$

$$\mu_p = 48 + \frac{447}{1 + \left(\frac{N_T}{6.3 \times 10^{16}}\right)^{0.76}}$$

Note that the total doping means **Nd + Na**, very different from net doping, which is Nd-Na.

Reason is simple, both ionized donors and acceptors can scatter the motion of electrons / holes through Columb force

Important resistances/resistivity in transistors

Thursday, August 16, 2012
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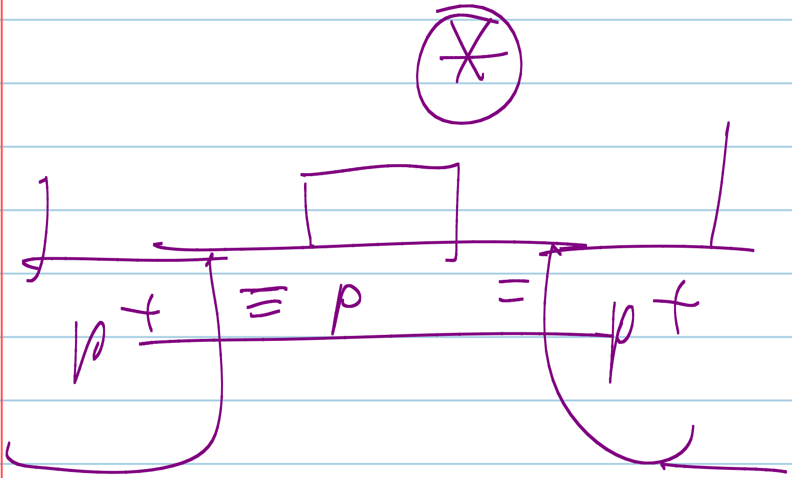
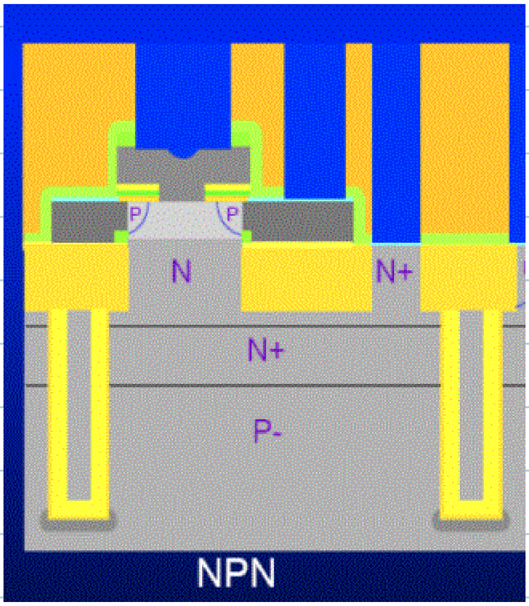
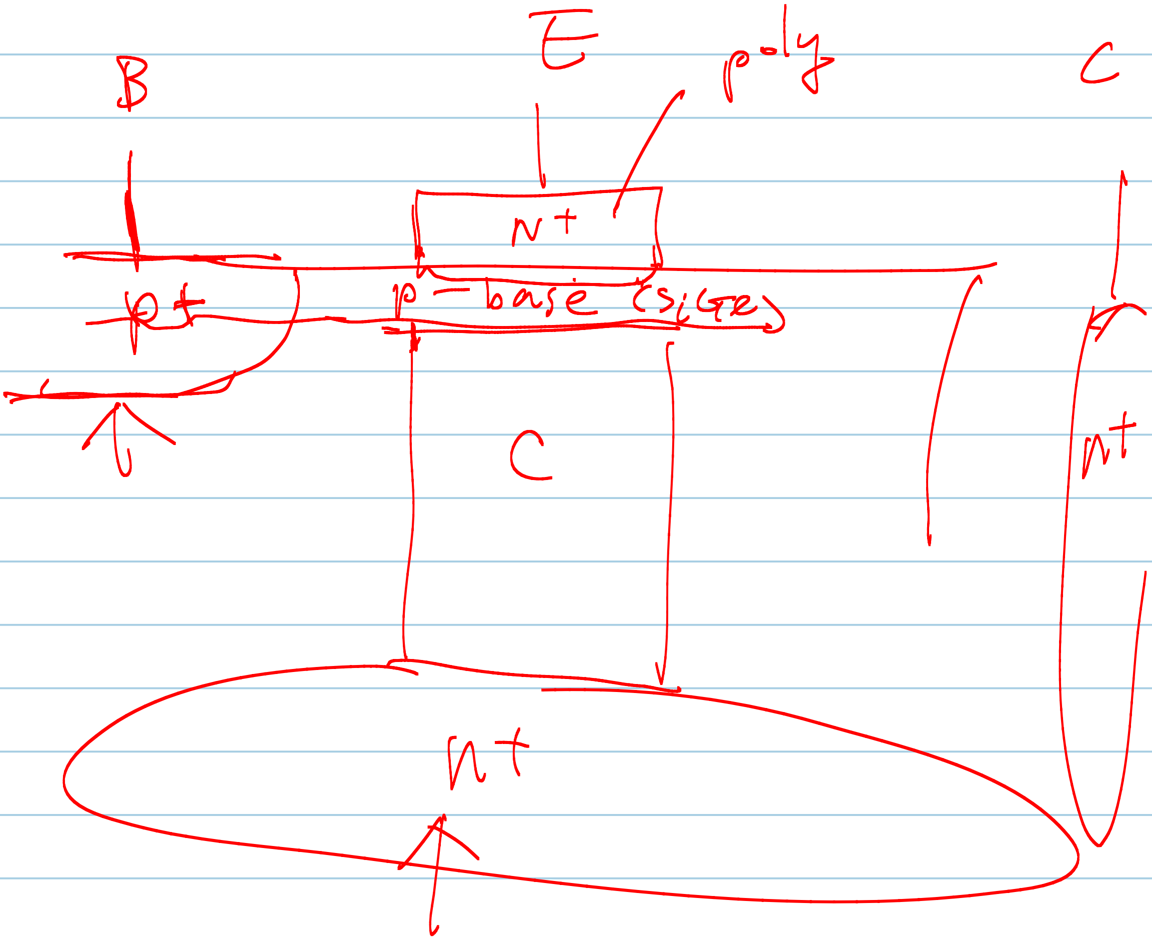
substrate resistivity (loss at RF etc)
intrinsic base sheet resistance
extrinsic base sheet resistance
n+ buried layer sheet resistance
collector sheet resistance






B
E
poly
C
n+
p+
p-base (SiGe)
C
n+
n+

examples

Wednesday, May 23, 2012
12:13 AM

Example of electron drift current density calculation.

n-type

In a uniform Si sample doped with arsenic at 10^{17}/cm3, find vn(drift) and Jn(drift) in an electric field of 2500V/cm. Assume electron mobility is 700cm^2/(V s).

$v_n = -\mu_n \cdot E$ $\qquad \mu_n = 700 \frac{cm^2}{V\cdot s}$ $\qquad E = 2500 \frac{V}{cm}$

$$= -700 \frac{cm^{\cancel{2}}}{\cancel{V}\cdot s} \cdot 2500 \frac{\cancel{V}}{\cancel{cm}} = -\square \ cm/s$$

$J_n = -q \cdot n v_n$ $\qquad$ n-type, $N_D = 10^{17}/cm^3 >> 2\times10^{10}/cm^3$

$n = ?$ $\qquad n = 10^{17}/cm^3$

$$v_n = -\mu_n \cdot E = -700 \frac{cm^2}{V\cdot s} \cdot 2500 \frac{V}{cm} = -1.75\times10^{6} \frac{cm}{s}$$

$J_n = -q \cdot n \cdot v_n$ $\qquad n = N_d$ because $10^{17} >> 2\cdot 10^{10}$

$$= -1.6\times10^{-19} C \cdot 10^{17} \frac{1}{cm^3} \left(-1.75\times10^{6}\right) \frac{cm}{s}$$

$$= 2.8 \times 10^4 \frac{C}{cm^2 \cdot s} = 2.8 \times 10^4 \, A/cm^2$$

In our text, $Q_n \overset{\Delta}{=} -qn$

$$Q_p \overset{\Delta}{=} qp$$

Example of hole drift current density calculation.

In a uniform Si sample doped with arsenic at 10^{17}/cm3, find vp(drift) and Jp(drift) in an electric field of 2500V/cm. Assume hole mobility is 200cm^2/(V s).

$$p = \frac{n_i^2}{n}$$

$$\vartheta_p = \mu_p \cdot E \qquad\qquad p = \frac{n_i^2}{n}$$

$$J = q \cdot p \cdot \vartheta_p \qquad\qquad 10^{20} \quad , \quad 3$$

$$J_p = q \cdot p \cdot v_p \qquad = \frac{10^{20}}{10^{17}} /cm^3$$

$$= 10^3 /cm^3$$

$\mu_p = 200\ cm^2/(V \cdot S)$, find v_p, J_p.

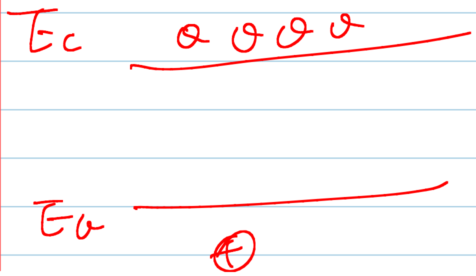
$$v_p = \mu_p E = 200 \frac{cm^2}{V \cdot S} \cdot 2500 \frac{V}{cm} = 5 \times 10^5\ cm/s$$

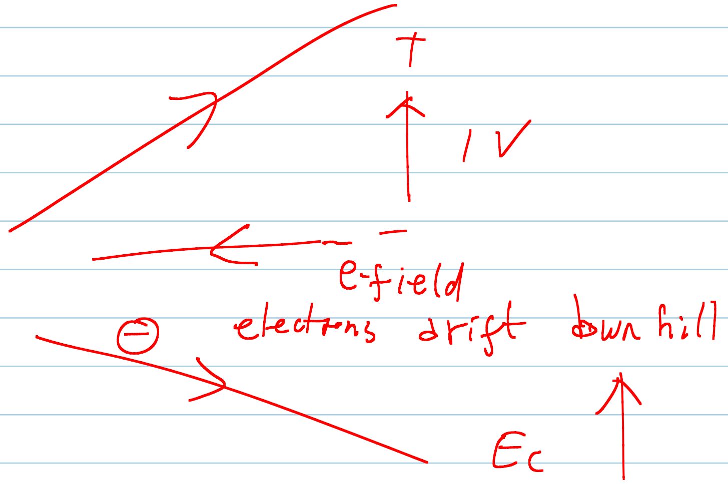
$$J_p = q \cdot p \cdot v_p \qquad p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d}$$

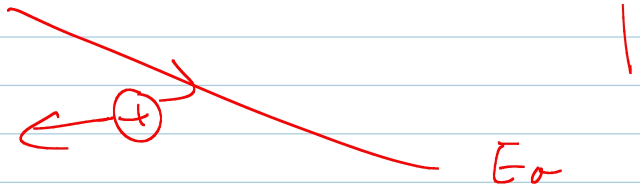
$$n_i = 10^{10} /cm^3$$

vp = 5 x 105 cm/s. Jp = 8 x 10-11 A/cm2

field 1V/µm







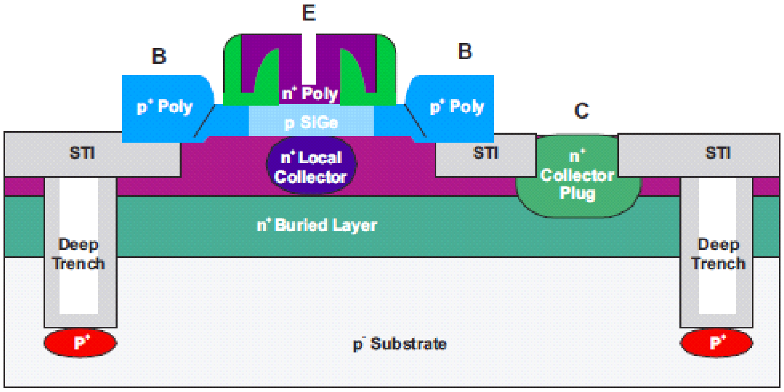
holes drift uphill

Example of collector resistance in HBT

Wednesday, August 15, 2012
5:32 PM

Consider a bipolar transistor. Emitter area is Ae. Collector epi layer doping is Nepi. Collector thickness (in the vertical direction along electron transport) is Wepi. Electron mobility is mun. Write down an expression for collector resistance.

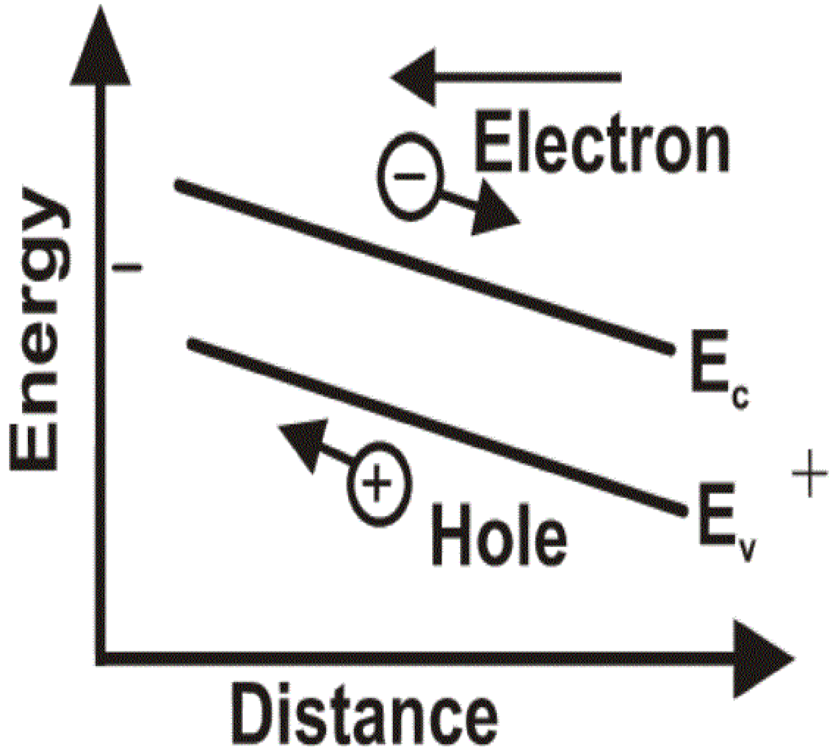
This is in fact the Rcv compact model parameter in Mextram.



Band diagram - with potential variation

Thursday, August 16, 2012
10:20 AM

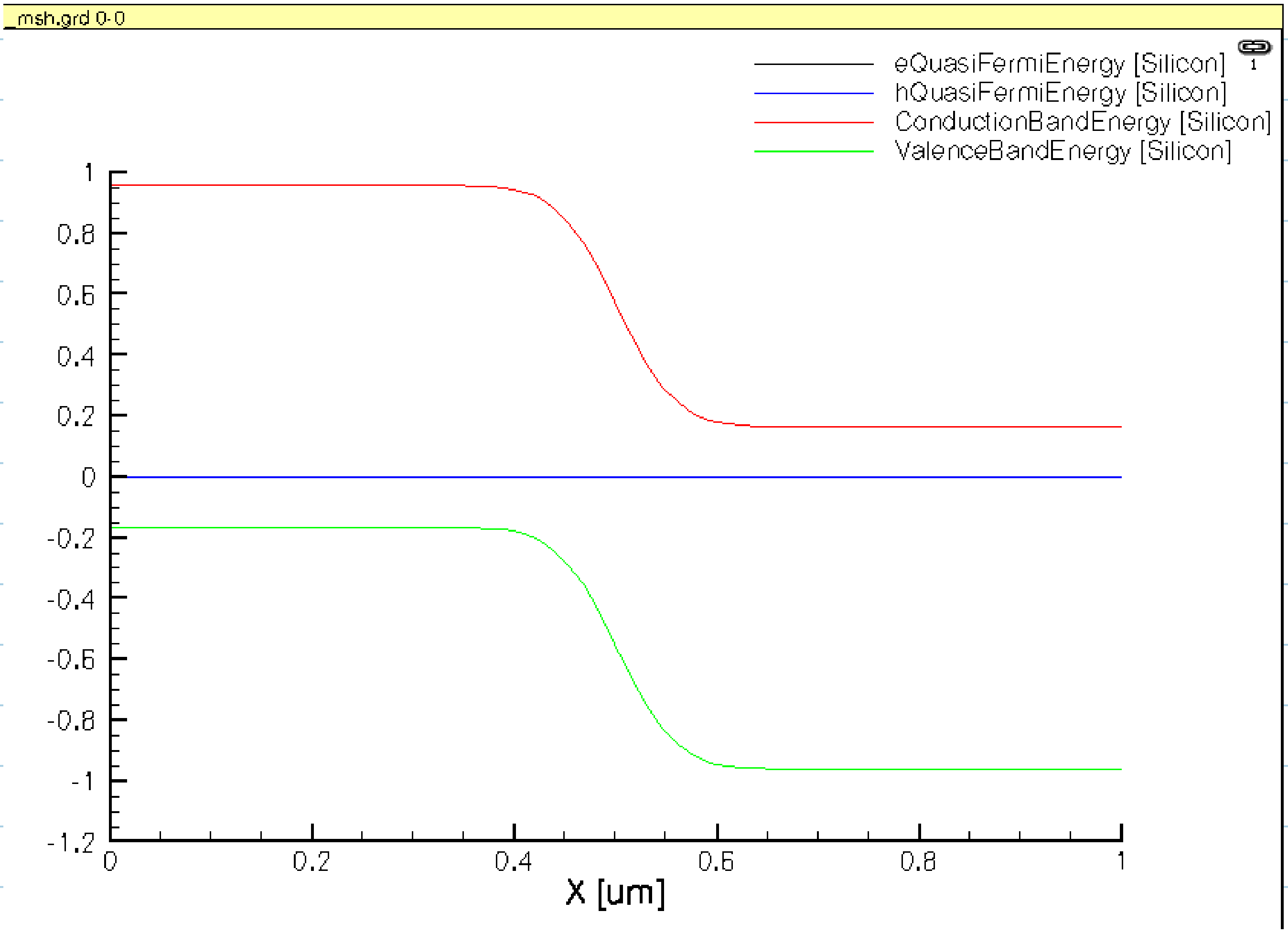
When an electric field is applied to a semiconductor, the bands are "tilted" and electrons and holes move in opposite directions leading to an electric current.

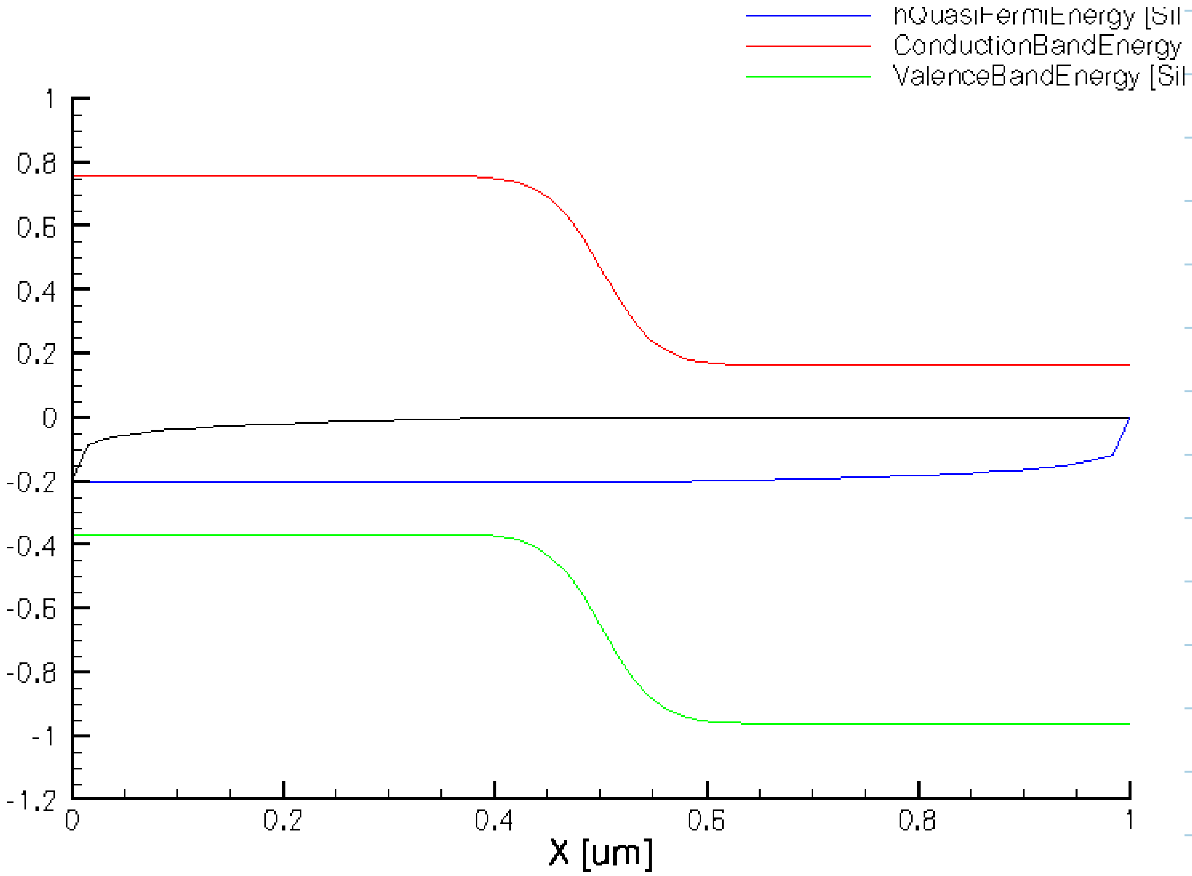


Electrons drift down hill

holes drift up hill

Pn junction under zero bias example:
Applied voltage V(p-contact) = 0 to 1.0V in 0.2V step.
V(n-contact) = 0 or ground.


_msh.grd 0-0
eQuasiFermiEnergy [Silicon]
hQuasiFermiEnergy [Silicon]
ConductionBandEnergy [Silicon]
ValenceBandEnergy [Silicon]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.2
0.4
0.6
0.8
1
X [um]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ConductionBandEnergy
ValenceBandEnergy [Si]
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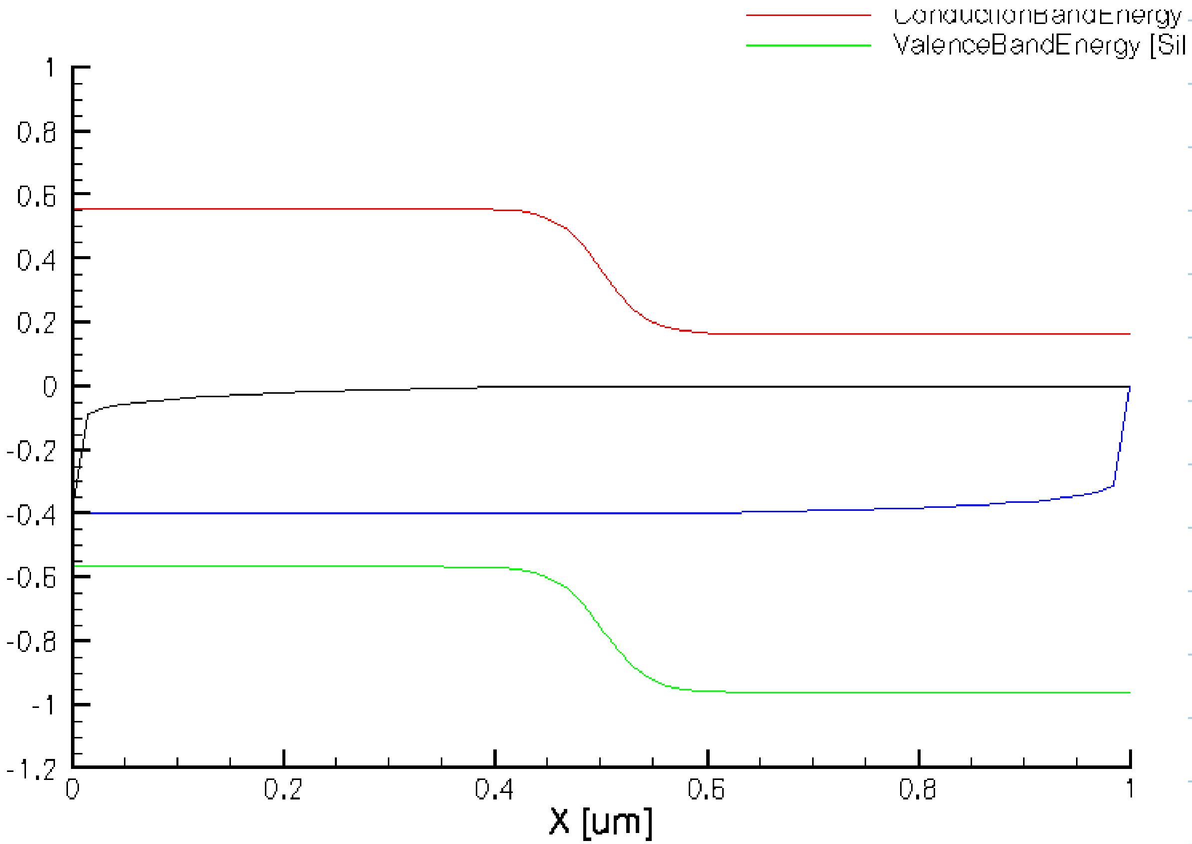
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.2
0.4
0.6
0.8
1
X [um]


ConductionBandEnergy
ValenceBandEnergy [Si]
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-0.8
-1
-1.2
0
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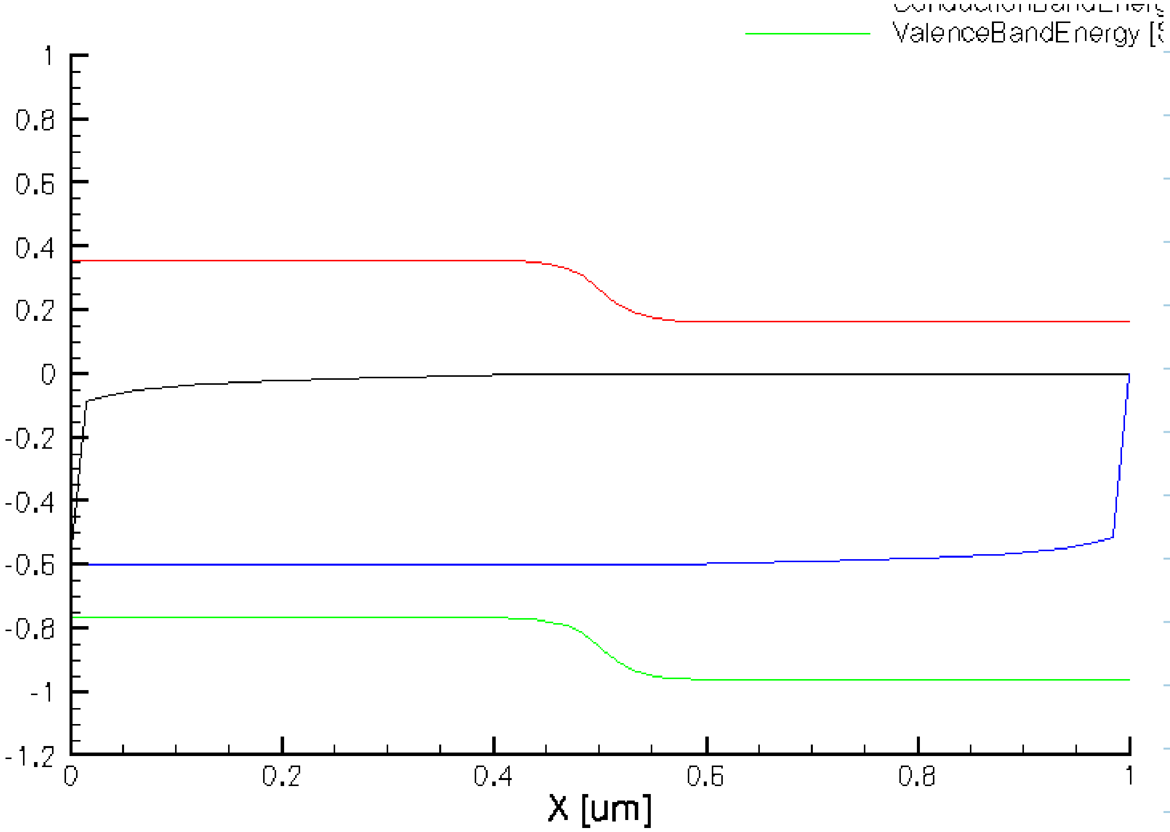
1
X [um]


ValenceBandEnergy [S
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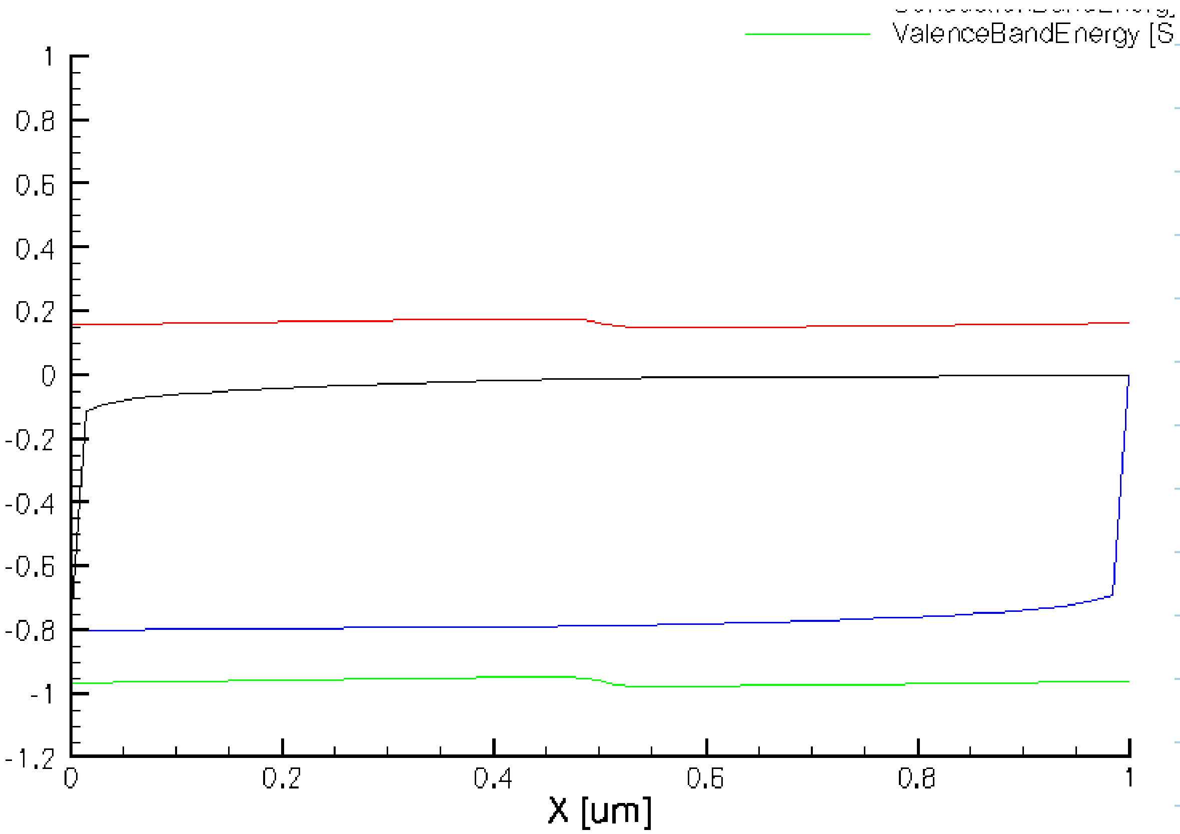
-1
-1.2
0
0.2
0.4
0.6
0.8
1
X [um]


ValenceBandEnergy [S
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0.4
0.2
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-0.2
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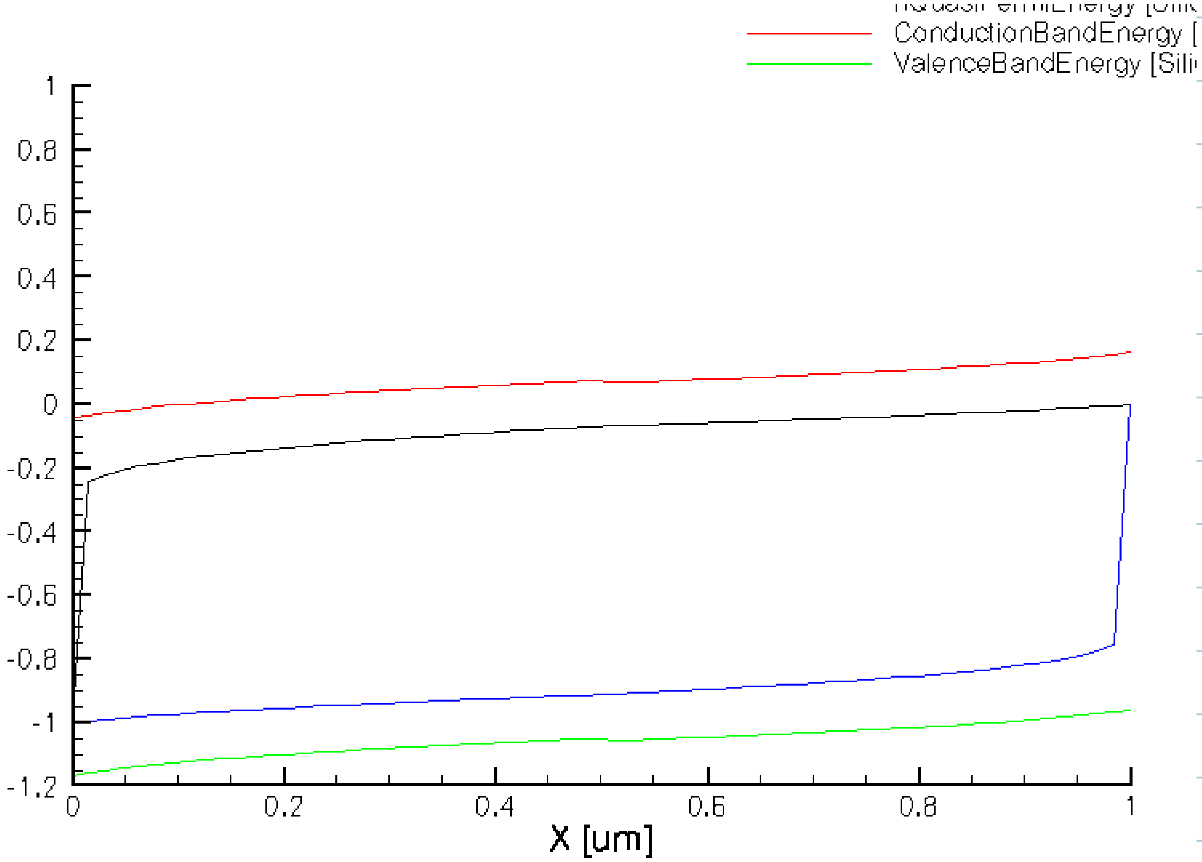
-1
-1.2
0
0.2
0.4
0.6
0.8
1
X [um]


nQuasiFermiEnergy [Silic
ConductionBandEnergy [
ValenceBandEnergy [Sili
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.2
0.4
0.6
0.8
1
X [um]

Band edges in presence of field

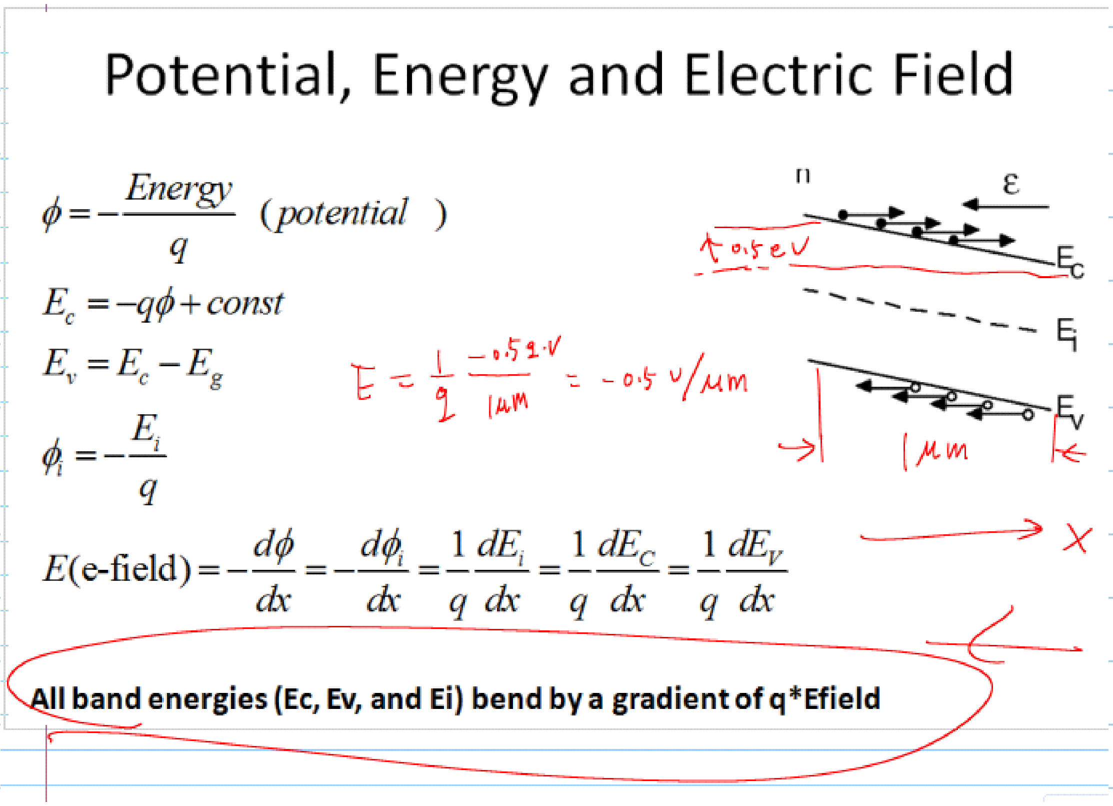
Thursday, August 16, 2012
10:34 AM

The bottom of the conduction band and the top of the valence band correspond to potential energies of electrons and holes, respectively. They are related to the electric potential, ϕ , as follows

$$E_c = -q\phi + const$$
$$E_v = -q\phi - E_g + const$$

Potential, energy and electric field

Thursday, August 16, 2012
10:26 AM



Potential, Energy and Electric Field

$$\phi = -\frac{Energy}{q} \quad (potential)$$

$$E_c = -q\phi + const$$

$$E_v = E_c - E_g$$

$$\phi_i = -\frac{E_i}{q}$$

$$E = \frac{1}{q}\frac{-0.5q \cdot V}{1\mu m} = -0.5\ V/\mu m$$

$$E(\text{e-field}) = -\frac{d\phi}{dx} = -\frac{d\phi_i}{dx} = \frac{1}{q}\frac{dE_i}{dx} = \frac{1}{q}\frac{dE_C}{dx} = \frac{1}{q}\frac{dE_V}{dx}$$

All band energies (Ec, Ev, and Ei) bend by a gradient of q*Efield

Diffusion

Sunday, April 22, 2012
1:54 PM

1. Diffusion concept
2. Diffusion current density expressions for electrons and holes
3. Electron diffusion illustration
4. Hole diffusion illustration
5. Einstein's relation
6. Numerical example

Diffusion Current Density

Sunday, May 20, 2012
9:55 AM

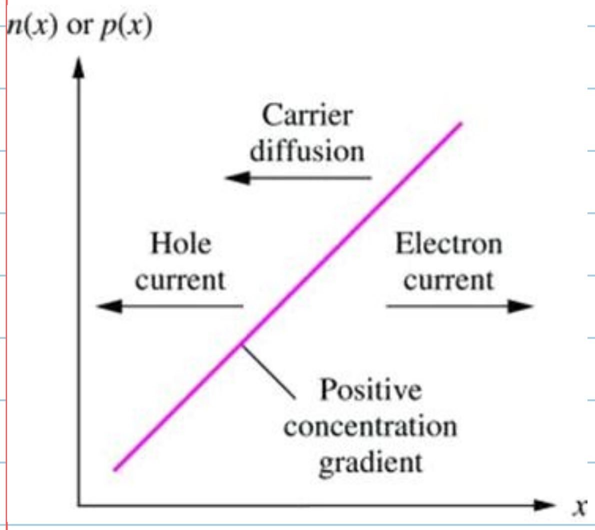
Diffusion Current Density

- Carriers diffuse from high concentration to low concentration due to random thermal motion
- diffusion current densities are proportional to the negative of the hole density gradient for holes and proportional to electron density gradient as follows:

$$J_n(diffusion) = qD_n \frac{dn}{dx}$$

$$J_p(diffusion) = -qD_p \frac{dp}{dx}$$

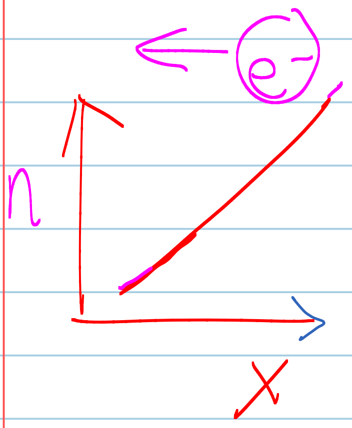
D_p and D_n are the hole and electron diffusivities with units cm^2/s.



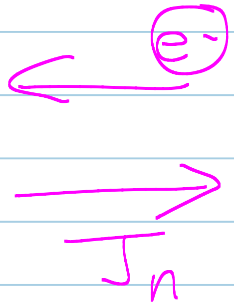
- **Minority carriers can produce large diffusion current even if their concentration is low!!! – this is the foundation of diodes and bipolar transistors!**
- **The gradient is what matters for diffusion current!!**

Electron diffusion

Sunday, May 20, 2012
9:59 AM



$$\frac{dn}{dx} > 0$$



$$J_n = q D_n \cdot \frac{dn}{dx}$$

Hole diffusion

Sunday, May 20, 2012
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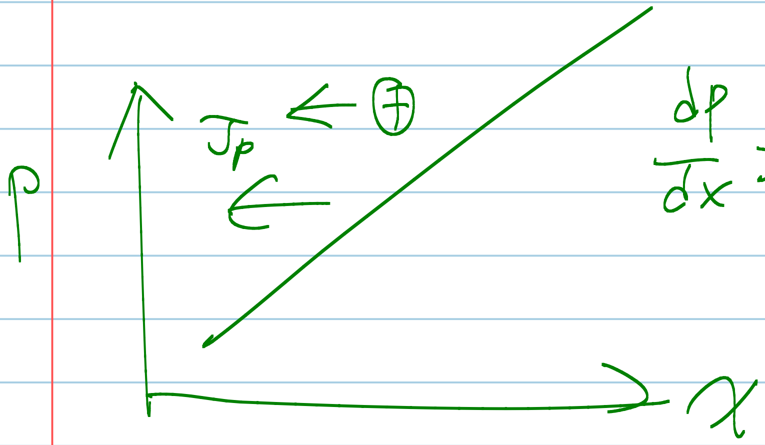
$$J_{p,diff} = -q D_p \cdot \frac{dp}{dx}$$



$$\frac{dp}{dx} > 0$$

Einstein's Relation

Sunday, May 20, 2012
10:00 AM

Einstein’s Relation

- Diffusivity and mobility are related by Einsteins’s relationship:

$$\frac{D_n}{\mu_n} = \frac{kT}{q} = V_T$$

$$\frac{D_p}{\mu_p} = \frac{kT}{q} = V_T$$

- The thermal voltage, $V_T = \frac{kT}{q}$, is approximately 25 mV at room temperature. This is worth remembering.

Example: Diffusion Constant Using Einstein's Relation

Find D for mu = 410cm^2/(V*s)

$$D = \mu \cdot \frac{KT}{q} = 410 \frac{cm^2}{V \cdot S} \cdot 25 \times 10^{-3} V$$

$$= 10 \ cm^2/S$$

$$= 10 \ cm^2/S$$

D in si $\quad 1 \sim 100 \ cm^2/S$

example

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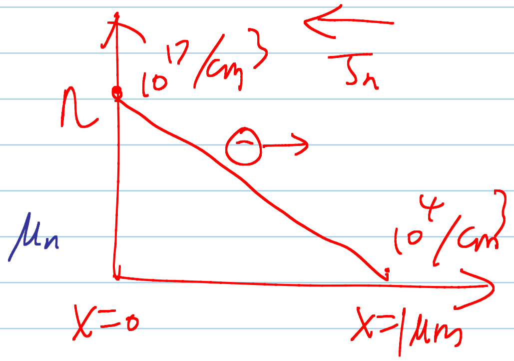
Example: diffusion current density calculation

- Assuming a linear decrease of n from $10^{17}/cm^3$ at x=0 to $10^4/cm^3$ at x=1um, a mobility of 350 $cm^2/(V\ s)$, find electron diffusion current density.



$$J_n = q \cdot D_n \cdot \frac{dn}{dx} \qquad q = 1.6 \times 10^{-19} C$$

$$D_n = \mu_n \cdot \boxed{\frac{kT}{q}} = 350 \frac{cm^2}{V \cdot S} \cdot 25 \times 10^{-3} V$$

$$\frac{dn}{dx}$$

$$J_{n.diff} = q D_n \cdot \frac{dn}{dx} \qquad D_n = \mu_n \cdot \frac{kT}{q}$$

$$\frac{kT}{q} = 25 mV \qquad \mu_n = 350\ cm^2 \frac{1}{V \cdot S} \qquad q = 1.6 \times 10^{-19} C$$

$$\frac{dn}{dx} = \frac{n_2 - n_1}{x_2 - x_1} = \frac{(10^4 - 10^{17})\frac{1}{cm^3}}{(1-0)\mu m} = \underset{\uparrow}{-}10^{17} \frac{1}{cm^3} \cdot \frac{1}{10^{-4} cm} = 10^{-21} \frac{1}{cm^4}$$

$\underline{1 \mu m = 10^{-4} cm}$

$$J_n = 1.6 \times 10^{-19} C \cdot 25 \times 10^{-3} V \cdot 350 \frac{cm^2}{V \cdot S} \cdot (-10^{21}) \frac{1}{cm^4}$$

$$= -1400\ A/cm^2$$

Always do unit Conversion

$$C \cdot V \cdot \frac{cm^2}{V \cdot S} \cdot \frac{1}{cm^4} = \frac{C}{S} \frac{1}{cm^2} = A/cm^2 \leftarrow$$

$$KT = 25\,\underline{\underline{meV}}$$

$$\left.\begin{aligned} J_{n.diff} &= q \cdot D_n \cdot \frac{dn}{dx} \\ D_n &= \mu_n \cdot \frac{KT}{q} \end{aligned}\right\} \Rightarrow \quad J_{n.diff} = q \cdot \mu_n \cdot \frac{KT}{q} \cdot \frac{dn}{dx}$$

$$= \mu_n \cdot \underset{\uparrow}{\underline{\underline{KT}}} \cdot \frac{dn}{dx}$$

$$\underline{\underline{25\ meV}}$$

$$0.025\ \underline{\underline{eV}}$$

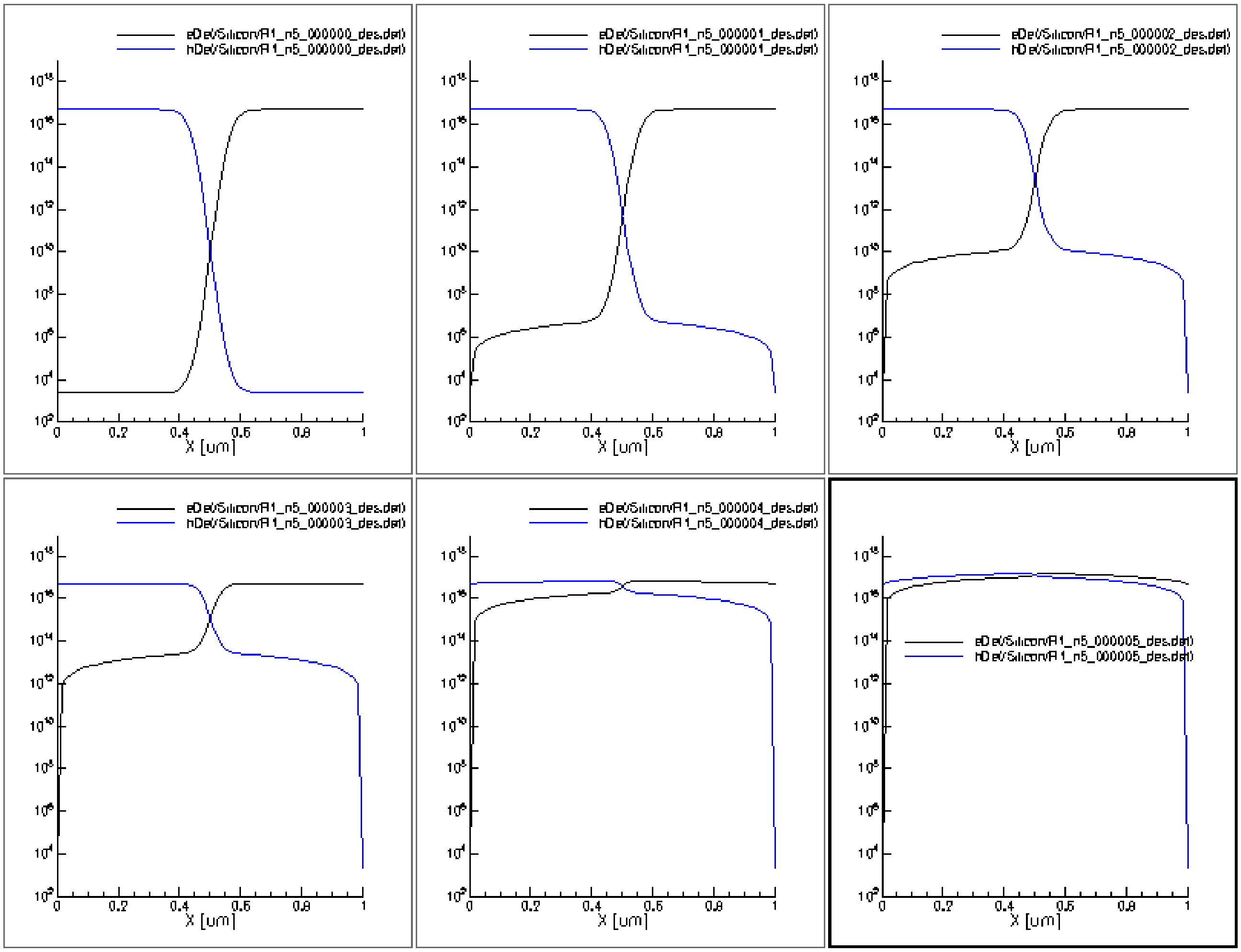
$$= \square\ \frac{cm^2}{V \cdot S}\ \square\ eV \cdot\ \square\ \frac{1}{cm^4}$$

$$\underline{\underline{1eV}} = \underline{\underline{1 \cdot q \cdot V}} = 1 \cdot \underline{\underline{1.6 \times 10^{-19}}}\, C \cdot V =$$

PN junction TCAD example

Monday, August 20, 2012

7:39 PM



Observe how gradient of p and n is created by applying forward bias

- At zero bias, no gradient
- P on n-side, and n on p-side both increase with forward bias
- At transition region ends, minority carrier concentration increases exponentially by exp(vf/pt) at small bias - more on this later from band diagram
- For biases < 0.6V, injected minority carrier << majority

carrier, this is called "low injection"

- For bias = 1.0V, minority carrier concentration is comparable to majority carrier concentration - "high injection"

Electron diffusion current in uniform sample

Wednesday, August 15, 2012
5:30 PM

Consider electrons diffusion in a given p-type sample with width Wp.

At x=0, electron concentration is n0.

At x=Wp, electron concentration is 0.

Write down an expression for electron diffusion current density, including sign.

Write down an expression for total electron charge per area, and an expression for the ratio of current density to total charge per area.

Diffusion current, charge and transit time

Wednesday, August 15, 2012
5:24 PM

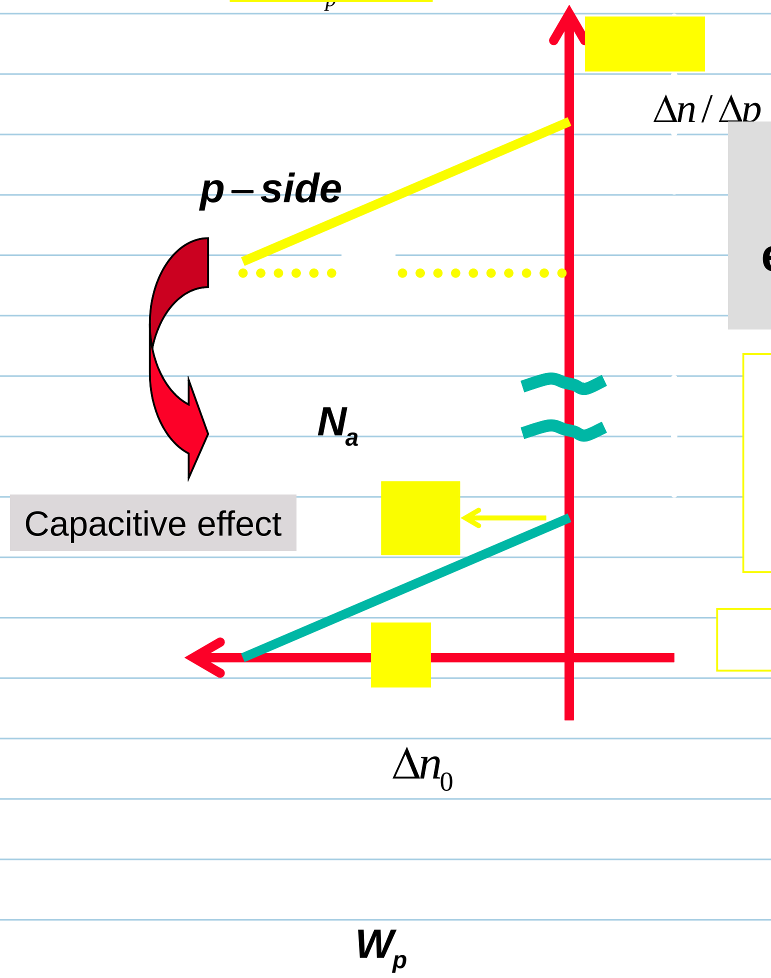
- In basic pn junctions, current control is made possible through charge modulation.
- Current control in PN junction is made possible through charge modulation
- Vf induces excess minority charge
- excess majority charge is also induced to keep "quasi-neutral" outside space charge regions, thereby causing "capacitance"
- This "capacitance" has ties to the diffusion origin of diode current, and is thus called "diffusion capacitance" – the positive and negative charges are at the same location – quasi-neutral
- Excess charge is more fundamental than "diffusion capacitance"

Excess (delta) Minority Carrier Charge

$$J_n = qD_n \frac{\Delta n_0}{W_p}$$

$$Q_{ex,\Delta n} = q\int_0^{W_p} \left[\Delta n(x)\right] dx$$

$$= q\frac{1}{2}W_p \Delta n_0$$



The ratio of current to excess charge becomes:

t Q $qW_p\Delta n_0$ 1 W_p^2

Charge control:

W_p

transit time

$$Q_{ex,\Delta n} = J_n t_n$$

☆?

Base transit time

Wednesday, August 15, 2012
5:53 PM

Electron mobility is on the order of a few hundred cm^2/V-second.

Vertical base width is on the order of 100nm in modern transistors.

Consider mobility is 200 cm^2/V-s, base width is 100nm, find the base transit time.

The drift transistor

Wednesday, August 15, 2012
5:40 PM

(Nobel Physics Winner Kromer Studied this in his early days, around your age - and modern SiGe HBT is a "drift" transistor with drift from bandgap engineering)

Minority carrier vs majority carrier mobility

Thursday, August 16, 2012
10:06 AM

For the same doping, electrons in n-type semiconductor have different mobility from electrons in p-type semiconductor.

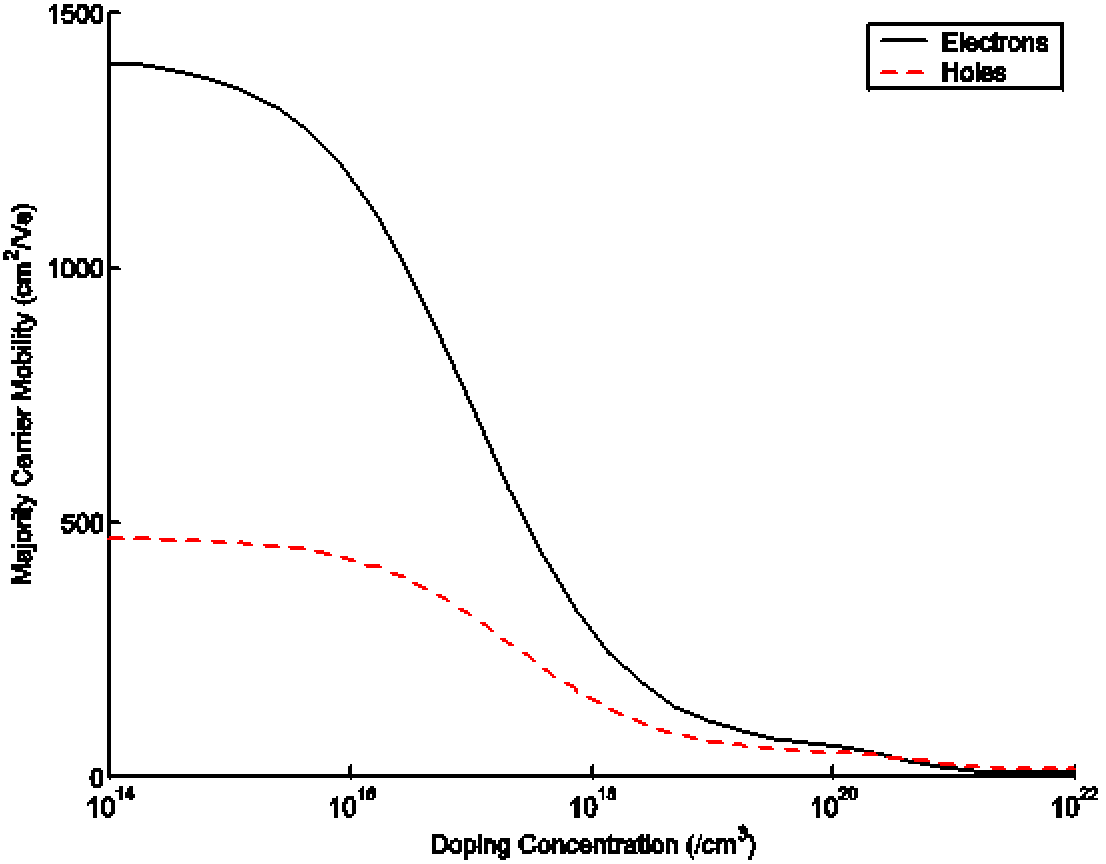
Same can be said for holes.

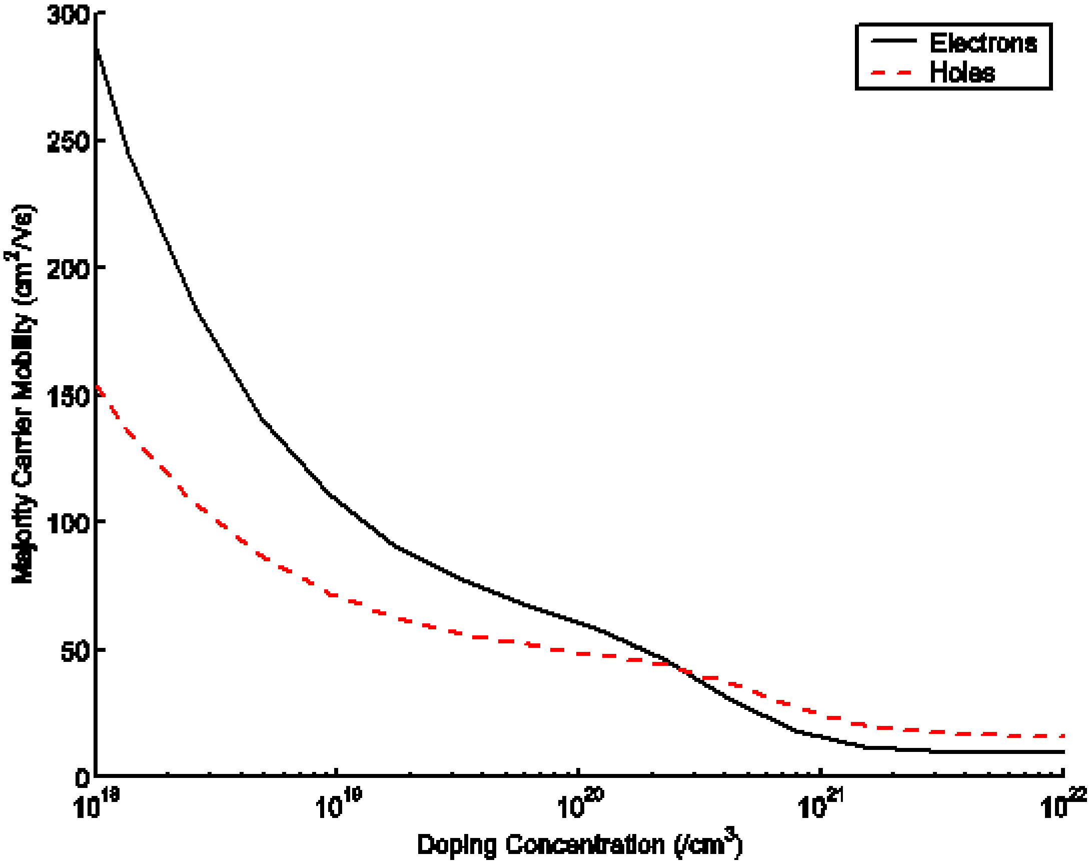
This is important to keep in mind.

The figures below are calculated at 300K with the most advanced bulk mobility model (Phumob in Sentaurus) - equations are very complicated so I'll not give them here (several pages)

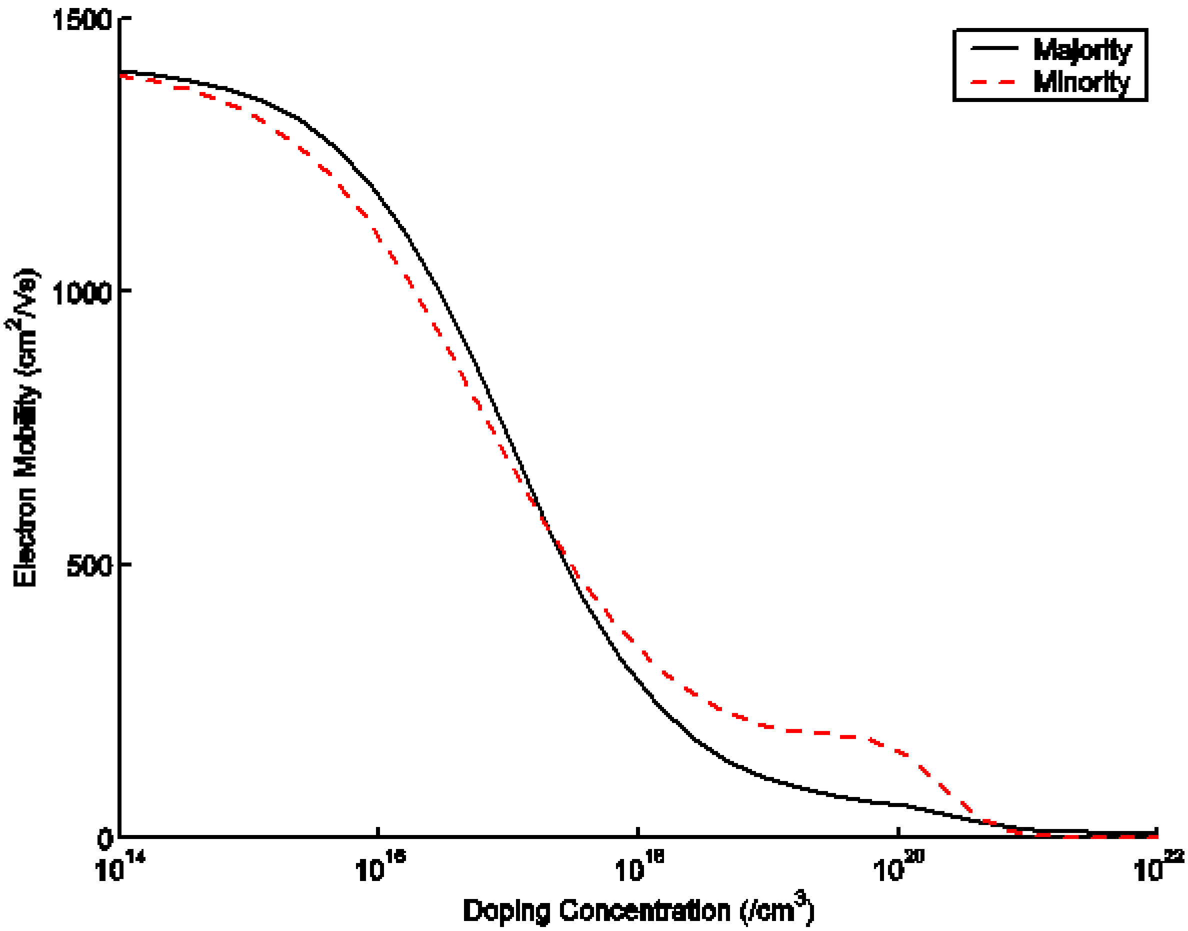
Pay attention to the doping dependence

Majority carrier mobility

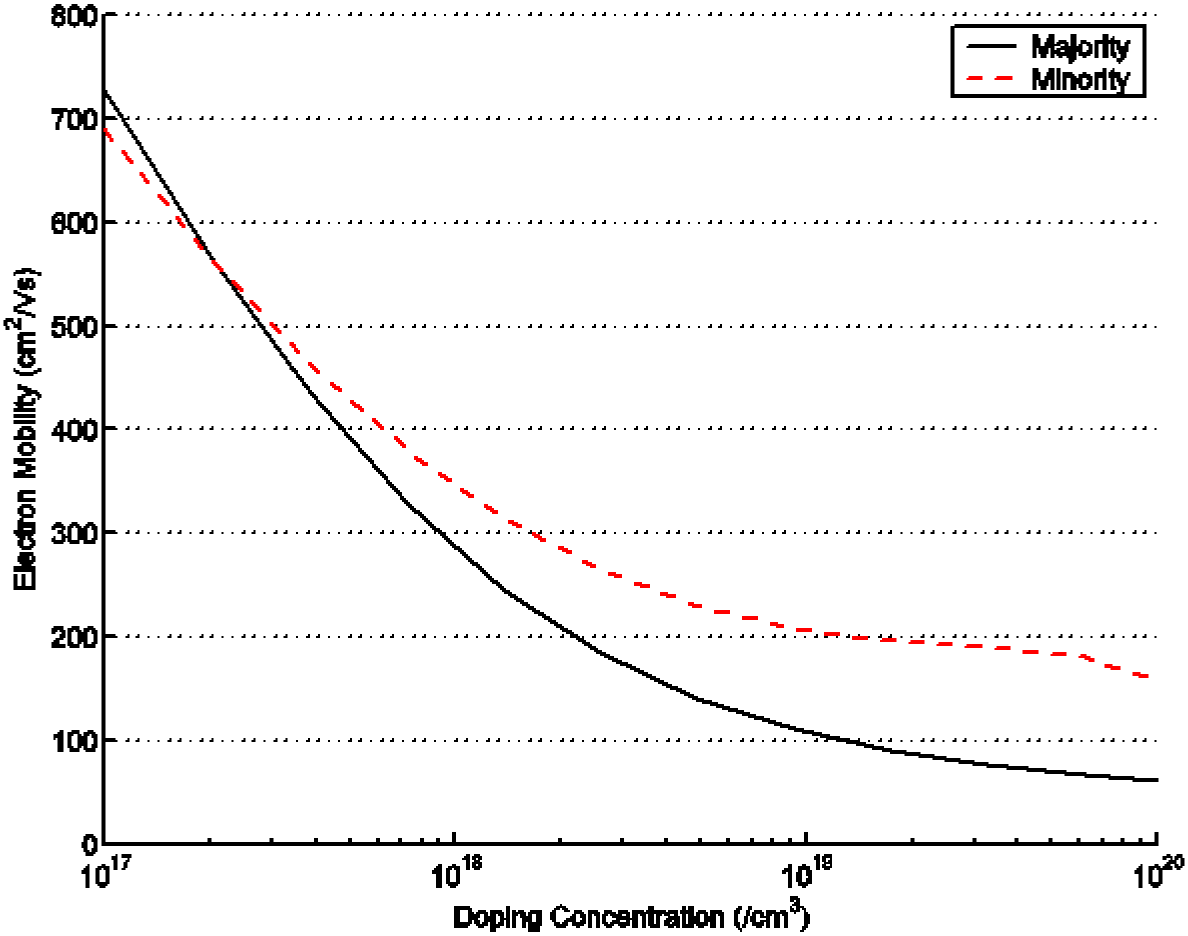

Electrons
Holes
Majority Carrier Mobility (cm^2/Vs)
1500
1000
500
0
10^{14}
10^{16}
10^{18}
10^{20}
10^{22}
Doping Concentration (/cm^3)


Electrons
Holes
Majority Carrier Mobility (cm^2/Vs)
300
250
200
150
100
50
0
10^{18}
10^{19}
10^{20}
10^{21}
10^{22}
Doping Concentration (/cm^3)

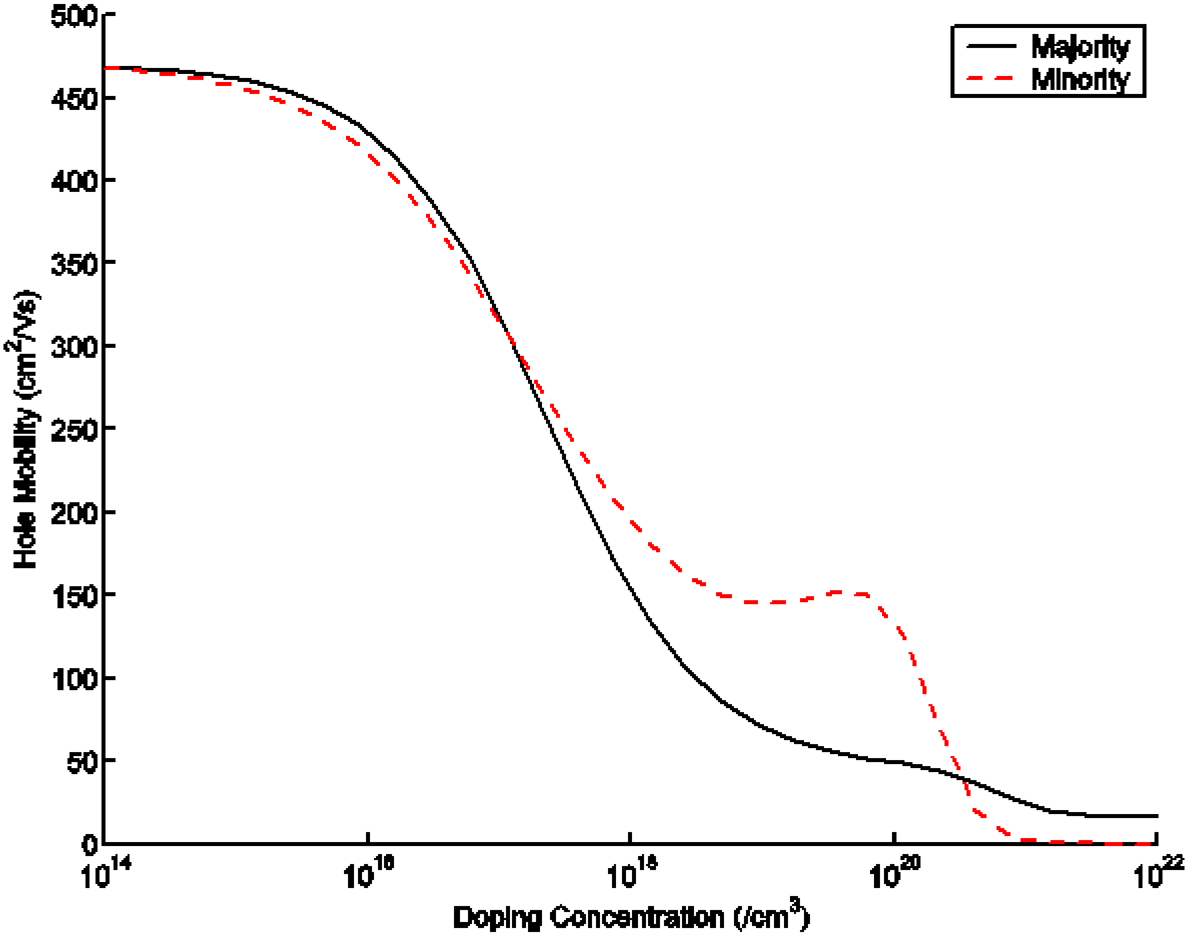
Electron mobility:



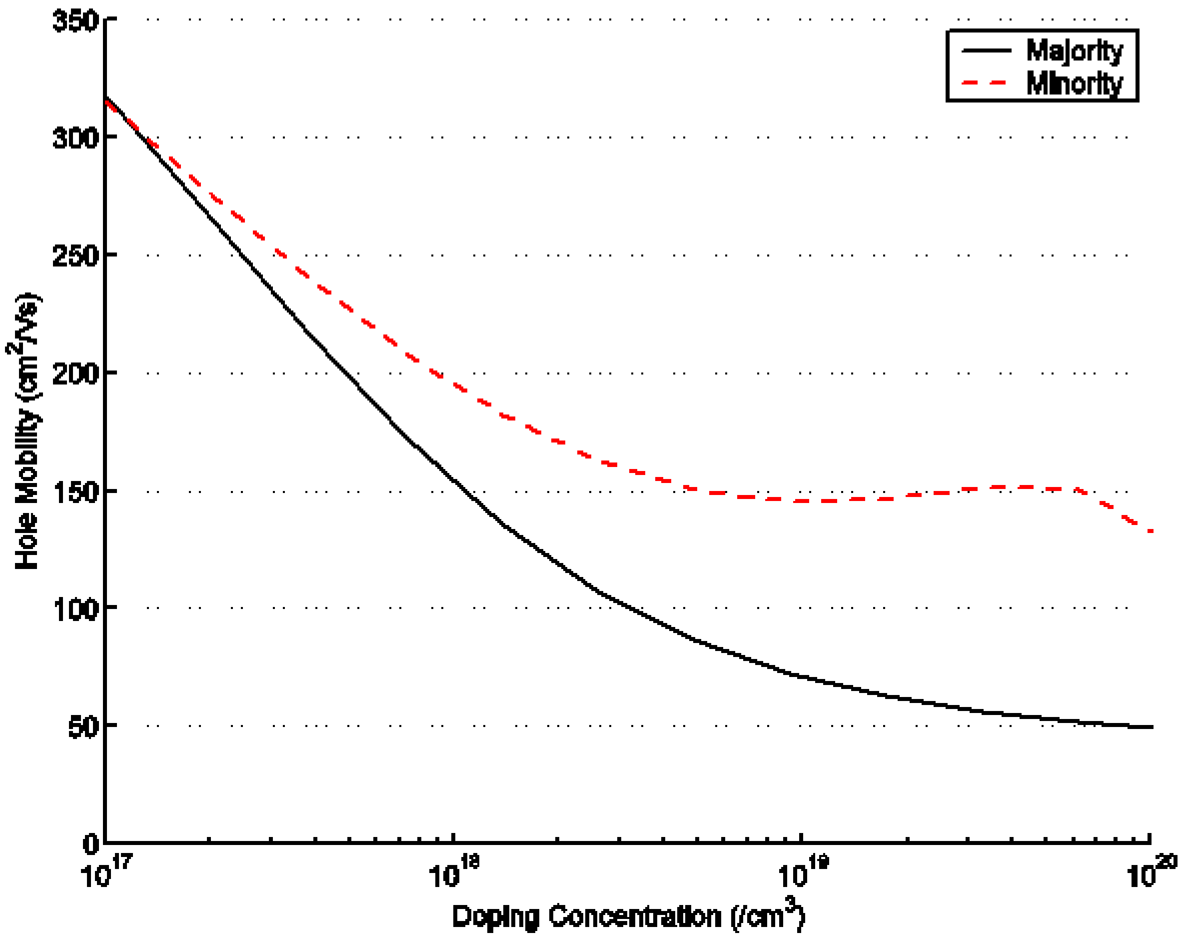
Zoom in to the more interesting region:



Hole mobility:



Zoom in to the high doping region of interest:



Majority
Minority
Hole Mobility (cm^2/Vs)
350
300
250
200
150
100
50
0
10^{17}
10^{18}
10^{19}
10^{20}
Doping Concentration (/cm^3)

Unit conversions for microelectronics

Sunday, April 22, 2012
1:57 PM

$1\,\mu m = 10^{-6}\,m.$ $\quad 1\,m = 100\,cm \quad 1\,\mu m = 10^{-4}\,cm$

$1\,\text{Å} = 10^{-10}\,m \quad 1\,nm = 10^{-9}\,m \quad 1\,nm = 10\,\text{Å}$

$1\,\mu m = 1000\,nm$

$130\,nm = 0.13\,\mu m$

$90\,nm = 0.09\,\mu m$

$$1\,\mu m^2 = (10^{-4}\,cm)^2 = 10^{-8}\,cm^2$$

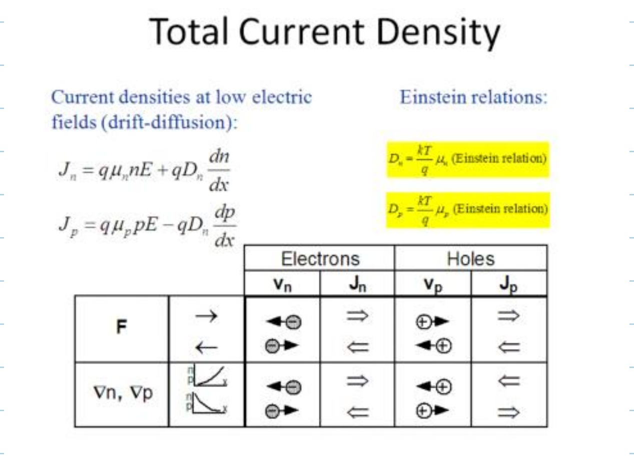
$$1\,mA/\mu m^2 = 1\times10^{-3}\,A/(10^{-8}\,cm^2) = 10^{5}\,A/cm^2$$

$$= 100\,KA/cm^2$$

Important
Unit
Conversion in
Microelectronics

Total current density

Sunday, April 22, 2012
1:59 PM



Total Current Density

Current densities at low electric fields (drift-diffusion):

$$J_n = q\mu_n nE + qD_n \frac{dn}{dx}$$

$$J_p = q\mu_p pE - qD_n \frac{dp}{dx}$$

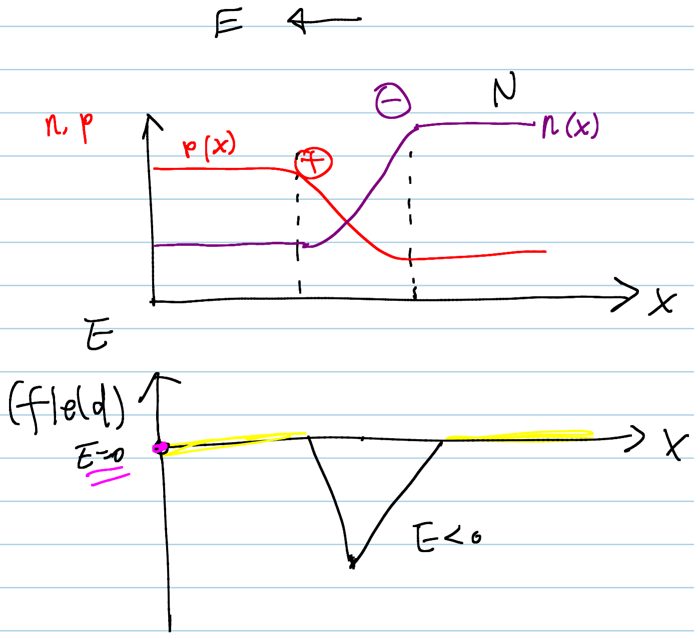
Einstein relations:

$D_n = \frac{kT}{q}\mu_n$ (Einstein relation)

$D_p = \frac{kT}{q}\mu_p$ (Einstein relation)

		Electrons		Holes	
		v_n	J_n	v_p	J_p
F	→ ←	◄⊖ ⊖►	⇒ ⇐	⊕► ◄⊕	⇒ ⇐
∇n, ∇p	n p x n p x	◄⊖ ⊖►	⇒ ⇐	◄⊕ ⊕►	⇐ ⇒

Consider this non-uniform semiconductor system:



Quiz:

indicate directions of
1) electron diffusion motion ←
2) electron drift motion →
3) electron diffusion current →
4) electron drift current ←
5) hole diffusion motion →
6) hole drift motion ←
7) hole diffusion current →
8) hole drift current ←

6700-8710 advanced Topics Start Here

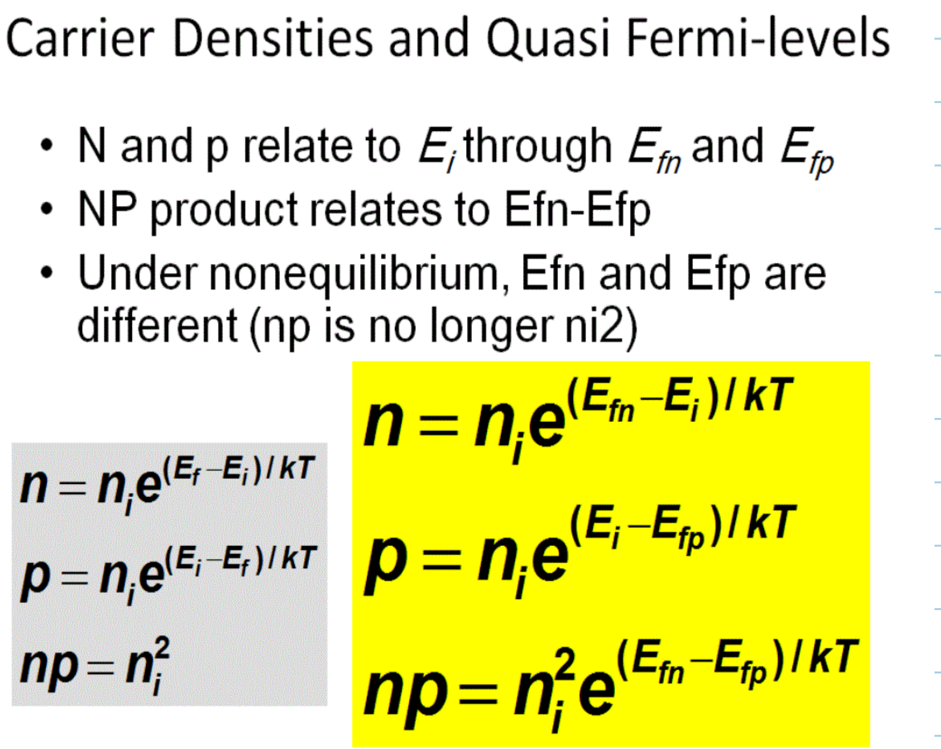
Friday, August 24, 2012
10:47 AM

Quasi-Fermi Levels

Thursday, August 16, 2012
10:27 AM

For non-equilibrium (applying field, light etc), we no longer have a spatially constant Fermi-level, rather electrons and holes each have their own "quasi-Fermi-level"

Furthermore, they are both position dependent (no longer spatially constant)



Carrier Densities and Quasi Fermi-levels

- N and p relate to E_i through E_{fn} and E_{fp}
- NP product relates to Efn-Efp
- Under nonequilibrium, Efn and Efp are different (np is no longer ni2)

$$n = n_i e^{(E_f - E_i)/kT}$$
$$p = n_i e^{(E_i - E_f)/kT}$$
$$np = n_i^2$$

$$n = n_i e^{(E_{fn} - E_i)/kT}$$
$$p = n_i e^{(E_i - E_{fp})/kT}$$
$$np = n_i^2 e^{(E_{fn} - E_{fp})/kT}$$

Another way to look at this is:

Quasi Fermi-levels (Nonequilibrium)

- E_{fn} and E_{fp} can be defined using n, p, and the intrinsic Fermi-level
- Efn and Efp are in general position dependent

$$n = n_i e^{(E_{fn} - E_i)/kT}$$

$$p = n_i e^{(E_i - E_{fp})/kT}$$

$$np = n_i^2 e^{(E_{fn} - E_{fp})/kT}$$

$$E_{fn} - E_i = kT \ln\left(\frac{n}{n_i}\right)$$

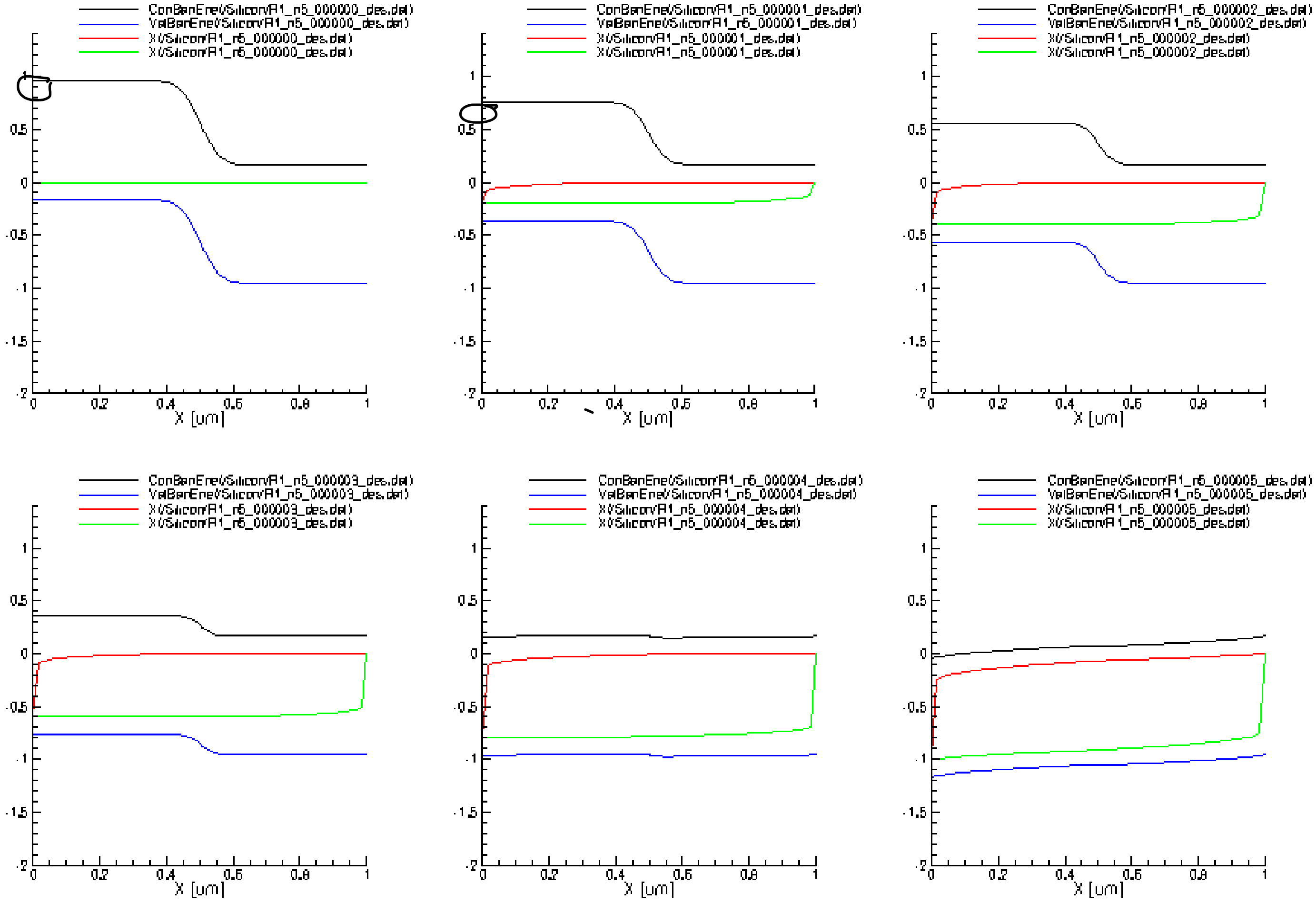
$$E_{fp} - E_i = -kT \ln\left(\frac{p}{n_i}\right)$$

Band diagrams of Pn junction under bias

Thursday, August 16, 2012
10:55 AM

The figure below shows PN junction band diagrams, Ec, Ev, Efn, Efp for Vf = 0 to 1.0 in 0.2V step.

Make observations on how the bias affects Ec, Ev, Efn and Efp.



PN junction p/n profiles evolution with bias

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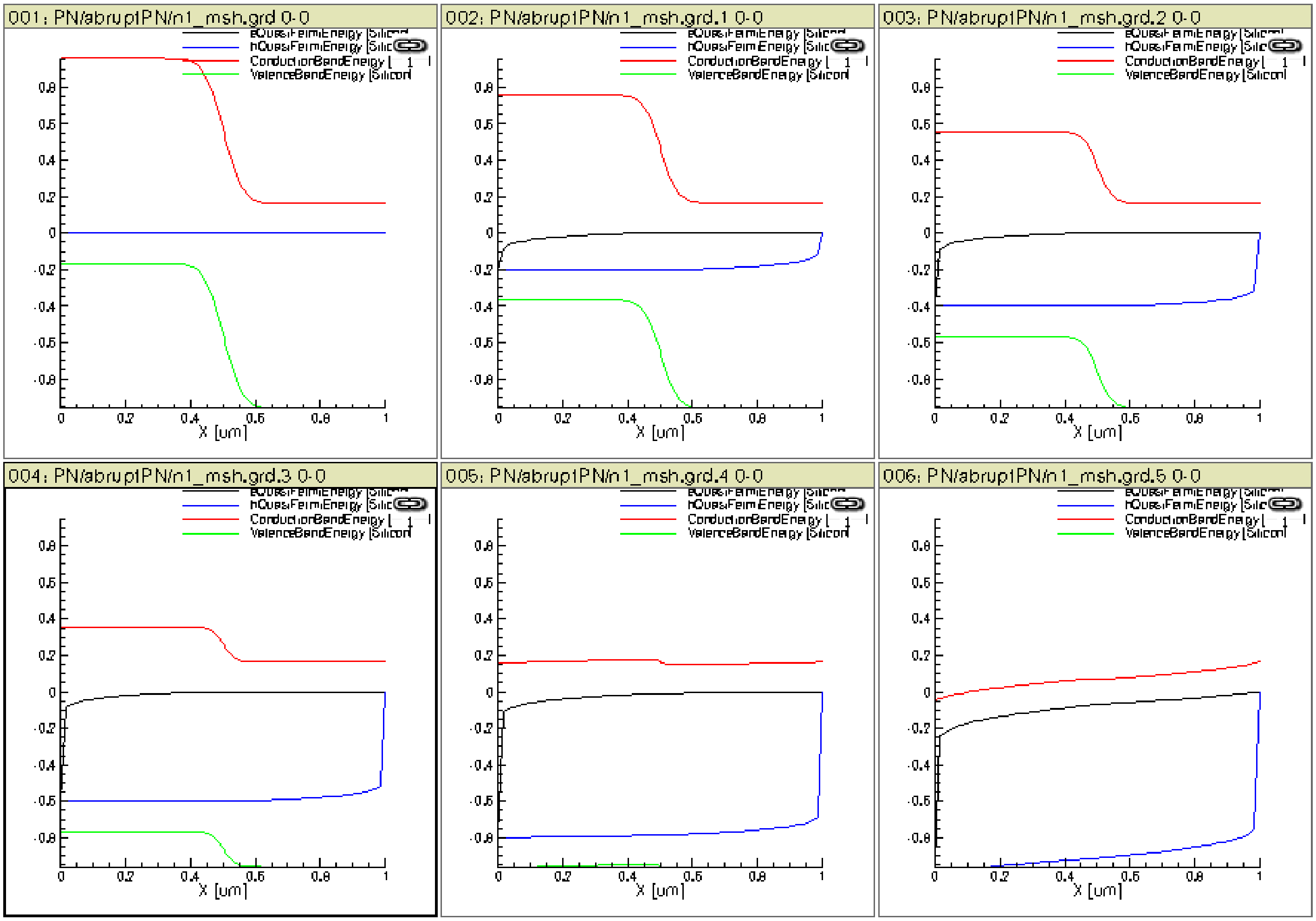
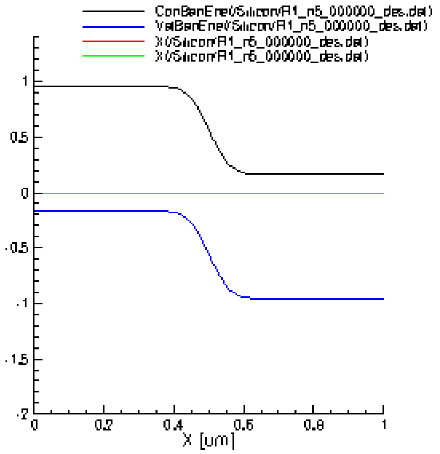
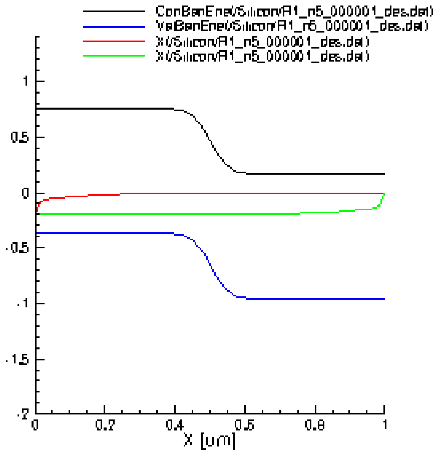
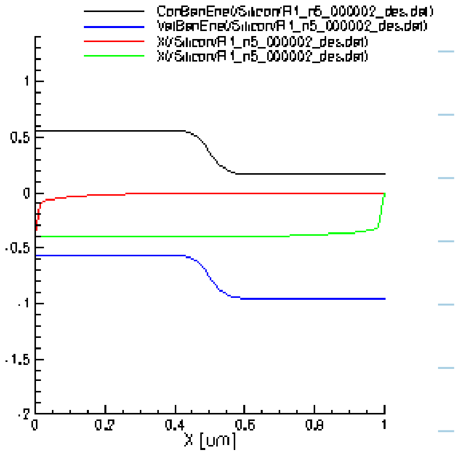
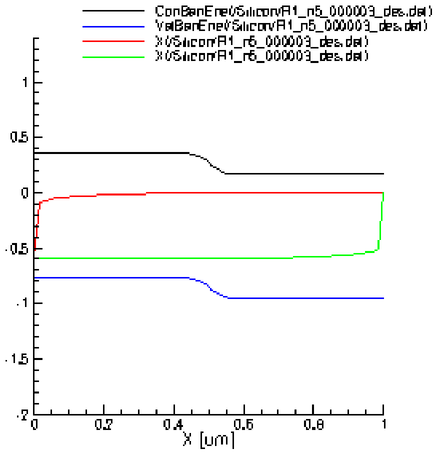
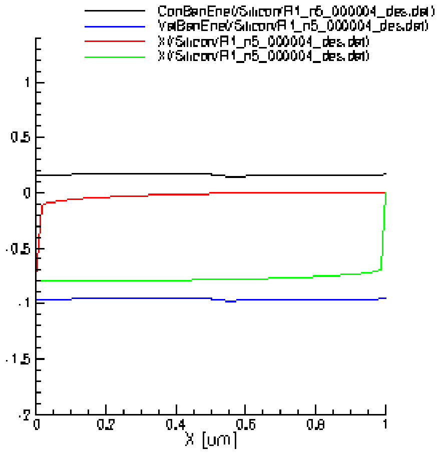


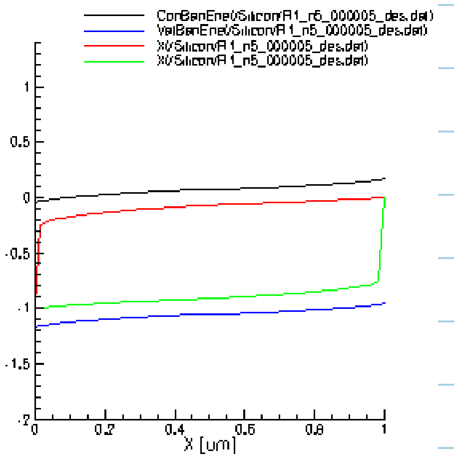




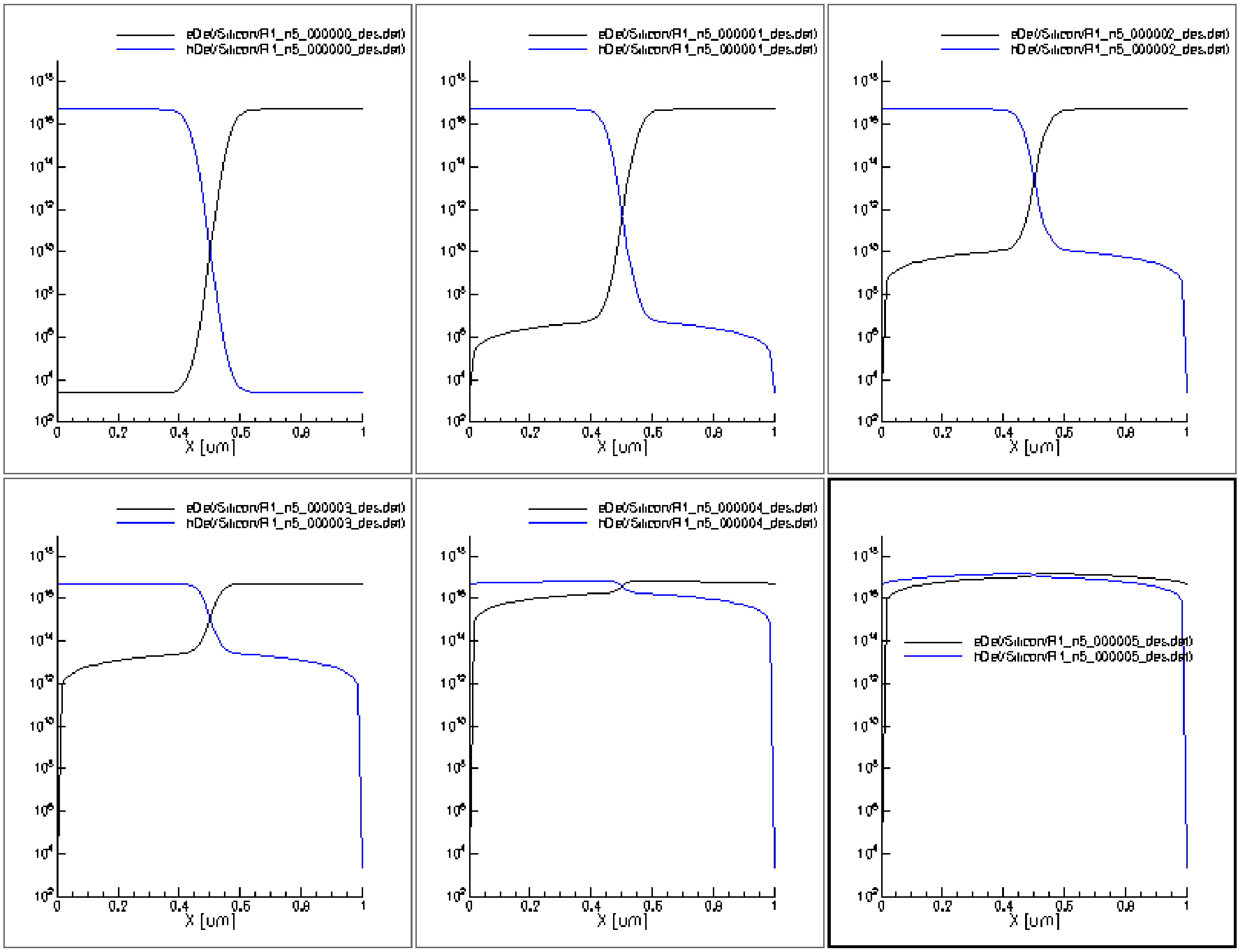








Now think about what the "p" and "n" profiles look like, compare your "visual picture" with simulation results: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hDe(/Silicon/R1_n5_000000_des.dat)
eDe(/Silicon/R1_n5_000001_des.dat)
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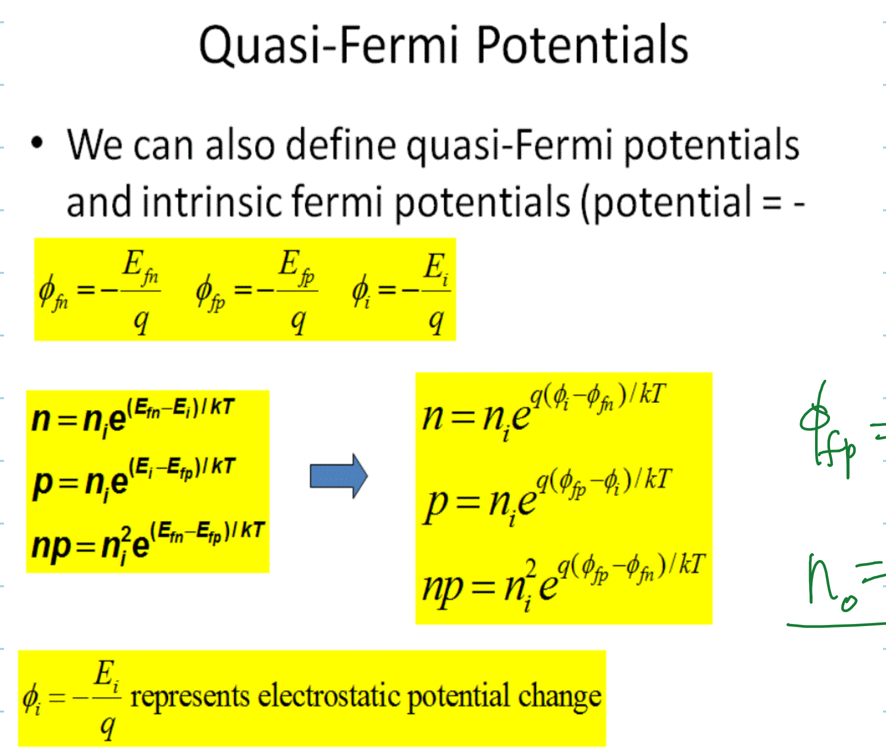
eDe(/Silicon/R1_n5_000002_des.dat)
hDe(/Silicon/R1_n5_000002_des.dat)
eDe(/Silicon/R1_n5_000003_des.dat)
hDe(/Silicon/R1_n5_000003_des.dat)
eDe(/Silicon/R1_n5_000004_des.dat)
hDe(/Silicon/R1_n5_000004_des.dat)
eDe(/Silicon/R1_n5_000005_des.dat)
hDe(/Silicon/R1_n5_000005_des.dat)
10^{18}
10^{16}
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
0
0.2
0.4
0.6
0.8
1
X [um]

Quasi-Fermi Potential

Thursday, August 16, 2012
10:29 AM



Quasi-Fermi Potentials

- We can also define quasi-Fermi potentials and intrinsic fermi potentials (potential = -

$$\phi_{fn} = -\frac{E_{fn}}{q} \qquad \phi_{fp} = -\frac{E_{fp}}{q} \qquad \phi_i = -\frac{E_i}{q}$$

$$n = n_i e^{(E_{fn}-E_i)/kT}$$
$$p = n_i e^{(E_i-E_{fp})/kT}$$
$$np = n_i^2 e^{(E_{fn}-E_{fp})/kT}$$

$$\Rightarrow$$

$$n = n_i e^{q(\phi_i-\phi_{fn})/kT}$$
$$p = n_i e^{q(\phi_{fp}-\phi_i)/kT}$$
$$np = n_i^2 e^{q(\phi_{fp}-\phi_{fn})/kT}$$

$\phi_i = -\frac{E_i}{q}$ represents electrostatic potential change

$\phi_{fp} = \phi_{fn} = 0$

$\underline{n_o = n_i e^{\phi_i/\phi_t}}$

When you later look at Sentaurus device simulation result, you will come across "electrostatic potential", you will find that this is actually defined with respect to the intrinsic semiconductor, more on this later after you learn built-in potential.

$n_o = n_i e^{\overset{\phi_i/\phi_t}{\uparrow}}$

$p_o = n_i e^{\overset{-\phi_i/\phi_t}{\uparrow}}$

$n_o p_o = n_i^2$

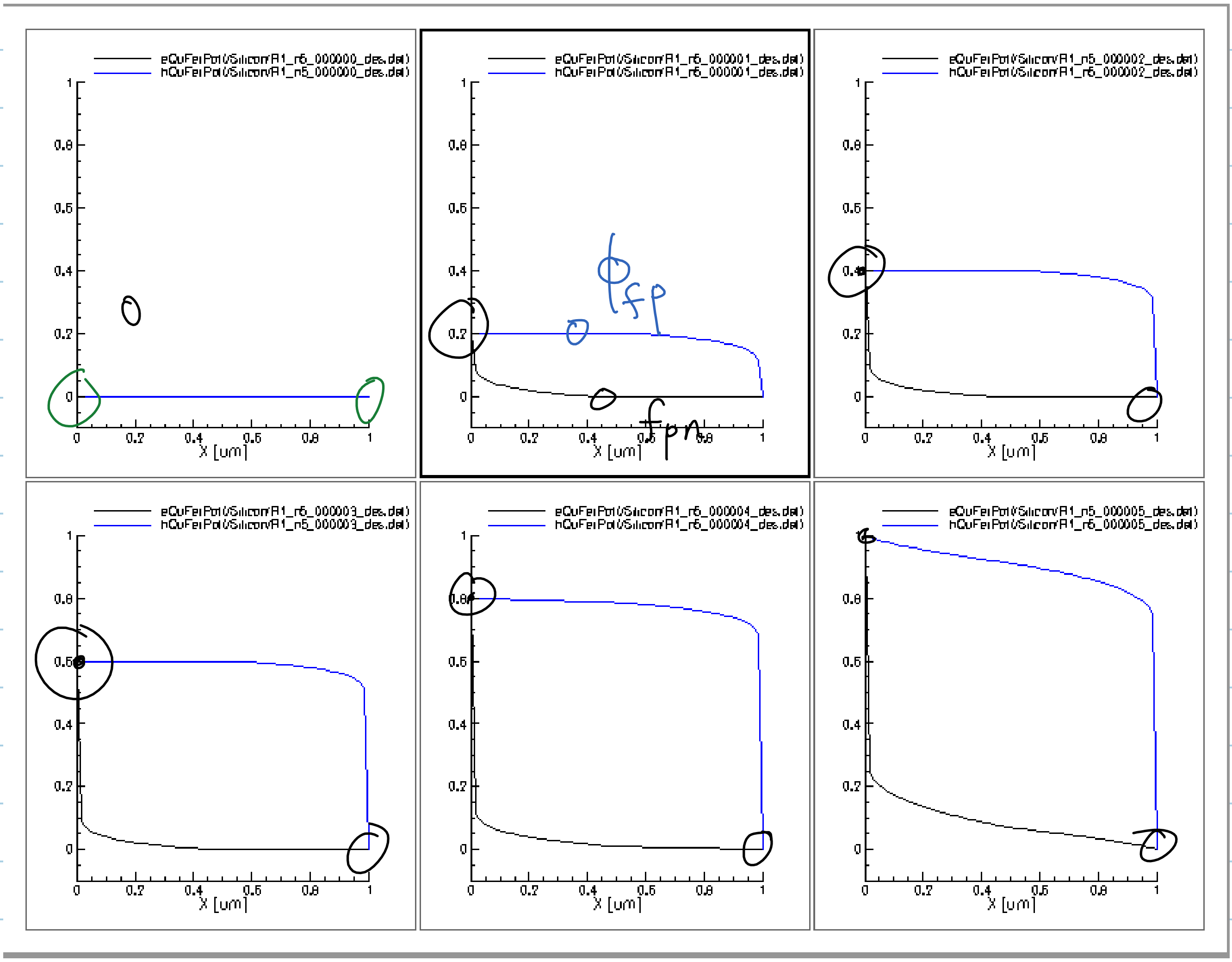
$\underline{\underline{\phi_{fn} - \phi_{fp}}}$ = Cont. applied voltage

= 0

at contact

$$\phi_{fn} = \phi_{fp} = V_{applied} \Leftarrow \begin{cases} p = p_0 \\ n = n_0 \\ \phi_i = \phi_{bi} \, (\text{w.r.t intrinsic}) + V_{applied} \end{cases}$$

PN junction example:



Note that the quasi-Fermi potentials at the ohmic contacts are equal to applied voltages.

- The n-contact voltage is set to ground in our circuit.

- The p-contact voltage increases from 0 to 1.0V in 0.2V step.

- Both eQuasiFermiPotential and hQuasiFermiPotential change by exactly the same amount between two contacts, equal to the applied voltage difference

- However, except near the n-contact, hQuasiFermiPotential changes very little

- Except near the p-contact, eQuasiFermiPotential changes very little

Homework Assignment 8/28 - due 8/30 class

Tuesday, August 28, 2012
1:22 PM

1. Read the next page right below this page on "Physical meaning of intrinsic Fermi potential" - which I wrote to address the physical meaning of phi_i we talked about today. (nothing to turn in)

2. optional proof, required for tcad2 group members

2. prove that at a contact

if we choose

$$\phi_i = \phi_{bi}(\text{intrinsic reference}) + V_{applied}$$

$$\phi_{fn} = \phi_{fp} = V_{applled}$$

then

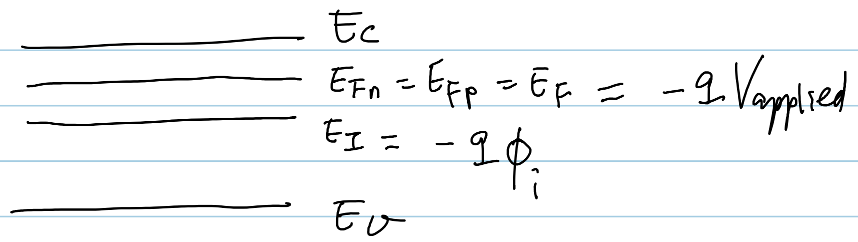
$$p = p_0$$

$$n = n_0$$

where p_0 and n_0 are equilibrium

hole and electron concentrations at

the contact。



3. Consider a NPN transistor with 3 contacts, collector, base, emitter. Doping level is 1e19, 1e18 and 1e17 at emitter, base and collector contacts. Doping is p-type for base, and n-type for emitter and collector.

   Using ipython, and the "basicsemi" program provided, determine
   - Electrostatic potential phi_i, phi_fn, phi_fp, Ei, Efn, Efp, p and n at all contacts at Vb=Vc=Ve=0
   - Electrostatic potential phi_i, phi_fn, phi_fp, Ei, Efn, Efp, p, and n at all contacts at Vb=0.7V, Vc=1.0V, Ve=0V

   You can also use your editor to create a file with extension .py, e.g. hwbjtcontact.py

   Put your commands in that, and then run it by typing "python hwbjtcontact.py".

   For example, the content can be like:

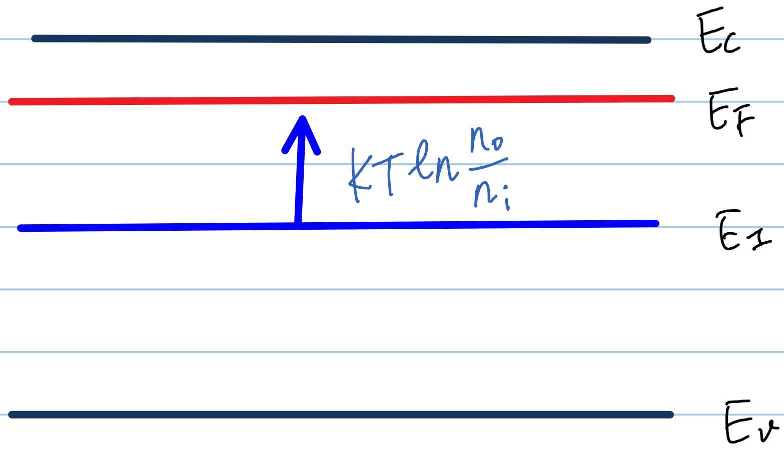
```
'''
Created on Aug 28, 2012

@author: GuofuNiu
```

```
'''

from basicsemi import *


# this is comment
# for emitter, it is n-type
se = Nsemi(1e19)

sb = Psemi(1e18)

sc = Nsemi(1e17)

print se, sb, sc
```

You should have all the information you can use to obtain all the potential and energy values using the equations we discussed today.

solution

Thursday, August 30, 2012
10:34 AM



$$\phi_{bi} = \frac{kT}{q} \ln \frac{n_0}{n_i} \quad \left(\text{or } \frac{E_F - E_I}{q}\right) = \phi_t \ln \frac{n_0}{n_i}$$

B.C. is $\phi_i = \phi_{bi} + V_{applied} \quad (\phi)$

$$\phi_{fn} = V_{applied}$$

$$\phi_{fp} = V_{applied}$$

n. p equations are:

$$n = n_i e^{(\phi_i - \phi_{fn})/\phi_t}$$

Physical meaning of intrinsic Fermi potential phi_i

Tuesday, August 28, 2012
1:09 PM

In class today, I was trying to have you visualize band diagram to see the physical meaning of phi_i, defined by -Ei/q, at ohmic contacts.

You can look at this using equations too (I think I lost most of you when I referred to band diagram):

So the built-in potential to an intrinsic reference is

$$\phi_{bi}(\text{intrinsic reference}) = \frac{E_F - E_I}{q} \quad \text{at contact}$$

$E_F = E_{Fn} = E_{Fp}$

recall that $\phi_{fn} \overset{\Delta}{=} -\frac{E_{Fn}}{q}$ $\phi_i \overset{\Delta}{=} -\frac{E_I}{q}$

$$\phi_{bi}(\text{intrinsic ref}) = -\phi_{fn} + \phi_i$$

that is $\phi_i = \phi_{bi}(\text{intrinsic reference}) + \phi_{fn}$

If we set $\phi_{fn} = \phi_{fp} = V_{applied}$

$\phi_i = \phi_{bi}$ (intrinsic ref) $+ V_{applied}$ at contacts

of course when $V_{applied} = 0$,

ϕ_i is equal to ϕ_{bi} (intrinsic ref)

Jn using Efn gradient

Thursday, August 16, 2012
10:32 AM

- Electron current (drift+diffusion) density is proportional to both n and Efn gradient
- For a constant Jn, Efn variation is small in regions of large n

$$J_n = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$= qn\mu_n\left(-\frac{d\phi}{dx}\right) + q\frac{kT}{q}\mu_n\frac{dn}{dx}$$

$$= n\mu_n\left(-q\frac{d\phi}{dx} + kT\frac{1}{n}\frac{dn}{dx}\right)$$

$$= n\mu_n\left(\frac{dE_i}{dx} + kT\frac{d}{dx}\ln n\right) = n\mu_n\frac{d}{dx}\left(E_i + kT\ln n\right) = n\mu_n\frac{d}{dx}\left(E_i + kT\ln n - kT\ln n_i\right)$$

$$= n\mu_n\frac{d}{dx}\left(E_i + kT\ln\frac{n}{n_i}\right) = n\mu_n\frac{dE_{fn}}{dx}$$

$$E_{fn} - E_i = kT\ln\left(\frac{n}{n_i}\right)$$

We used:

PN junction example:

Jp Using Efp gradient

Thursday, August 16, 2012
10:37 AM

- Hole current density is also in proportion to p and dEfp/dx
- Jn and Jp share the same equation form when related to quasi Fermi levels
- Spatial gradient of quasi Fermi level is the driving force of current flow

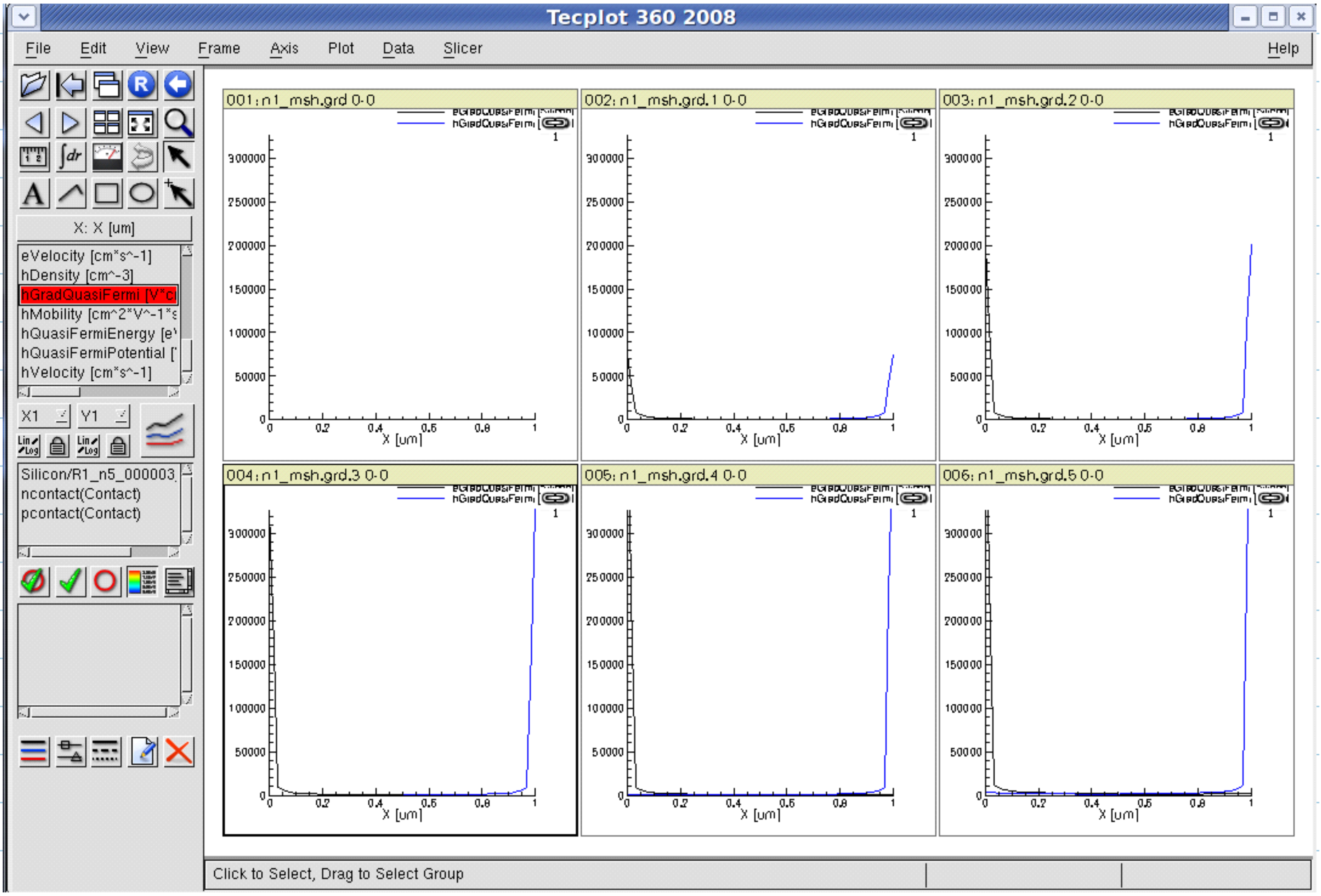
$$J_p = p\mu_p \frac{dE_{fp}}{dx}$$
$$J_n = n\mu_n \frac{dE_{fn}}{dx}$$

E_{fn} variation is small in regions of large n

E_{fp} variation is small in regions of large p

Let us look at TCAD result:

(go to axis, adjust range of y1)



Observe that:

- eGradQuasiFermi (V/cm) is largely zero except near the p-contact (where n is very small)
- hGradQuasiFermi is large zero except near the n-contact (where p is very small)

Thermodynamic Equilibrium

Thursday, August 16, 2012
10:39 AM

- Drift and diffusion balances exactly
- Efn=Efp=const (that const level is defined as the Fermi level for the whole system)
- Zero current requires electrons and holes to share the same Fermi-level, and that Fermi level must be spatially constant

$$J_p = p\mu_p \frac{dE_{fp}}{dx} = 0$$
$$J_n = n\mu_n \frac{dE_{fn}}{dx} = 0$$

Furthermore,
$E_{fp} = E_{fn}$

This is an extremely important result – spatially constant EF at equilibrium

Electrostatic Potential distribution in PN junction and bias impact

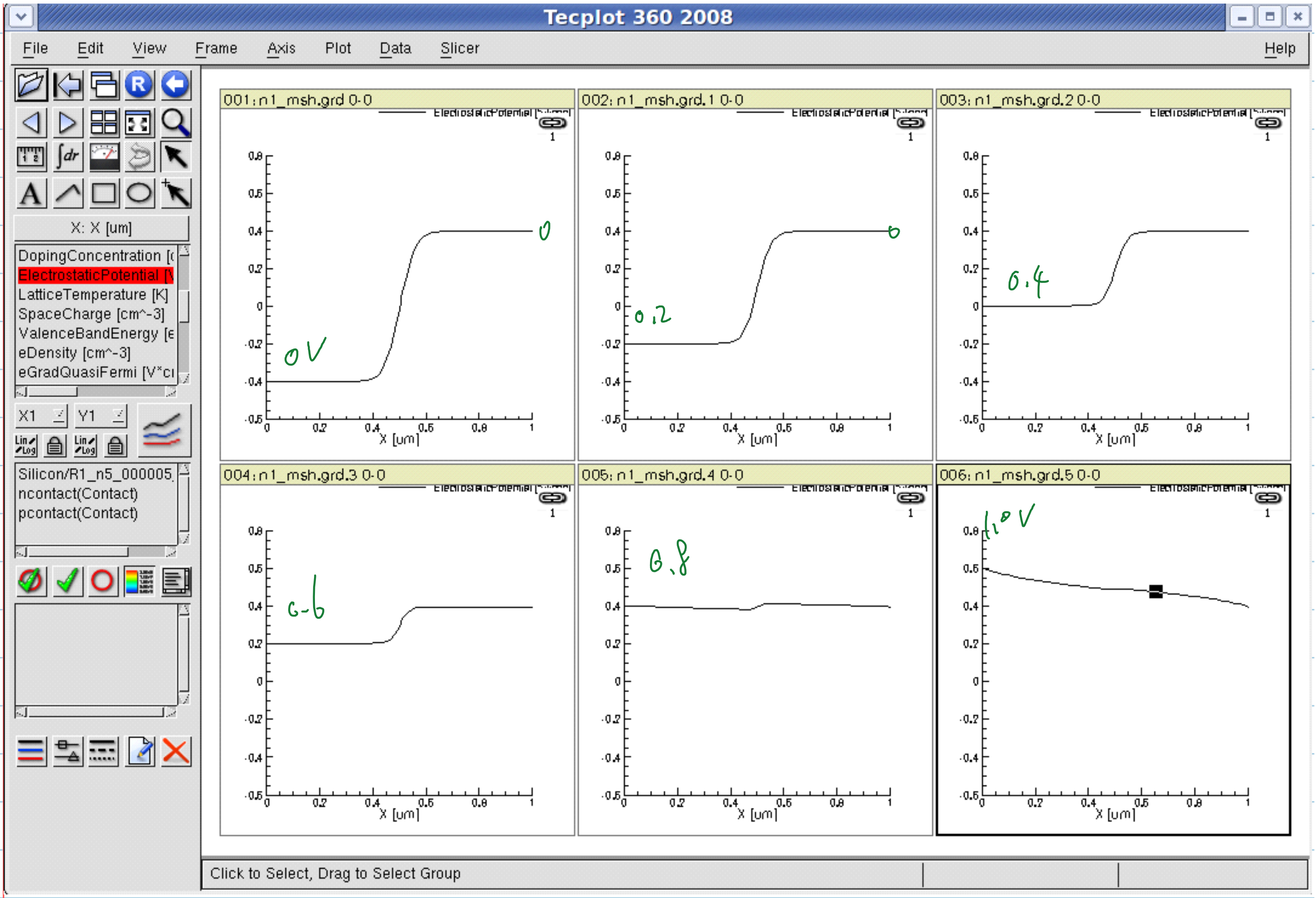
Thursday, August 16, 2012
12:45 PM

Here we **fix the n-contact applied voltage at 0V**.

We **ramp up the p-contact voltage in step of 0.2V, from 0 to 1.0V**.

Observe that the **contact potential in TCAD simulation is the equilibrium contact potential + applied voltage at the contact**, this is known as Ohmic contact.

At the ohmic contact, the p and n values are also fixed at their equilibrium values, meaning they are not affected by the applied biases.


Tecplot 360 2008
File Edit View Frame Axis Plot Data Slicer Help
X: X [um]
DopingConcentration [
ElectrostaticPotential [V
LatticeTemperature [K]
SpaceCharge [cm^-3]
ValenceBandEnergy [e
eDensity [cm^-3]
eGradQuasiFermi [V*c
X1 Y1
Silicon/R1_n5_000005
ncontact(Contact)
pcontact(Contact)
001: n1_msh.grd 0-0
002: n1_msh.grd.1 0-0
003: n1_msh.grd.2 0-0
004: n1_msh.grd.3 0-0
005: n1_msh.grd.4 0-0
006: n1_msh.grd.5 0-0
ElectrostaticPotential
X [um]
0V
0
0.2
0
0.4
0.6
0.8
1.0V
Click to Select, Drag to Select Group

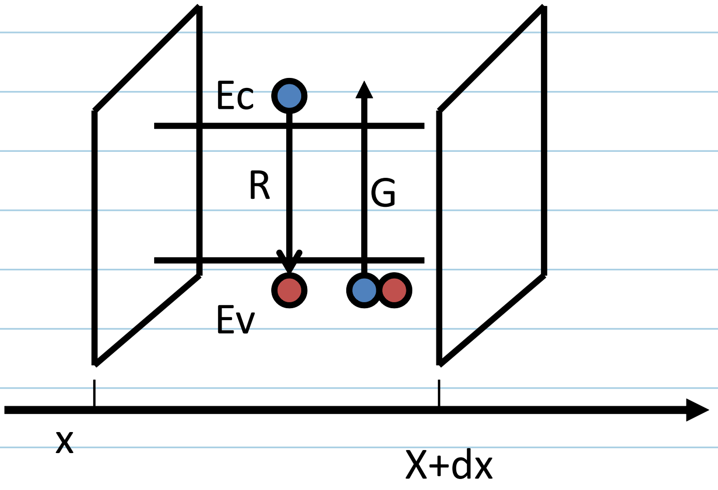
Electron and Hole Continuity Equations

Sunday, August 26, 2012
12:07 PM

Continuity Equation for General Particles

$$\frac{\partial C}{\partial t} = -\frac{\partial F}{\partial x} + (G - R)$$

- **Flux density F: # of particles flowing per unit area per unit time**
- Applying particle conservation to a small slice between x and x+dx (with cross-sectional area of A):



Particle Conservation

- Consider the change of # of particles within a small slice between x and x+dx, with cross-sectional area of A,

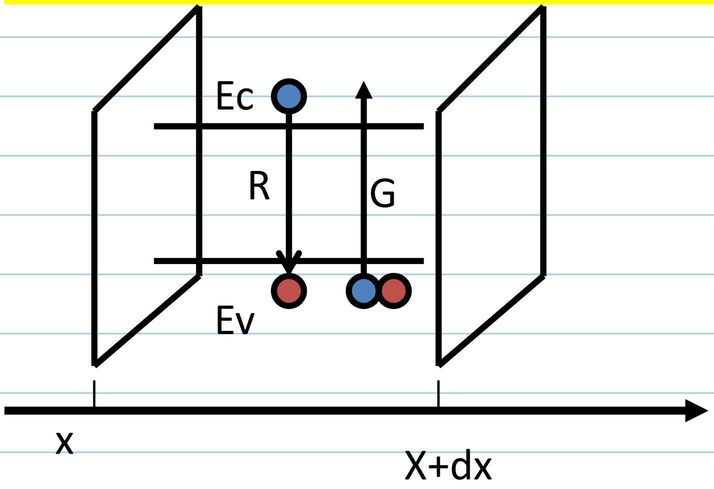
$$\overbrace{\frac{\partial C}{\partial t}}^{\text{net increase per sec}}\underset{\text{volume of slice}}{Adx} = \overbrace{AF(x)}^{\#\text{ flowing in per sec}} - \overbrace{AF(x+dx)}^{\#\text{ flowing out per sec}} + \overbrace{\underset{\text{volume of slice}}{Adx}\,(G-R)}^{\#\text{ due to net generation per sec}}$$

C: concentration #/cm^3

F: #/cm^2/sec

G: generation rate, # of particles generated within unit volume per second #/cm^3/sec

R: recombination rate, # of particles recombined within unit volume per second #/cm^3/sec



$$\frac{\partial C}{\partial t} = -\frac{\partial F}{\partial x} + (G - R)$$

- **The flux density F is proportional to current density by –q for electrons and q for holes:**

$$J_n = -qF_n$$

$$J_p = qF_p$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G - R)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + (G - R)$$

G and R are the same for electrons and holes since they are always generated/recombined in pairs

Fundamental Semiconductor Equations

Friday, August 24, 2012
11:04 AM

It is helpful and necessary at this point to have a concrete feel of the basic equations solved in TCAD tools, and boundary conditions.

We will focus on the basic drift diffusion equations for simplicity. That is, in the solve part, you only have "Poisson Electron Hole".

Poisson refers to the Poisson's equation
Electron - electron continuity equation
Hole - hole continuity equation

Fundamental Semiconductor Equations

A set of mathematical models governing operation of arbitrary semiconductor structure or devices

Maxwell Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{}$$

$\vec{E}$ - indepen

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_{cond} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

$\left.\begin{matrix}\vec{E} \\ \vec{H}\end{matrix}\right\}$ indepen

$\vec{B}$

$\vec{D}$

* Auxilary equations

$\vec{D} = \varepsilon \vec{E}$ ε, μ

$\vec{B} = \mu \vec{H}$ material specific

* Current continuity equations

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = q\,(G-R) \qquad q>0$$

For electrons

$$-\frac{1}{q}\nabla\cdot\vec{J}_n + \frac{\partial n}{\partial t} = G_n - R_n$$

For holes

$$\frac{1}{q}\nabla\cdot\vec{J}_p + \frac{\partial p}{\partial t} = G_p - R_p$$

For steady state, the time dependence is set to zero.

For transient problems, we need initial conditions, typically we start from a given initial "DC" solution.

For frequency domain solution, we use exp(j*w*t) to describe time dependence.

* Poisson's Equations

define scalar potential ψ

$$\vec{E} = -\nabla\psi$$

$$\nabla\cdot\vec{D} = \nabla\cdot(\varepsilon\vec{E}) = \rho$$

For homogeneous material ε = const.

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \quad \text{or} \quad \nabla^2 \psi = -\rho/\varepsilon$$

For semiconductor

$$\rho = q\left(p - n + N(x,y,z)\right)$$

$$N \triangleq N_d^+ - N_a^- \qquad q > 0$$

* Drift - diffusion model for $\vec{J}$

$\vec{J} \propto \vec{E}$ drift

$\vec{J} \propto \nabla \left(\begin{smallmatrix} p \\ \text{or} \\ n \end{smallmatrix} \right)$ diffusion

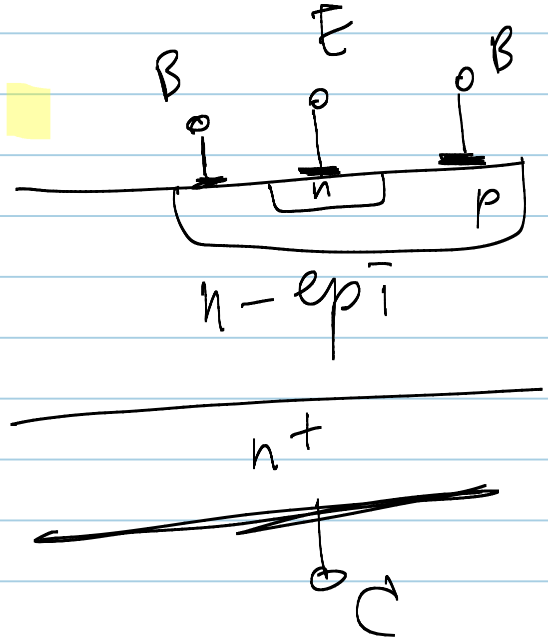
*

$$\vec{J}_n = q\left(n \mu_n \vec{E} + D_n \nabla n\right)$$

$$\vec{J}_p = q\left(p \mu_p \vec{E} - D_p \nabla p\right)$$

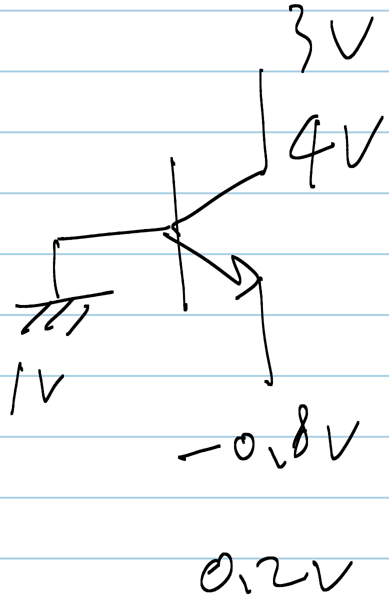
* Einstein relation $\frac{D}{\ } = \frac{KT}{\ }$ (

* Einstein relation $\frac{D}{\mu} = \frac{kT}{q} = \phi_t$



① doping

② applied voltage

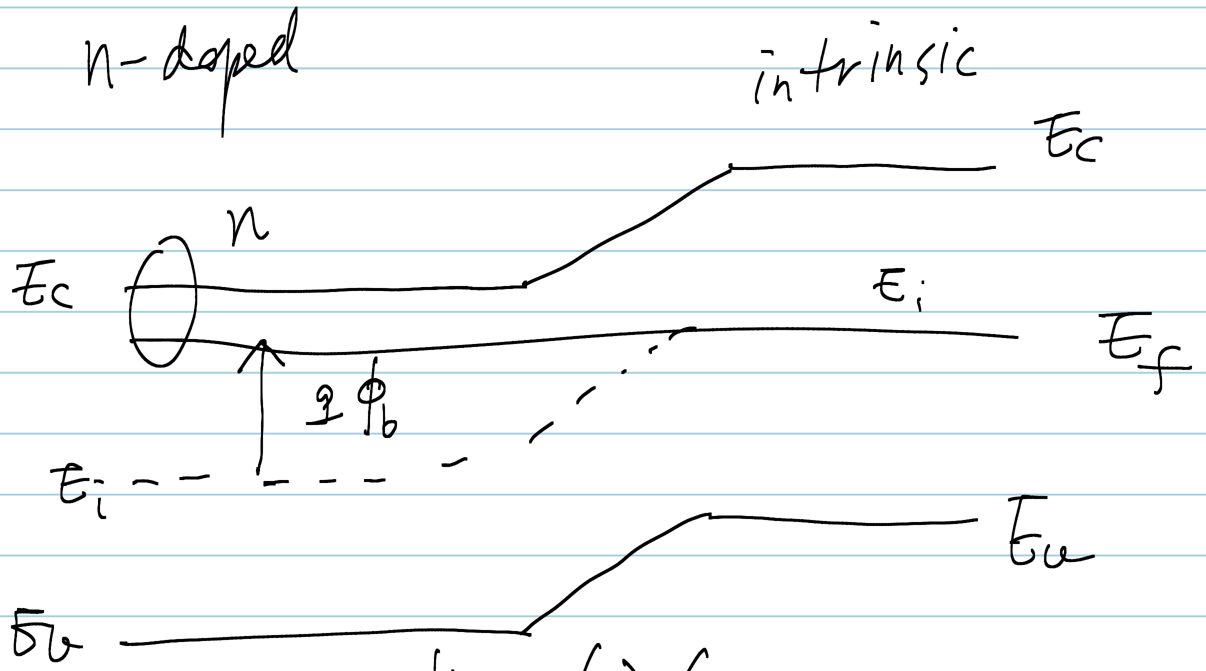


ohmic contact

n. p are at equi. values

Built-in potential of a doped material to the intrinsic reference:

The following uses n-type as example, the equations apply to p-type as is too.



$$n = n_i e^{(\psi - \phi_n)/\phi_t}$$

$$\phi_n \triangleq -\frac{E_{fn}}{q}$$

$$\underline{\phi_b = \phi_t \ln \frac{n_0}{n_i}}$$

built-in

doping

$$n_0 p_0 = n_i^2$$

$$\phi_b = -\phi_t \ln \frac{p_0}{n_i} = \phi_t \ln \frac{n_0}{n_i}$$

So we have listed 3 equations, Poisson, Electron and Hole continuity equations (called Electron and Hole in Sdevice).

We have 3 fundamental variables (well, that is for every grid poin), and 3 equations, that is, potential, p, and n. Everything else is a function of them, e.g. mobility might be a function of efield, which is derivative of potential wrt position, n, and p.

Let us consider the boundary conditions for potential, p and n.

In particular, we care about the **Ohmic contacts** where we apply voltages to our transistors. Recall that we have previously defined the built-in potential with respect to an intrinsic reference phi_b, ϕ_b:

* B.C. for ψ at ohmic contacts

$$\psi = \phi_b + V_{applied}$$

$$n = n_0$$

$$p = p_0$$

$$\begin{cases} n_0 p_0 = n_i^2 \\ p_0 + N_d^+ = n_0 + N_a^- \end{cases}$$ equilibrium

$$N \overset{0}{=} N_d^+ - N_a^-$$ net doping

$$p_0 - n_0 + N = 0$$

$$p_0 - \frac{n_i^2}{p_0} + N = 0$$

$$p_0^2 - n_i^2 + N p_0 = 0$$

$a=1$ c $b=N$

$$N \pm \sqrt{N^2 + 4n_i^2}$$

$a=1 \qquad c \qquad b=N$

$$\left\{\begin{aligned} p=p_0 &= \frac{-N+\sqrt{N^2+4n_i^2}}{2} \\ n=n_0 &= \frac{n_i^2}{p_0} \\ \psi &= V_{applied}+\phi_b \end{aligned}\right.$$

$$= V_{applied}+\phi_t \ln\frac{n_0}{n_i}$$

$$= V_{applied}-\phi_t \ln\frac{p_0}{n_i}$$

$$\phi_n \triangleq -\frac{E_{fn}}{q}$$

$$\phi \triangleq -\frac{E}{q}$$

$$\phi_p \triangleq -\frac{E_{fp}}{q}$$

$$\left\{\begin{matrix} \psi \\ n \\ p \end{matrix}\right. \qquad\qquad \left\{\begin{matrix} \psi \\ \phi_n \\ \phi_p \end{matrix}\right.$$

$$\left\{\begin{array}{l}\psi\\ n\\ p\end{array}\right. \qquad\qquad \left\{\begin{array}{l}\psi\\ \phi_n\\ \phi_p\end{array}\right.$$

$$\left.\begin{array}{l}\psi = \phi_b + V_{applied}\\ n = n_0\\ p = p_0\end{array}\right\} \Leftrightarrow \left\{\begin{array}{l}\psi = \phi_b + V_{applied}\\ \phi_n = V_{applied}\\ \phi_p = V_{applied}\end{array}\right.$$

If you have a good understanding of the band diagram, you can see the equivalence visually. You can of course prove it too:

if we let $\psi = -\frac{E_i}{q}$

$$n = n_i e^{(E_{fn} - E_i)/kT} = n_i e^{(\psi - \phi_n)/\phi_t}$$

As

$$n = n_i e^{(\phi_b + V_{applied} - V_{applied})/\phi_t} = n_i e^{\phi_b/\phi_t}$$

As

$n = n_i e \qquad = n_i e^{\phi_b/\phi_t}$

$$= n_i e^{\phi_t \ln \frac{n_o}{n_i} / \phi_t}$$

$$= n_i \cdot \frac{n_o}{n_i}$$

$$= n_o$$

Similarly $\quad p = p_o$

Quasi Fermi potential and

$\phi_n, \phi_p \sim n, p$ transforms

- These variables are based on statistics of carriers.
- We can also understand them from a pure mathematical treatments

Recall that $\vec{J}_n$ and $\vec{J}_p$ are

$$\frac{\nabla n}{n}$$

$$\vec{J}_n = q\left(n\mu_n \vec{E} + D_n n \frac{\nabla n}{n}\right)$$

$$\langle J_{n1}\rangle \qquad = q\, n\, \mu_n \left(-\nabla \psi + \frac{kT}{q} \nabla \ln n\right)$$

$$\vec{J}_p = q\, p\, \mu_p \left(-\nabla \psi - \frac{kT}{q} \nabla \ln p\right)$$

Define ϕ_n and ϕ_p as

$$\langle J_{n2}\rangle \qquad \vec{J}_n = q\, n\, \mu_n \left(-\nabla \phi_n\right)$$

$$\vec{J}_p = q\, p\, \mu_p \left(-\nabla \phi_p\right)$$

Compare Eq. (J_{n1}) and Eq (J_{n2})

$$\vec{J}_n = q\, n\, \mu_n (-\nabla \phi_n)$$

$$= q\, n\, \mu_n \left(-\nabla \psi + \frac{kT}{q} \nabla \ln n\right)$$

$$\phi_n = \psi + \frac{kT}{q} \ln n + \text{const}$$

We used $\boxed{\begin{gathered} \frac{D}{\mu} = \frac{kT}{q} \\ \frac{d \ln n}{dx} = \frac{1}{n} \frac{dn}{dx} \end{gathered}}$

Compare both sides of "="

$$\phi_n = \psi - \frac{kT}{q} \ln n + \text{const}$$

Similarly, for holes,

$$\vec{J}_p = q p \mu_p (-\nabla \phi_p)$$

$$= q p \mu_p \left(-\nabla \psi - \frac{KT}{q} \nabla \ln p\right)$$

$$\phi_p = \psi + \frac{KT}{q} \ln p + \text{const}$$

Choosing proper values for constants we can always relate n, p to the newly defined variables ϕ_n ϕ_p by

$$n = n_i e^{(\psi - \phi_n)/\phi_t}$$

$$(\phi_p - \psi)/\phi_t$$

$p = n_i e$

Recall that for Boltzman statistics

$$n = n_i e^{(E_{fn} - E_i)/kT} \qquad \psi = -\frac{E_i}{q}$$

$$n = n_i e^{(\psi - \phi_n)/\phi_t} \qquad \phi_n = -\frac{E_{fn}}{q}$$

Similarly, for holes

$$p = n_i e^{(E_i - E_{fp})/kT} \rightarrow$$ Eq. you are more familiar with

$$= n_i e^{(\phi_p - \psi)/\phi_t}$$

$$\Rightarrow \phi_p = -\frac{E_{fp}}{\ } \qquad \psi = -\frac{E_i}{q}$$

$$\Rightarrow \quad \phi_p = -\frac{E_{fp}}{q} \qquad \psi = -\frac{E_i}{q}$$

Therefore $\psi = -\dfrac{E_i}{q}$

$$\phi_n = -\frac{E_{fn}}{q}$$

$$\phi_p = -\frac{E_{fp}}{q}$$

and the ϕ_n, ϕ_p defined are the quasi Fermi-potential of the

Boltzmann statistics.

However the derivation for Fermi

Dirac statistics is more involved.

In many device simulators. ϕ_n ϕ_p are solved as the fundamental variables, as opposed to n and p.

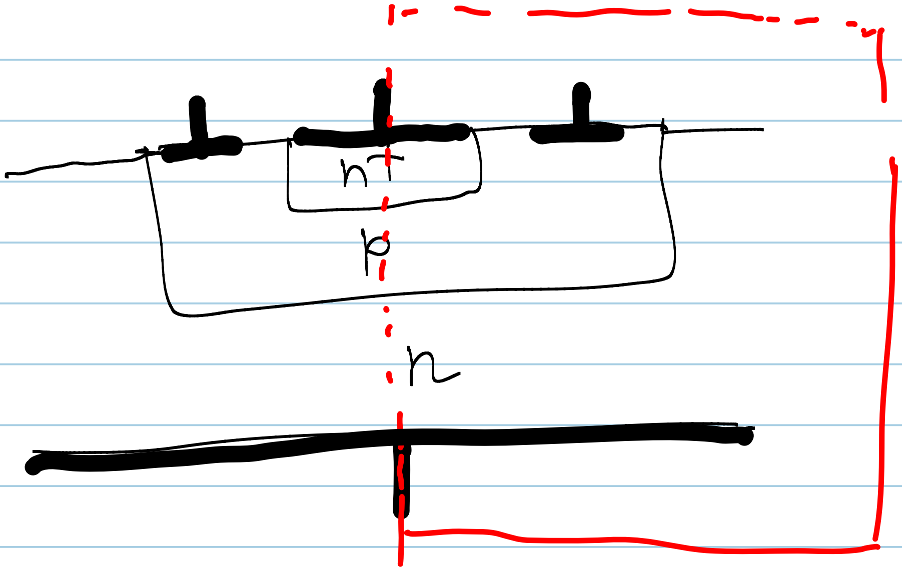
Symmetry Boundary Condition

- * Devices we simulate often have a given geometrical structure.

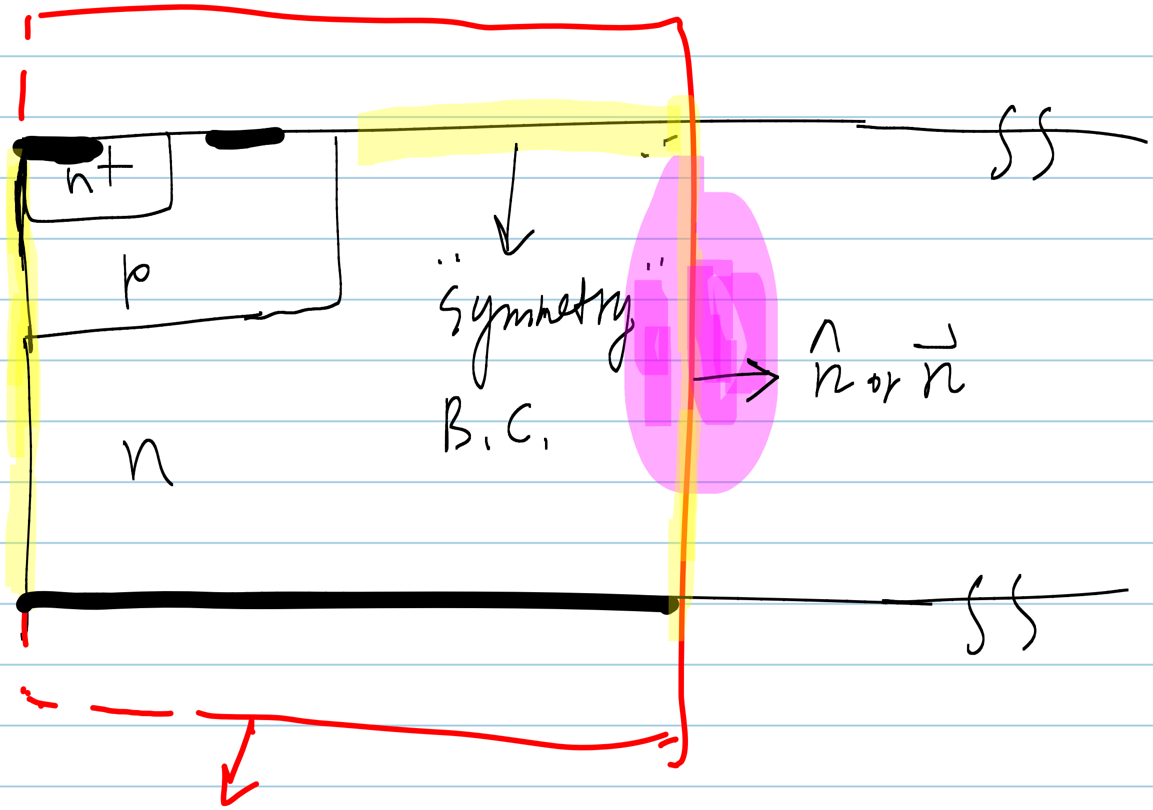
- * We can only simulate the "core" part where device action exists. to save memory and time

- * For devices with symmetry, only half of the device is necessary

* At boundary — well, artificial Boun. Symmetry is useful too, e.g.



* Far away from "core" there is often no "action" — $\vec{J}_n$, $\vec{J}_p$, or $\vec{E}$ field normal to the boundary



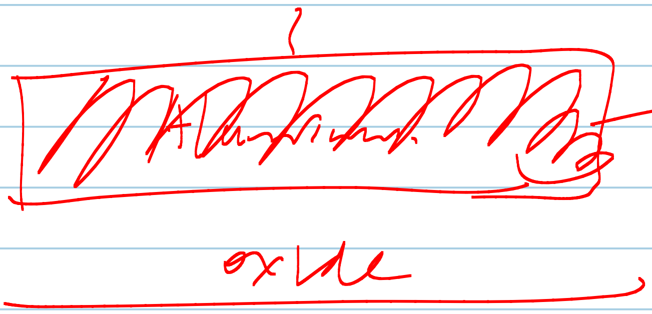
Simulation structure

B) Schottky Contact

$$\psi = \phi_b + V_{applied}$$

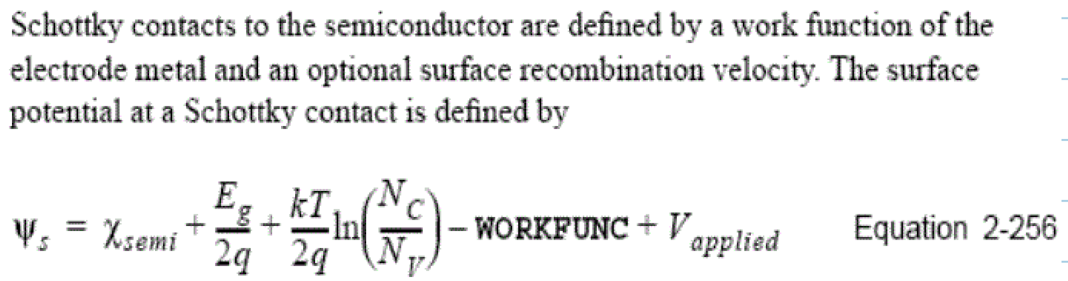
↳ includes schottky barrier height

n, p depend on J_n J_p model



$\phi_b + V_{applied}$

workfunction



Schottky contacts to the semiconductor are defined by a work function of the electrode metal and an optional surface recombination velocity. The surface potential at a Schottky contact is defined by

$$\psi_s = \chi_{semi} + \frac{E_g}{2q} + \frac{kT}{2q}\ln\left(\frac{N_C}{N_V}\right) - \mathbf{WORKFUNC} + V_{applied} \qquad \text{Equation 2-256}$$

Bipolar transistor

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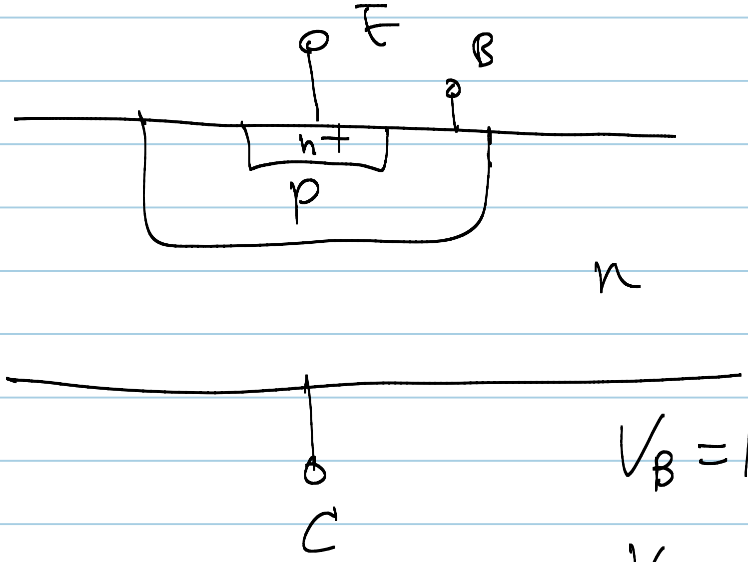
Example: Bipolar

Given



N_E (donor)

N_B (acceptor)

N_C (d)

$V_B = 1V$ $V_C = 2V$

$V_E = 0V$

Find the B.C. at contacts

ψ. ϕ_n. ϕ_p

base $\psi = V_b + \left(-\phi_t \ln \frac{p_0}{n_i}\right)$ $\quad p_0 \approx N_a$

$= N_B$

$= 1V - \phi_t \ln \frac{N_B}{n_i}$

$\phi_n = \phi_p = V_b = 1V$

emitter $\psi = V_e + \left(\phi_t \ln \frac{n_0}{n_i}\right)$ $\quad n \approx N_d^+$

$= N_E$

$= 0V + \phi_t \ln \frac{N_E}{n_i}$

$\phi_n = \phi_p = V_E = 0V$

Collector: $\psi = 2V + \phi_t \ln \frac{N_c}{n_i}$

$\phi_n = \phi_p = 2V$

PN example

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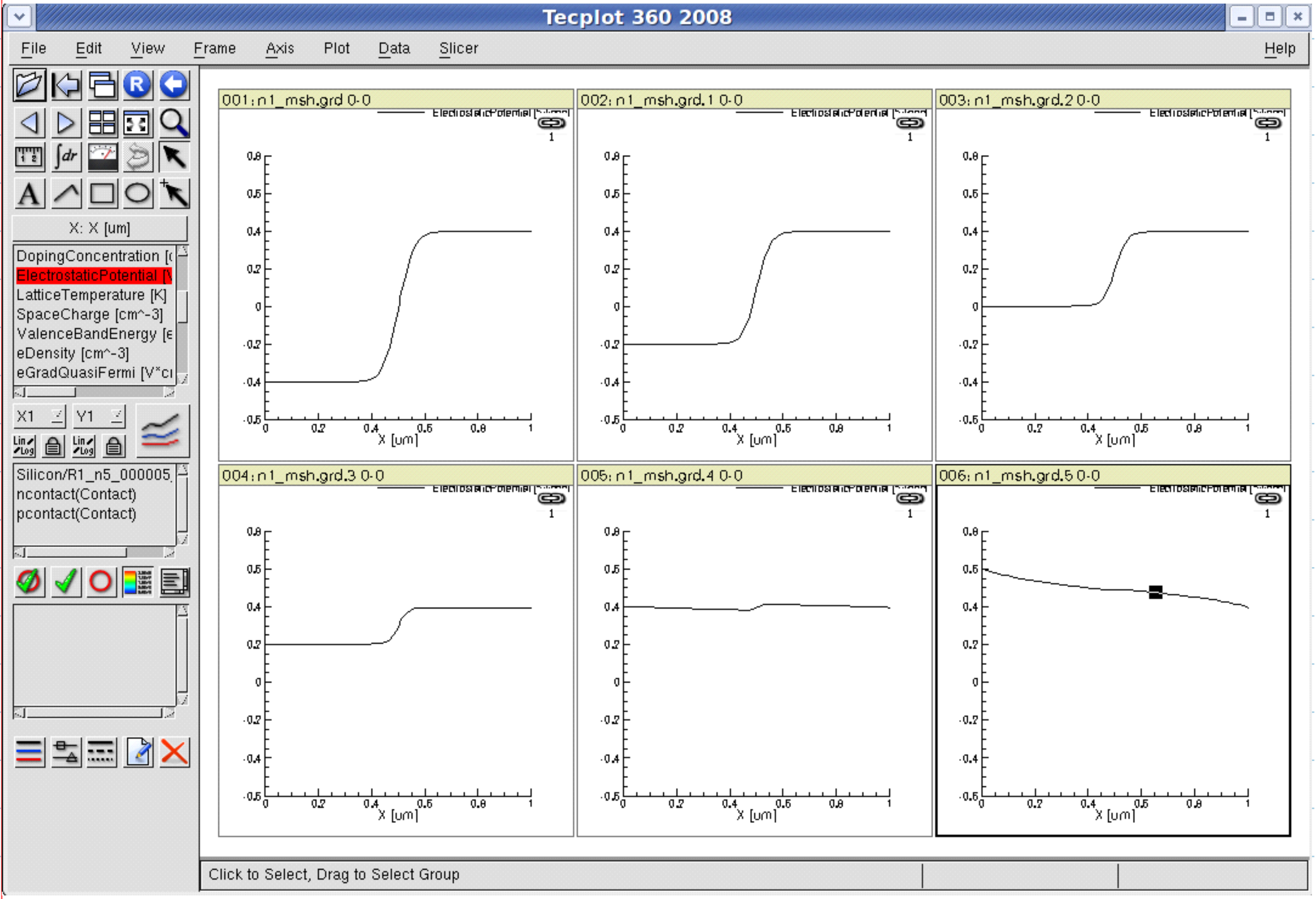
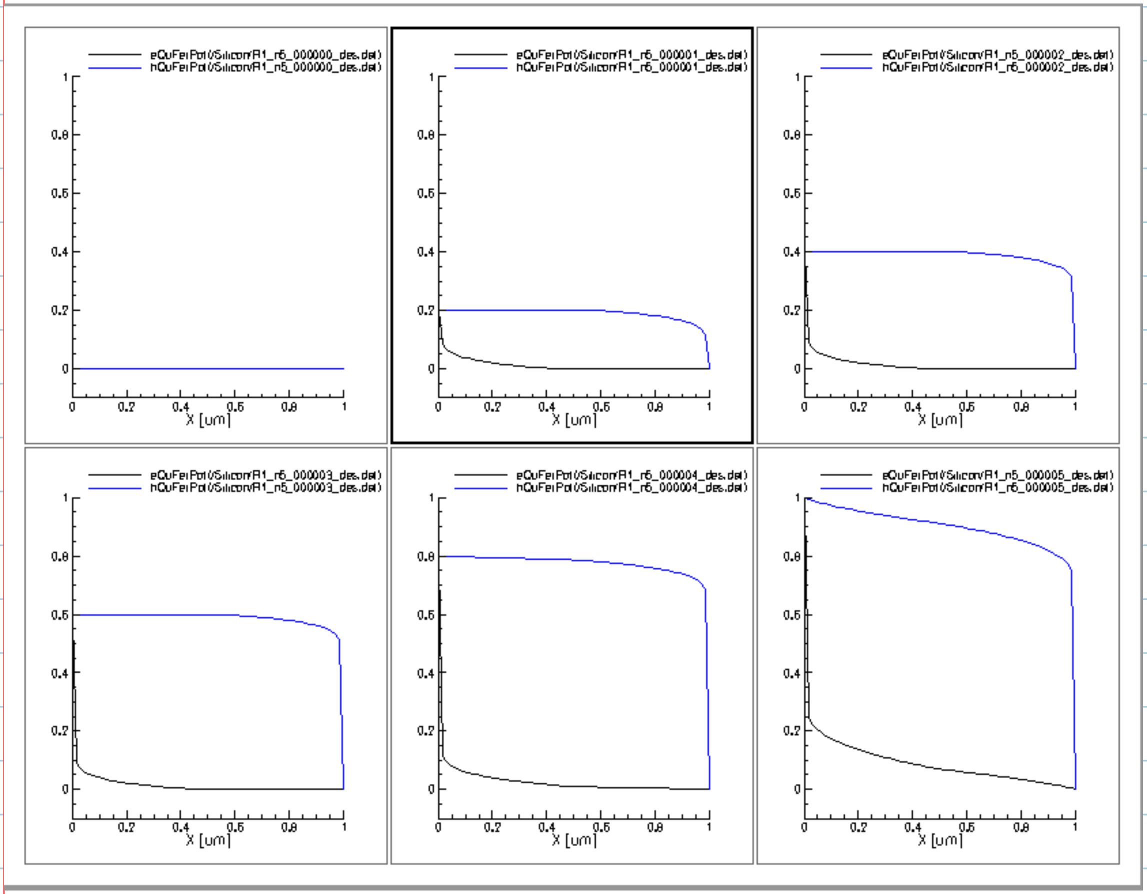
Recall the effective doping on the p-side is 5e16, n-side is 5e16, assume ni=1e10

- Calculate with python the p0 and n0 at both contacts (pcontact and ncontact), when ncontact is grounded, pcontact voltage increases from 0 to 1.0V in 0.2V step. Compare with simulation results at both contacts.



- Calculate the eQuasiFermiPotential and hQuasiFermiPotential at pcontact when pcontact voltage is at 0 - 1.0V in 0.2V step, compare your results with the simulated results.



MOSFET

Sunday, August 26, 2012
12:03 PM

Consider that Ei is approximately in the middle of bandgap. A NMOS transistor has a p-type substrate doping of 1e17/cm^3, and a heavily doped n+ poly gate (treat Poly as Si for our purpose). Find the electrostatic potential at the gate and substrate ohmic contacts in TCAD tools. Assume for simplicity that the Ef is at Ec for N+ poly gate. Eg = 1.12 eV. Ni=1e10/cm3.

physical models

Physical models are needed for solving equations:

For electrons

continuity equations

$$-\frac{1}{q}\nabla\cdot\vec{J}_n + \frac{\partial n}{\partial t} = G_n - R_n$$

For holes

$$\frac{1}{q}\nabla\cdot\vec{J}_p + \frac{\partial p}{\partial t} = G_p - R_p$$

$$\left.\begin{array}{l}\vec{J}_n \\ \vec{J}_p\end{array}\right\} \qquad \begin{array}{l}\vec{J}_n = q\left(n\mu_n\vec{E} + D_n\nabla n\right) \\ \vec{J}_p = q\left(p\mu_p\vec{E} - D_p\nabla p\right)\end{array}$$

Einstein relation $\frac{D}{\mu} = \frac{KT}{q} = \phi_t$

Q We need models for G/R rate

- $\boxed{G/R}$ due to Shockley-Read-Hall (SRH) recombination $\underline{\underline{pn > n_i^2}}$ $\underline{R}$.
- $\underline{\underline{R}}$ due to Auger recombination
- $\underline{\underline{G}}$ due to external light (photo effect)
- $\underline{\underline{G}}$ due to impact ionization
- $\underline{\underline{G}}$ due to heavy ion or alpha particle

* physical models for each mechanism have been developed

The model parameters, however, may

* depend on the fabrication process.

G/R Physics and Models

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SRH

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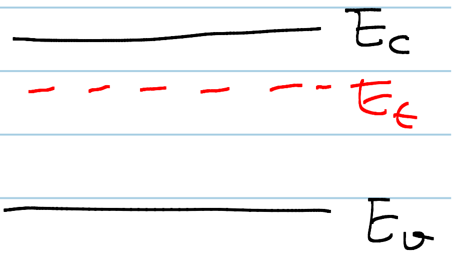
SRH Recombination

* Sometimes the model parameters must be "tuned" to produce meaningful results

$$R_{SRH} = \frac{p \cdot n - n_i^2}{\tau_p(n + \underset{n_t}{\underset{\downarrow}{n_i}}) + \tau_n(p + \underset{p_t}{\underset{\downarrow}{n_i}})}$$

#/cm³/s



SHOCKLEY/READ/HALL Recombination via traps with energy levels in the intrinsic Fermi level - the most effective G/R center energy level. Two parameters are involved, electron lifetime and hole lifetime, they are in general a strong function of doping and processing condition.

For other energy levels, ni and pi are replaced by nt and pt, the n and p values obtained when the Fermi-level is set to the trap energy level.

Auger Recombination

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Auger recombination is important at high doping level or at high injection level found at high VBE in bipolar devices.

$$R_{Aug} = \left(C_n n + C_p p\right) \cdot \left(pn - n_i^2\right)$$

Impact ionization

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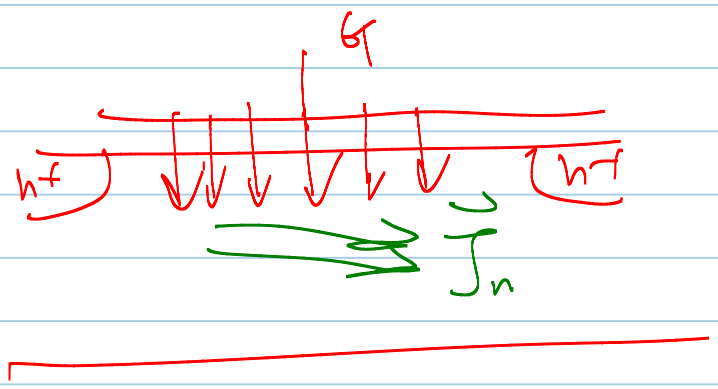
Impact ionization or avalanche multiplication induced Generation of electrons and holes.

Note that only the component of electric field parallel to the current flow direction should be counted for impact ionization, not the total strength of the electric field.

This is extremely important for MOSFET, where the current flow is lateral, and a strong vertical field exists because of the large gate-to-source and gate-to-drain voltages.



Avalache Multiplication

Generation rate due to II or AV is proportional to the local current density, as well as the so called ionization coefficient, the number of e-h pairs generated per distance

$$G_n^{II} = \alpha_n \frac{|\vec{J}_n|}{q} \qquad G^{II} = G_n^{II} + G_p^{II}$$

$$G_p^{II} = \alpha_p \frac{|\vec{J}_p|}{q}$$

ionization rates for electrons and holes defined as generated e-h pairs per unit length of travel and per electron, respectively

For instance, an electron generates over a distance of 1/alpha_n one e-h pair on average.

Both theory and experiment show an exponential dependence of the ionization rates on the electric field component E in direction of current flow

$$\alpha_n = \alpha_n^{\infty} \cdot \exp\left(-\left(\frac{E_n^{crit}}{E}\right)^{\beta_n}\right)$$

Impact ionization or avalanche multiplication induced Generation of electrons and holes.

Note that only the component of electric field parallel to the current flow direction should be counted for impact ionization, not the total strength of the electric field.

This is extremely important for MOSFET, where the current flow is lateral, and a strong vertical field exists because of the large gate-to-source and gate-to-drain voltages.

Mobility

Friday, August 24, 2012
11:25 AM

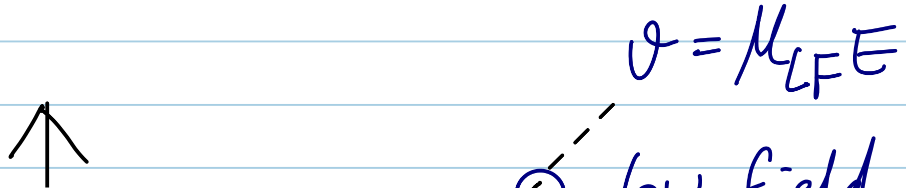
② We need models for mobility, μ, which sets diffusivity as well.

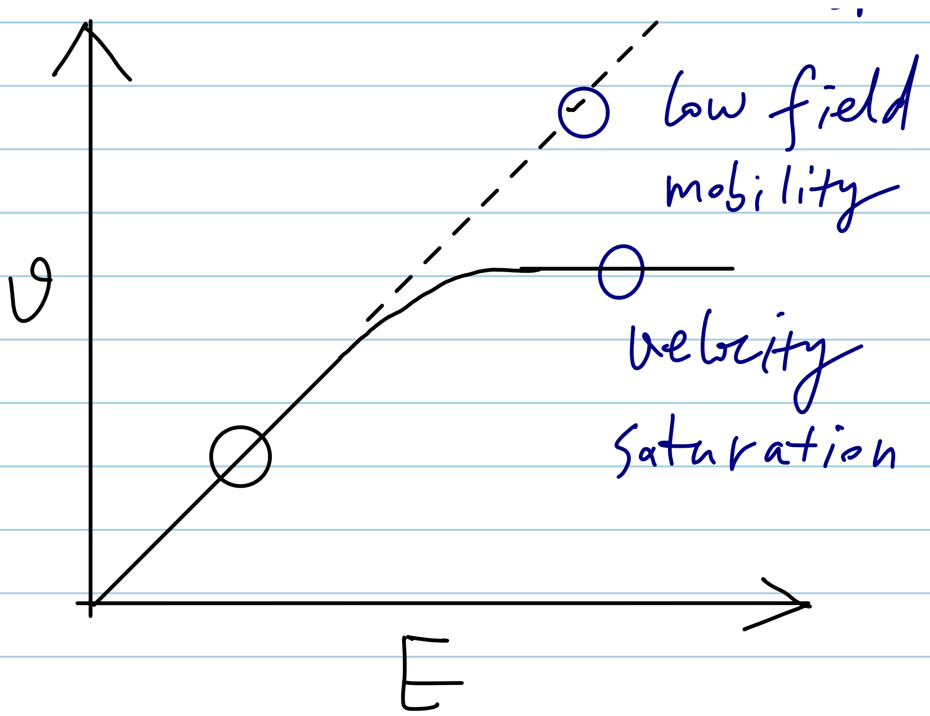
* physical basis, or definition

$$v = \mu E \quad \text{for scalar case}$$

μ is the coefficient of $v \sim E$ relation for pure drift of electrons/holes in a uniform e-field

* Low-field mobility





* Typically, mobility models in simulators refer to the low-field mobility, μ_{LF}

Low field mobility model

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High field mobility model

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* The mobility for high field is modeled separately, this often involves the saturation velocity.

* physics of mobility is extremely complicated and all existing mobility models are empirical or semi empirical.

* For different devices, we shall choose different mobility models

- MOSFET surface mobility must deal with surface roughness scattering

Mobility model selection

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- Bipolar mobility model must deal with the difference between majority carrier mobility and minority carrier mobility
- Temperature, doping dependence must be modeled for all models
- It is important to understand the physical basis and validity range of the mobility model you choose in your simulation
- For bipolar, phllips unified mobility model

is a good one. PHUMOB in MEDICI.

* Mathiessen's rule is often used in mobility modeling

assuming two mechanisms of scattering are completely independent, the net chance of scattering is the sum of the two individual scattering mechanisms

$$\frac{1}{\mu_{total}} = \frac{1}{\mu_A} + \frac{1}{\mu_B}$$

An example using Sah's model

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* mobility model example

Sah model

$$\boxed{\mu_n^I} = 90\,\frac{cm^2}{V\cdot s}\left(1+\frac{2\cdot 10^{18}\,cm^{-3}}{C_I}\cdot\left(\frac{T}{300K}\right)\right)$$

$C_I \rightarrow N_D^+ + N_A^-$

mobility due to ionized dopants scattering alone

$$\boxed{\mu_n^L} = \frac{1}{\dfrac{1}{4195\,\frac{cm^2}{Vs}\left(\frac{T}{300}\right)^{-1.5}} + \dfrac{1}{2153\,\frac{cm^2}{Vs}\left(\frac{T}{300}\right)^{-3.13}}}$$

mobility due to lattice scattering (lattice vibration)

The net mobility due to both lattice & ionized dopants scattering is

$$\mu_n^{LI} = \frac{1}{\frac{1}{\mu_n^L} + \frac{1}{\mu_n^I}}$$

* Strictly speaking, lattice scattering and ionized dopants are correlated, and mathiessen's rule cannot be applied but this mode is actually good for many applications in practice.

for doping $[10^{11}, 10^{20}]$ $/cm^3$

Minority vs Majority Carrier Mobility

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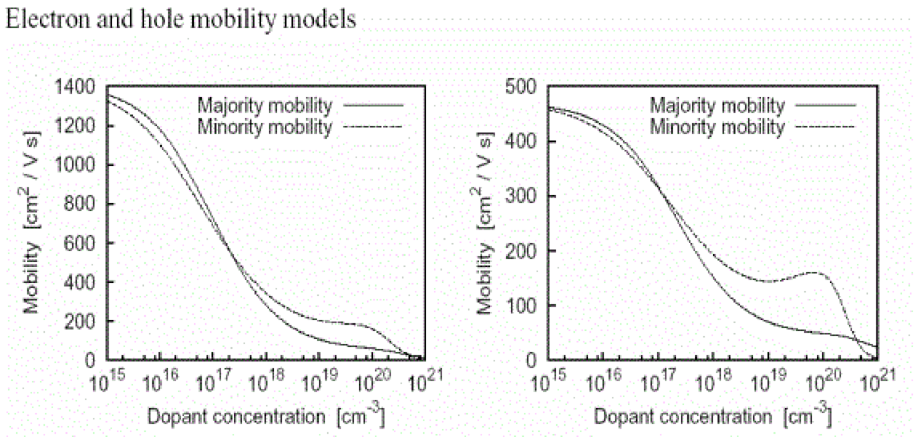
— electrons in n-type (majority)

- — - -- p-type (minority)

can have different mobility

minority versus majority carrier mobility difference

This difference-is typically small for low doping, but becomes significant at high doping levels



Electrons (left), holes (right)

Driving force for high "field" velocity saturation

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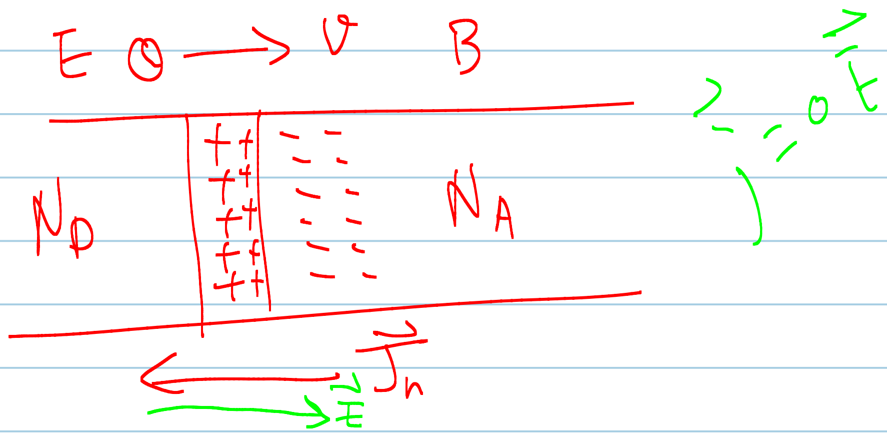
(or decrease of effective mobility)

Consider drift velocity value

$$|\vec{v}_n| = -\mu_n^* \cdot \frac{\vec{E} \cdot \vec{J}_n}{|\vec{J}_n|}$$

note that the $\vec{E}$ field direction is not always the same as the carrier velocity !



The above expression holds only for negligible diffusion current component

* A more accurate way is to use the gradient of ϕ_n as the driving force

$$|\vec{v}_n| = \mu_n^* \cdot |\nabla \phi_n|$$

Q: which one is the driving force of electron current? and thus responsible for velocity saturation?

$\nabla \phi_n$ not $\nabla \psi$

Unfortunately in most books, papers, velocity saturation is attributed to high-electric field using a $v-E$ relation, as we described before, it only holds for homogeneous E-field case

Now define the driving force

$$\vec{E_n} = -\nabla \phi_n$$

"High field" mobility (non-equilibrium)

μ^{HF} $\quad$ μ_n^{LF}

$$\mu_n^{HF} = \frac{\mu_n}{\left(1+\left(\frac{E_n}{E_{n,crit}}\right)^{\beta}\right)^{1/\beta}}$$

$E_{n,crit}$, and β are model parameters

This is the most widely used "Non-equilibrium" mobility model.

$$v_{n,p}^{sat}(T) = \frac{2.4\times 10^{7}}{1+0.8\cdot\exp\left(\frac{T}{600}\right)}$$