

# Relating Local Vision Measurements to Global Navigation Satellite Systems Using Waypoint Based Maps

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**Abstract**—Roughly 50% of all the traffic fatalities are due to lane departures [2]. There is great interest in advanced driver assistance systems that prevent unintended lane departure. Currently there are passive lane detection systems that warn the driver of unintended lane departure know as lane departure warning (LDW) systems which rely on cameras to track lane markings. LDW systems base solely off camera measurements are prone to failures due to poor environmental lighting or poor lane marking coverage. Combining the measurements from multiple sensors will create a much more robust LDW system that is not prone to failures.

The purpose of this paper is to present a method that involves combining measurements from global navigation satellite systems (GNSS) with camera and Light Detection and Ranging (LiDAR) measurements for lane level positioning. These measurements are blended with IMU data using a Kalman filter. When using a Kalman filter [7] to blend the data, the states of the filter are based in a global coordinate frame because GNSS measurements are given in a global coordinate frame. Lane position measurements are given in a local coordinate frame; therefore, lane position measurements must be related to the global coordinate frame in order to incorporate them into the navigation filter.

This paper presents a method of relating local vision measurements to the global coordinate frame. This method can be used in conjunction with pre-existing global based navigation filters. The vision measurements are related to the global coordinate frame using a waypoint map. This method is dependent on the choice of global coordinate frame. Navigation filters based in two popular global coordinate frame are discussed in this paper. The first is the North, East, Down (NED) coordinate frame. The second is the Earth Centered Earth Fixed (ECEF) coordinate frame.

## I. INTRODUCTION

Many vehicle manufacturers are developing lane departure warning (LDW) systems to reduce the number of traffic fatalities that occur due to unintentional lane departures. LDW systems alert the driver when the vehicle has driven outside the bounds of the current lane of travel. Most of the LDW systems in production now are solely based off camera measurements. A LDW camera uses feature extraction to determine lateral position in the current lane. The feature used for a camera-based LDW system is the painted lane lines.

Lane positioning can also be accomplished using a Light Detection and Ranging (LiDAR) scanner. Unlike the camera, a LiDAR scanner can provide three-dimensional ranging information; however, LiDAR does not provide color information. To overcome this, some LiDAR scanners also provide reflectivity data. The reflectivity data can be used to extract lane markings; and the ranging information can be used to provide an estimate of a vehicle's lateral position in the lane.

The objective of this paper is to provide a method of using a lateral lane position measurement to aid a traditional navigation system. Two types of vision integration are discussed in this paper. The first is integration of vision to aid a position, speed, and heading navigation filter. The position, speed, and heading navigation filter is a three degree of freedom (DOF) filter. The states of this filter are based in the NED coordinate frame. The second is integration of vision to aid a navigation filter based in the ECEF coordinate frame. The ECEF based navigation filter is a six DOF filter.

This paper will start with a brief overview of how the camera and LiDAR scanner are used to measure a vehicles lateral position in a lane. Next, the necessary coordinate frame rotations are discussed. The next sections cover what information is needed from the waypoint maps. The final two sections discuss integration and results for the two different navigation filters.

## II. LANE POSITIONING METHODS

Two methods of lane positioning are presented in this section. The first is using a LiDAR scanner for lane positioning. Currently, LiDAR scanners are very expensive. The cost of LiDAR scanners has prevented implementation of LiDAR based LDW systems on civilian vehicles; however, the cost of LiDAR scanners is expected to decrease as demand for these devices increases. The second method of lane positioning involves using a camera. Ideally, both cameras and LiDAR scanners will be used in future LDW systems. Each device has different strengths and weaknesses. Using both devices will improve the robustness of both the LDW system and navigation system.

### A. Lane Positioning with LiDAR

Light Detection and Ranging (LiDAR) measures the range to an object by pulsing a light wave at the object. The light wave for LiDAR applications is a laser. LiDAR is very similar to sonar, but instead of using a sound wave, LiDAR uses a light wave. LiDAR scanners combine the laser with a moving mirror that rotates the laser's beam. This can provide ranging information in three dimensions. LiDAR can also provide reflectivity information on the surface the laser is intersecting. Surface reflectivity in units of echo width. Reflectivity data can be used to classify objects the LiDAR encounters.

A LiDAR scanner with reflectivity measurements can be used to search for and range off of lane markings. Painted lane markings are more reflective than the asphalt that surrounds the lane marking, therefore the echo width data from a LiDAR scanner can be used to find lane markings, and then the ranging information from the LiDAR can be used to determine the vehicles lateral distance from the lane marking. More information regarding how LiDAR can be used to determine lane position can be found in [3].

One advantage of using a LiDAR scanner based LDW system is the robustness of LiDAR scanners to varying lighting and weather conditions. Unlike a camera, LiDAR scanners work independent of surrounding lighting conditions. Currently, the largest disadvantage of LiDAR based LDW systems is the cost of the hardware. LiDAR is a relatively new technology, and there are a limited number of manufactures. Also, using LiDAR scanners to detect lane markings is a new and undeveloped science.

### B. Lane Positioning with Camera

Cameras are the most popular type of hardware used to determine lane position for LDW systems. Camera based LDW systems are available as an option on some production vehicles. The first step of camera based lane positioning is lane line extraction. This involves searching the image for a lane marking. Thresholding is the process of filtering out unwanted features in the image. Thresholding is used to extract areas of the image that are white to yellow in color. Then, edge detection and the Hough transform are used to extract areas of the image that have lines or edges. Other camera-based lane detection systems employ optical flow, neural networks, and alternatives to the Hough transform such as peak and edge finding. The vehicles position in the lane can

be estimated after the image is searched for lane markings. One simple method of estimating lateral lane position is counting the number of pixels from the center of the image to the lane marking. The number of pixels is multiplied by a scale factor to determine the distance from the center of the vehicle to the lane line. More information regarding how cameras can be used to determine lane position can be found in [8].

One advantage of using a camera based LDW system is the cost of the hardware involved. Digital cameras have been in production for decades, and the cost of these devices is relatively cheap. Also, methods of lane positioning using a camera were pioneered in the 90's; therefore, the algorithms used for lane positioning using a camera are well established. Some disadvantages to camera based LDW systems include vulnerability to lighting and weather conditions. At dawn and dusk, when the sun is low in the sky, a camera may be blinded by the sun. Also, camera based lane detection can be difficult in urban environments where lane markings are in poor condition or visibility of lane markings are blocked by surrounding traffic.

## III. COORDINATE FRAME ROTATION AND TRANSLATION

Lane position measurements are given in the road based coordinate frame. The road based coordinate frame can be approximated with a waypoint based map. A rotation matrix is need to rotate coordinates in the navigation's coordinate frame to the road coordinate frame because the navigation filter's coordinate frame is not oriented the same way as the road coordinate frame.

A rotation matrix is a matrix that if multiplied by a vector of values expressed in an initial coordinate frame will result in a vector of values expressed in a new coordinate frame. The new coordinate frame has some different attitude from the initial coordinate frame. The difference in attitude will govern the values in the rotation matrix. A rotation matrix can be constructed using Euler angles. The rotational direction of Euler angles are based off the right handed coordinate system. In short, rotation about an axis that points towards the observer results in a counter-clockwise positive rotation. Rotation about an axis that points away from the observer results in a clockwise positive rotation. Rotation matrices are orthogonal; therefore, the inverse of a rotation matrix is equal to its transpose. Equation (1) describes this principle. Rotation matrices are denoted by  $C_{\alpha}^{\beta}$ . This rotation matrix maps coordinates in the  $\alpha$  coordinate frame to the  $\beta$  coordinate frame.  $e$  denotes the ECEF coordinate frame.  $n$  denotes the North East coordinate frame.  $r$  denotes the road coordinate frame.

$$C_{\alpha}^{\beta^{-1}} = C_{\alpha}^{\beta T} = C_{\beta}^{\alpha} \quad (1)$$

### A. Two-Dimensional Rotation

The position, speed, and heading navigation filter employs a two-dimensional navigation coordinate frame. Two-Dimensional coordinate frame rotation matrices are 2x2 matrices based off one attitude angle. Equation (2) shows a two-dimensional rotation matrix based off the attitude angle  $\theta$ .

$$C_{(i,j)}^{(i',j')} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \quad (2)$$

### B. Three-Dimensional Rotation

Three-Dimensional coordinate frame rotation matrices are 3x3 matrices based off three Euler angles [6]. A three-dimensional coordinate frame rotation can be thought of as a series of three two-dimensional rotations. The first rotation is about the z (k) axis. The second rotation is about the new y (j') axis. The third rotation is about the new x (i'') axis. Equation (3) shows how the three-dimensional rotation matrix is constructed. s1 is the sine of  $\theta_1$  and c1 is the cosine of  $\theta_1$ . s2 is the sine of  $\theta_2$  and c2 is the cosine of  $\theta_2$ . s3 is the sine of  $\theta_3$  and c3 is the cosine of  $\theta_3$ .

$$C_{(i,j)}^{(i'',j'',k''')} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & s_1 \\ 0 & -s_1 & c_1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{bmatrix} \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_2 c_3 & & \\ s_1 s_2 c_3 - c_1 s_3 & & \\ c_1 s_2 c_3 + s_1 s_3 & & \end{bmatrix} \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{bmatrix} \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

### C. Coordinate Frame Translation

The origin of the road based coordinate frame is not located at the same point in space as the origin of the navigation coordinate frame origin. When mapping coordinates in the navigation coordinate frame to the road coordinate frame, the coordinate frame must be moved and rotated. Fig. 1 shows a two dimensional example of this.

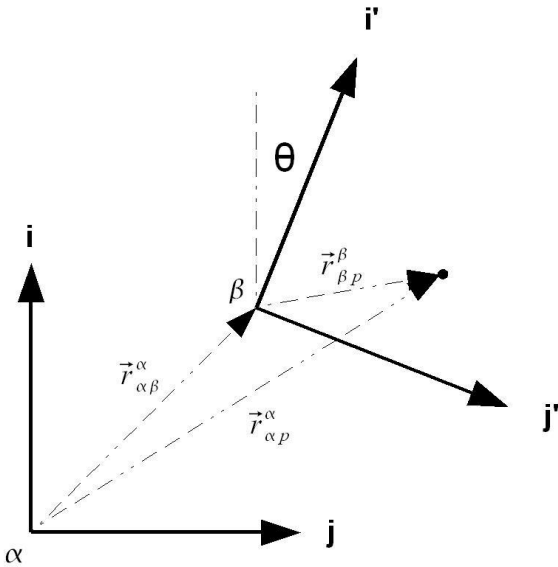


Figure 1: An example of a two-dimensional coordinate frame translation and rotation

Equation (4) shows how to map coordinates in coordinate frame that has been moved and rotated  $\vec{r}_{\alpha p}^{\alpha}$  is the position vector expressed in the initial coordinate frame ( $\alpha$ ), and  $\vec{r}_{\beta p}^{\beta}$  is the position vector expressed in the rotated and translated coordinate frame ( $\beta$ ).  $\vec{r}_{\alpha\beta}^{\alpha}$  is a vector expressed in the initial coordinate that points from the initial coordinate frame to the

new coordinate frame.  $C_{\alpha}^{\beta}$  is the rotation matrix from coordinate frame alpha to coordinate frame beta.

$$\vec{r}_{\beta p}^{\beta} = C_{\alpha}^{\beta} [\vec{r}_{\alpha p}^{\alpha} - \vec{r}_{\alpha\beta}^{\alpha}] \quad (4)$$

Coordinates in the final coordinate frame can be mapped back to the initial coordinate frame using (5).

$$\vec{r}_{\alpha p}^{\alpha} = C_{\beta}^{\alpha} \vec{r}_{\beta p}^{\beta} + \vec{r}_{\alpha\beta}^{\alpha} \quad (5)$$

### D. Global Coordinate Frame Rotations

Sometimes it may be necessary to map coordinates in the Earth Centered Earth Fixed (ECEF) coordinate frame to the North East Down (NED) coordinate frame. The North East plane of the NED coordinate frame is tangential to a reference ellipsoid. The reference ellipsoid is an ellipsoid that mimics the surface of the earth. Using the latitude ( $\phi$ ) and longitude ( $\lambda$ ) of the origin of the NED coordinate frame, a rotation matrix that maps ECEF coordinates to NED coordinates can be constructed. Equation (6) shows the rotation matrix that maps coordinates in the ECEF coordinate frame to the NED coordinate frame. The transpose of this matrix will map coordinates from the NED coordinate frame to the ECEF coordinate frame.

$$C_{\epsilon}^n = \begin{bmatrix} -\sin(\phi) \cos(\lambda) & -\sin(\phi) \sin(\lambda) & \cos(\phi) \\ -\sin(\lambda) & \cos(\lambda) & 0 \\ -\cos(\phi) \cos(\lambda) & -\cos(\phi) \sin(\lambda) & -\sin(\phi) \end{bmatrix} \quad (6)$$

Equation (7) converts ECEF position coordinates ( $\vec{r}_{\epsilon p}^{\epsilon}$ ) to NED position coordinates ( $\vec{r}_{np}^n$ ). Notice that, along with the latitude and longitude of the origin of the NED coordinate frame, the position of the origin expressed in the ECEF coordinate frame ( $\vec{r}_{\epsilon n}^{\epsilon}$ ) must be known. Methods of going between geodetic (lat, lon, height) coordinates and ECEF coordinates can be found in [4].

$$\vec{r}_{np}^n = C_{\epsilon}^n [\vec{r}_{\epsilon p}^{\epsilon} - \vec{r}_{\epsilon n}^{\epsilon}] \quad (7)$$

Equation (8) converts NED position coordinates to ECEF position coordinates.

$$\vec{r}_{\epsilon p}^{\epsilon} = C_{\epsilon}^{nT} \vec{r}_{np}^n + \vec{r}_{\epsilon n}^{\epsilon} \quad (8)$$

Equation (9) converts ECEF velocity coordinates to NED velocity coordinates. Notice that velocity mapping is independent of the position of the NED coordinate frame. The latitude and longitude of the NED coordinate frame is still need to construct the rotation matrix.

$$\vec{v}_n = C_{\epsilon}^n \vec{v}_{\epsilon} \quad (9)$$

Equation (10) converts NED velocity coordinates to ECEF velocity coordinates.

$$\vec{v}_{\epsilon} = C_{\epsilon}^{nT} \vec{v}_n \quad (10)$$

## IV. WAYPOINT BASED MAPS

The road coordinate frame is the coordinate frame in which the vision measurements are given. The road frame can be approximated using a waypoint map. The waypoints lie in the

center of the lane which is being mapped. The distance between waypoints should be defined by the road geometry. Complex road geometry will require waypoints to be close together. For example, in a turn, the waypoints will need to be close together to match the geometry of the road. On a straightaway, the waypoints can be very spread out due to the lack of change in road geometry.

In order to use the techniques discussed in this paper, the lane map must contain three pieces of information for each waypoint. The first is the position of the waypoint expressed in the navigation coordinate frame. The second piece of information is the attitude of the road with respect to the navigation coordinate frame. The attitude of the road is determined by looking at the attitude of the line segment created between the waypoint and the next waypoint. The last piece of information is the distance between the waypoint and the next waypoint. The position of the waypoints are assumed to be given in the ECEF coordinate frame from a survey of the lane.

#### A. 3 DOF Map

$$\text{Map Database} = \begin{bmatrix} N_{r,1} & E_{r,1} & \psi_{r,1} & d_{r,1} \\ \vdots & \vdots & \vdots & \vdots \\ N_{r,m} & E_{r,m} & \psi_{r,m} & d_{r,m} \end{bmatrix} \quad (11)$$

Equation (11) shows an example map database that can be used to aid the position, speed, and heading (3 DOF) navigation system.  $N_{r,i}$  and  $E_{r,i}$  are the position coordinates of the  $i^{\text{th}}$  waypoint. The position of a waypoint in the NED coordinate frame can be derived from the position of the waypoint in the ECEF coordinate frame using techniques covered in the global coordinate frame rotations section. The position of the navigation coordinate frame in ECEF coordinates must be known.

$\psi_{r,i}$  is the heading of the  $i^{\text{th}}$  road coordinate frame. The heading of the road coordinate frame is measured from the north axis with clockwise rotation being positive. The heading of the road can be solved using (12).

$$\psi_{r,i} = \text{atan2}([E_{r,i+1} - E_{r,i}], [N_{r,i+1} - N_{r,i}]) \quad (12)$$

$d_{r,i}$  is the distance from waypoint  $i$  to waypoint  $i+1$ . The distance between the waypoints can be determined using the distance equation (13).

$$d_{r,i} = \sqrt{(E_{r,i+1} - E_{r,i})^2 + (N_{r,i+1} - N_{r,i})^2} \quad (13)$$

#### B. 6 DOF Map

Equation (14) represents an example map database that can be used for the ECEF based (6 DOF) navigation filter.  $\vec{r}_{er,i}^e$  is the position vector of waypoint  $i$  in the ECEF coordinate frame. Since the waypoint map is assumed to be surveyed in the ECEF coordinate frame,  $\vec{r}_{er,i}^e$  will come directly from the survey.

$$\text{Map Database} = \begin{bmatrix} \vec{r}_{er,1}^e & \vec{\varphi}_1 & d_{r,1} \\ \vdots & \vdots & \vdots \\ \vec{r}_{er,m}^e & \vec{\varphi}_m & d_{r,m} \end{bmatrix} \quad (14)$$

$\vec{\varphi}_i$  is the attitude vector of the road coordinate frame. The attitude is represented the same way as the attitude states of the vehicle, with three Euler angles. The form of the rotation matrix from the ECEF coordinate frame to the road coordinate frame is given in (15). The elements from the road coordinate frame attitude ( $\vec{\varphi}_i$ ) are used to construct the rotation matrix.  $c_1$  is the cosine of the first attitude angle in the attitude vector.  $s_1$  is the sine of the first attitude angle in the attitude vector. Similarly,  $c_2$  and  $s_2$  are the trigonometric functions of the second angle, and  $c_3$  and  $s_3$  are the trigonometric functions of the third angle.

$$C_e^r = \begin{bmatrix} c_2 c_3 & c_2 s_3 & -s_2 \\ s_1 s_2 c_3 - c_1 s_3 & s_1 s_2 s_3 + c_1 c_3 & s_1 c_2 \\ c_1 s_2 c_3 + s_1 s_3 & c_1 s_2 s_3 - s_1 c_3 & c_1 c_2 \end{bmatrix} \quad (15)$$

The rotation matrix (15) can be thought of as a sequence of two rotations (16). The first rotation is from the ECEF coordinate frame to the north, east, down (NED) coordinate frame ( $C_n^e$ ). This rotation matrix is based off two angles.  $\phi_i$  is the latitude of the  $i^{\text{th}}$  coordinate frame,  $\lambda_i$  is the longitude of the  $i^{\text{th}}$  coordinate frame. The origin of the NED coordinate frame is at the  $i^{\text{th}}$  waypoint ( $\vec{r}_{er,i}^e$ ). The second rotation is from the NED coordinate frame to the road coordinate frame ( $C_e^n$ ). This rotation matrix is also based off two angles.  $\psi_i$  is the heading of the  $i^{\text{th}}$  coordinate frame,  $\theta_i$  is the pitch of the  $i^{\text{th}}$  coordinate frame. Since the geometry of the waypoints does not contain any information on the road bank, this angle is assumed to be zero.

$$C_e^r = C_n^e C_e^n \quad (16)$$

Equation (17) shows the form of the rotation matrix that maps coordinates in the ECEF coordinate frame to the NED coordinate frame. The longitude and latitude of the  $i^{\text{th}}$  road coordinate frame correspond to the global position of the  $i^{\text{th}}$  waypoint. The latitude and longitude can either be surveyed, or solved for using the position of the  $i^{\text{th}}$  waypoint. Transformations between ECEF coordinates and geodetic coordinates can be found in [4].

$$C_e^n = \begin{bmatrix} -\sin(\phi_i) \cos(\lambda_i) & -\sin(\phi_i) \sin(\lambda_i) & \cos(\phi_i) \\ -\sin(\lambda_i) & \cos(\lambda_i) & 0 \\ -\cos(\phi_i) \cos(\lambda_i) & -\cos(\phi_i) \sin(\lambda_i) & -\sin(\phi_i) \end{bmatrix} \quad (17)$$

Equation (18) shows the form of the rotation matrix that maps coordinates in the NED coordinate frame to the road coordinate frame. The pitch and heading angles can be solved by looking at the change in position between waypoint  $i$  and waypoint  $i+1$ . The first step in solving for these angles is to express the position of the  $i+1$  waypoint in the NED coordinate frame based at the  $i^{\text{th}}$  waypoint. This can be done using (19).

$$C_e^n = \begin{bmatrix} \cos(\theta_i) \cos(\psi_i) & \cos(\theta_i) \sin(\psi_i) & -\sin(\theta_i) \\ -\sin(\psi_i) & \cos(\psi_i) & 0 \\ \sin(\theta_i) \cos(\psi_i) & \sin(\theta_i) \sin(\psi_i) & \cos(\theta_i) \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} N_{r,i} \\ E_{r,i} \\ D_{r,i} \end{bmatrix} = C_e^n [\vec{r}_{er,i+1}^e - \vec{r}_{er,i}^e] \quad (19)$$

Once the position of waypoint  $i+1$  is expressed in the NED coordinate frame based at waypoint  $i$ , (20) and (21) can be used to solve for the road coordinate frame heading and pitch.

$$\psi_i = \arctan2(E_{r,i}, N_{r,i}) \quad (20)$$

$$\theta_i = \arctan\left(\frac{-D_{r,i}}{\sqrt{N_{r,i}^2 + E_{r,i}^2}}\right) \quad (21)$$

The longitude ( $\lambda_i$ ), latitude ( $\phi_i$ ), pitch ( $\theta_i$ ), and heading ( $\psi_i$ ) are all plugged into their corresponding rotation matrices. The result of (16) will be a rotation matrix that maps coordinates in the ECEF coordinate frame to the road coordinate frame. The three attitude angles in the map database ( $\vec{\varphi}_i$ ) can be extracted from this rotation matrix. Equations (22) through (24) show how to solve for the road coordinate frame attitude using the longitude, latitude, pitch (road grade), and heading angles. The relationship between attitude angles and rotation matrices is derived in [5].

$$\vec{\varphi}_{i,1} = \arctan2(-\cos(\phi_i) \sin(\psi_i), \cos(\phi_i) \sin(\theta_i) \cos(\psi_i) - \sin(\phi_i) \cos(\theta_i)) \quad (22)$$

$$\vec{\varphi}_{i,2} = \arcsin(\cos(\phi_i) \cos(\theta_i) \cos(\psi_i) + \sin(\phi_i) \sin(\theta_i)) \quad (23)$$

$$\vec{\varphi}_{i,3} = \arctan2(-\sin(\phi_i) \sin(\lambda_i) \cos(\theta_i) \cos(\psi_i) + \cos(\lambda_i) \cos(\theta_i) \sin(\psi_i) + \cos(\phi_i) \sin(\lambda_i) \sin(\theta_i), -\sin(\phi_i) \cos(\lambda_i) \cos(\theta_i) \cos(\psi_i) - \sin(\lambda_i) \cos(\theta_i) \sin(\psi_i) + \cos(\phi_i) \cos(\lambda_i) \sin(\theta_i)) \quad (24)$$

$d_{r,i}$  is the distance from waypoint  $i$  to waypoint  $i+1$ . This value is used to check if the vehicle has passed the next waypoint. The distance between the waypoints can be determined using the distance equation (25).

$$d_{r,i} = \sqrt{(\vec{r}_{er,i+1}^e - \vec{r}_{er,i}^e)^T (\vec{r}_{er,i+1}^e - \vec{r}_{er,i}^e)} \quad (25)$$

## V. 3DOF MEASUREMENT STRUCTURE

The position, speed, and heading navigation system is a three degree of freedom navigation filter based in a two-dimensional, rectangular coordinate frame. The navigation coordinate frame for this filter is the north east coordinate frame. The north east coordinate frame is denoted by  $n$ . The  $x$ -axis of the navigation coordinate frame points north and the  $y$ -axis of the coordinate frame points east. The road coordinate frame for the position, speed, and heading navigation system is also a two-dimensional coordinate frame. The road coordinate

frame is denoted by  $r$ . The  $x$ -axis of the road coordinate frame points from the last waypoint passed to the next waypoint. The  $y$ -axis of the road coordinate frame is perpendicular to the  $x$ -axis. If facing with the  $x$ -axis, the  $y$ -axis points to the left.

$$C_n^r = \begin{bmatrix} \cos(\psi_r) & \sin(\psi_r) \\ -\sin(\psi_r) & \cos(\psi_r) \end{bmatrix} \quad (26)$$

The rotation matrix in (26) represents the rotation matrix that maps coordinates in the north east (navigation) coordinate frame to the road coordinate frame. The angle  $\psi_{r,i}$  is the heading of the  $i^{\text{th}}$  road coordinate frame. This angle should not be confused with the heading state of the filter.

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = C_n^r \begin{bmatrix} \hat{N} - N_r \\ \hat{E} - E_r \end{bmatrix} \quad (27)$$

In order to use lateral lane position measurements, the lateral lane position with respect to the lane map must be estimated using the current states of the navigation filter.  $\hat{N}$  and  $\hat{E}$  denote the current position estimate in the north east coordinate frame.  $N_r$  and  $E_r$  denote the position of the last waypoint passed in the north east coordinate frame.  $\hat{x}$  and  $\hat{y}$  denote the estimated position in the road coordinate frame. Equation (27) is used to find the position estimates in the road coordinate frame.

$$h(x) = \hat{y} = -(\hat{N} - N_{r,i}) \sin(\psi_{r,i}) + (\hat{E} - E_{r,i}) \cos(\psi_{r,i}) \quad (28)$$

Equation (28) shows the measurement equation. The measurement equation is a function of the states of the filter and the map parameters. The matrix in (30) shows the measurement model, assuming the state matrix takes the form of (29). The measurement model is created by taking the partial derivative of the measurement equation with respect to each state.  $\psi_{r,i}$  is the heading of the current ( $i^{\text{th}}$ ) road coordinate frame, not the head state of the filter.

$$x = [v \quad b_a \quad \psi \quad b_r \quad N \quad E]^T \quad (29)$$

$$H(x) = [0 \quad 0 \quad 0 \quad 0 \quad -\sin(\psi_{r,i}) \quad \cos(\psi_{r,i})] \quad (30)$$

After every measurement update and time update, the states of the navigation filter will change. Every time the states of the filter are updated, the distance into the current road frame must be checked to insure that the vehicle has not passed into the next road coordinate frame. Plugging the position states into (31) will result in the current longitudinal distance into the road coordinate frame. If this value is larger than  $d_{r,i}$ , then the vehicle has passed into the next coordinate frame. The map index  $i$  should then be incremented by one.

$$\hat{x} = (N - N_{r,i}) \cos(\psi_{r,i}) + (E - E_{r,i}) \sin(\psi_{r,i}) \quad (31)$$

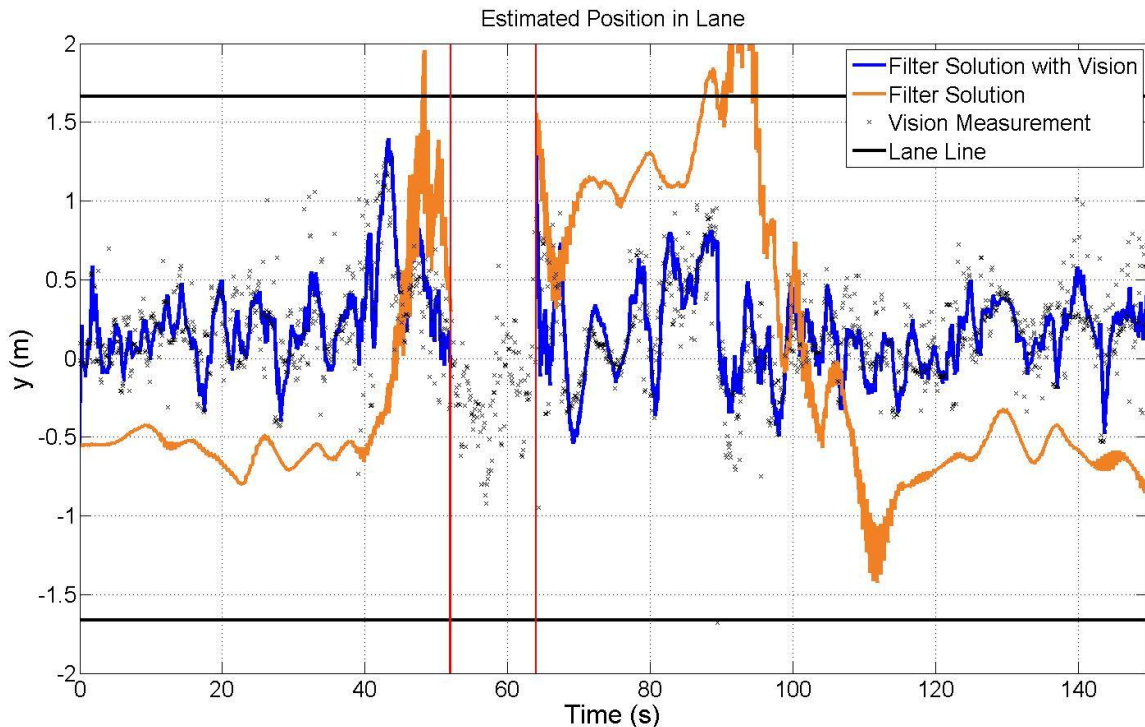


Figure 2: Estimated Lane Position of the 3DOF Navigation Filters

Fig. 2 shows the utility of using vision to aid a GPS based navigation filter. Fig. 2 is a plot of the lateral lane position estimates. This data for these results was collected at the NCAT test track, which is a 1.8 mile, two lane oval. The track is set up very similar to the interstate system. The orange line represents the navigation filter's estimated lane position when only GPS measurements are used to update the filter. GPS position is a bias measurement. These biases make it difficult to compare a GPS position measurement and a map to get vehicle position within a lane. The blue line is the result of the vision aided navigation filter. Unlike the GPS only aided filter, the estimated lane position reported by the vision aided navigation filter is not biased. The section in between the red lines represents when the vehicle passed into a section of the track that was not surveyed. Since the navigation filter is not vision dependent, the filter is still operating without a map using only GPS updates.

## VI. 6DOF MEASUREMENT STRUCTURE

The ECEF based navigation filter is a six degree of freedom navigation filter. The filter tracks the global position and velocity of a vehicle. The position and velocity estimates are based in the earth centered earth fixed (ECEF) coordinate frame. The ECEF coordinate frame is a three degree of freedom coordinate frame and is denoted by  $e$ . The closely coupled navigation filter also tracks the attitude of the vehicle. The attitude is represented by the three Euler angles. Along with the pose of the vehicle, the navigation filter tracks accelerometer biases, gyro biases, GPS clock bias, and GPS clock drift. A more in depth look at the structure of the ECEF based navigation filter can be found in [5].

The goal of this section is to describe how a lateral lane position measurement can be incorporated into the ECEF based navigation filter. Similar to the position speed and heading filter, a road map is necessary to include lane position measurements. The road coordinate frame for the closely coupled navigation filter is similar to the road coordinate frame for the position, velocity, heading filter. The road coordinate frame for the closely coupled navigation filter is a three degree of freedom coordinate frame and is denoted by the  $r$ . The  $x$ -axis of the road coordinate frame points from the last waypoint passed to the next waypoint. The  $y$ -axis of the road coordinate frame is perpendicular to the  $x$ -axis. If facing with the  $x$ -axis, the  $y$ -axis points to the left. The road coordinate frame is assumed to have no bank; therefore, the  $y$ -axis is always parallel with the plane tangent to earth's reference ellipsoid. The  $z$ -axis is perpendicular to the  $x$ - $y$  plane and points down.

In order to use lateral lane position measurements, the lateral lane position with respect to the lane map must be estimated using the current states of the navigation filter.  $\vec{r}_{eb}^e$  denotes the current position estimate in the ECEF coordinate frame.  $\vec{r}_{er,i}^e$  denotes the position of the last waypoint passed in the north east coordinate frame.  $C_e^r$  is the rotation matrix given in (15). Equation (32) is used to find the position estimates in the road coordinate frame.  $\hat{y}$  is the lateral lane position estimate,  $\hat{x}$  is the distance into the current road coordinate frame, and  $\hat{z}$  is the vertical position in the current road coordinate frame. The  $z$ -axis of the road coordinate frame points down.

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = C_e^r [\vec{r}_{eb}^e - \vec{r}_{er,i}^e] \quad (32)$$

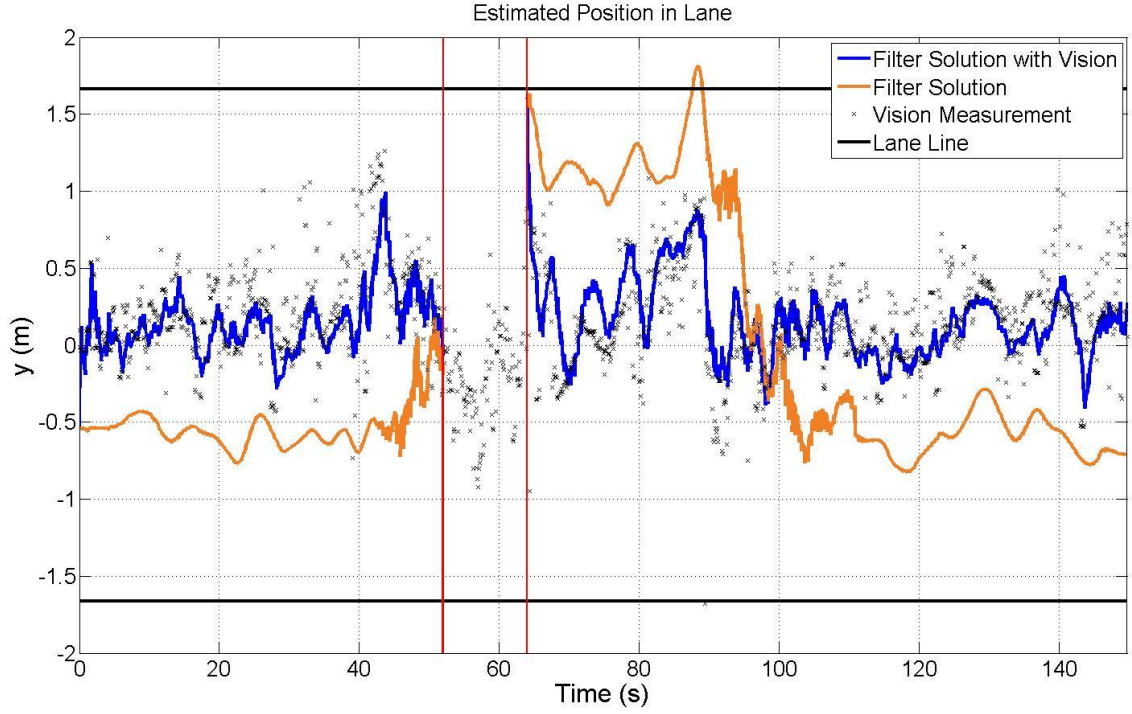


Figure 3: Estimated Lane Position of the 6DOF Navigation Filters

Equation (33) shows the measurement equations. The measurement equations are a function of the states of the filter and the map parameters. The first measurement equation is for the lateral lane position. This measurement is assumed to be provided by a camera or LiDAR. The second measurement equation is the height above the road coordinate frame. Since this navigation filter structure is for a ground vehicle, this measurement can be assumed to be a constant. The height measurement will be a negative number since the z-axis of the road coordinate frame points down.

$$h(x) = \begin{bmatrix} y \\ h \end{bmatrix} = \begin{bmatrix} C_{\epsilon}^r(2,1)(\vec{r}_{\epsilon b,1}^e - \vec{r}_{\epsilon r,i,1}^e) + C_{\epsilon}^r(2,2)(\vec{r}_{\epsilon b,2}^e - \vec{r}_{\epsilon r,i,2}^e) + C_{\epsilon}^r(2,3)(\vec{r}_{\epsilon b,3}^e - \vec{r}_{\epsilon r,i,3}^e) \\ C_{\epsilon}^r(3,1)(\vec{r}_{\epsilon b,1}^e - \vec{r}_{\epsilon r,i,1}^e) + C_{\epsilon}^r(3,2)(\vec{r}_{\epsilon b,2}^e - \vec{r}_{\epsilon r,i,2}^e) + C_{\epsilon}^r(3,3)(\vec{r}_{\epsilon b,3}^e - \vec{r}_{\epsilon r,i,3}^e) \end{bmatrix} \quad (33)$$

The matrix in (37) shows the measurement model, assuming the state matrix takes the form of (34). The measurement model is created by taking the partial derivative of the measurement equations with respect to each state.

$$x = [\vec{r}_{\epsilon b}^e \quad \vec{v}_{\epsilon b}^e \quad \vec{a} \quad \vec{b}_f \quad \vec{b}_g \quad c\delta t \quad c\delta t]^T \quad (34)$$

$$e_1 = [C_{\epsilon}^r(2,1), C_{\epsilon}^r(2,2), C_{\epsilon}^r(2,3)] \quad (35)$$

$$e_2 = [C_{\epsilon}^r(3,1), C_{\epsilon}^r(3,2), C_{\epsilon}^r(3,3)] \quad (36)$$

$$H(x) = \begin{bmatrix} e_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ e_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (37)$$

After every measurement update and time update, the states of the navigation filter will change. Every time the states of the filter are updated, the distance into the current road frame must be checked to insure that the vehicle has not passed into the next road coordinate frame. Plugging the position states into

(38) will result in the current longitudinal distance into the road coordinate frame. If this value is larger than  $d_{r,i}$ , then the vehicle has passed into the next coordinate frame. The map index  $i$  should then be incremented by one.

$$\hat{x} = C_{\epsilon}^r(1,1)(\vec{r}_{\epsilon b,1}^e - \vec{r}_{\epsilon r,i,1}^e) + C_{\epsilon}^r(1,2)(\vec{r}_{\epsilon b,2}^e - \vec{r}_{\epsilon r,i,2}^e) + C_{\epsilon}^r(1,3)(\vec{r}_{\epsilon b,3}^e - \vec{r}_{\epsilon r,i,3}^e) \quad (38)$$

Fig. 3 shows the utility of using vision to aid a GPS based navigation filter. Fig. 2 is a plot of the lateral lane position estimates. This data for these results was collected at the NCAT test track, which is a 1.8 mile, two lane oval. The track is set up very similar to the interstate system. The orange line represents the navigation filter's estimated lane position when only GPS measurements are used to update the filter. GPS position is a biased measurement. These biases make it difficult to compare a GPS position measurement and a map to get vehicle position within a lane. The blue line is the result of the vision aided navigation filter. Unlike the GPS only aided filter, the estimated lane position reported by the vision aided navigation filter is not biased. The section in between the red lines represents when the vehicle passed into a section of the track that was not surveyed. Since the navigation filter is not vision dependent, the filter is still operating without a map using only GPS updates.

## VII. CONCLUSIONS

It is possible to use road based position measurements in a traditional navigation filter. In order to relate the local road based position to the global based navigation filter, a waypoint map of the lane is needed. GPS position measurements are biased measurements. A navigation filter that only uses GPS measurement updates will reflect these biases in the filter's position states. Road based position measurements are not

biased; therefore, it is possible to remove some of the global position bias by using road based measurements. However, if the map is not globally accurate, then the global position estimates will reflect the bias of the map. In general, if the global accuracy of the filter's position estimate is not important, then the quality of the lane map can be poor. If the global accuracy of the filter's position estimate is important, then the lane map must be very accurate.

Another advantage of using road based measurements is the added observability to the system. For example, if lateral lane position is measured, then estimated position is constrained in one axis. Since the vehicle is assumed to be at a constant height above the ground, the estimated position is constrained in another axis. Using these measurements and constraints, it is possible to have a fully observable navigation with as few as two GPS ranges instead of the typical four GPS ranges needed to maintain observability [1].

### VIII. ACKNOWLEDGMENTS

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