Numerical Integration and Differentiation Techniques

Please don't memorize these formulas. Understand what they are doing – if you don't, come see me and I'll explain them with some simple examples. These numerical techniques are methods for ***approximating*** the integrals and derivatives!

Integration (area under the curve) $x(t) = \int_0^t \dot{x}(t)dt$

Euler Integration:

$$x_{k+1} = x_k + \dot{x}_k \times \Delta t$$

Trapezoidal Integration:

$$x_{k+1} = x_k + \left(\frac{\dot{x}_k + \dot{x}_{k+1}}{2} \right) \times \Delta t$$

Numerical Integration for solving differential equation (numerical simulation)

given: $a\ddot{x} + b\dot{x} + cx = F$

define = $v = \dot{x}$
$Ts = \Delta t$

$$\ddot{x} = \dot{v} = \frac{F - bv - cx}{a}$$

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for k=1:100
      v_dot(k+1)=(F(k)-b*v(k)-c*x(k))/a;               % calculate ẍ
      v(k+1)=v(k)+0.5*Ts*(v_dot(k+1)+v_dot(k);         % numerically integrate ẍ
      x(k+1)=x(k)+0.5*Ts*(v(k+1)+v(k);                 % numerically integrate ẋ
      t(k+1)=t(k)+Ts;                                  % record time
end
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Differentiation (slope of the curve) $\dot{y} = \frac{dy}{dt}$ $\ddot{y} = \frac{d\dot{y}}{dt} = \frac{d^2y}{dt^2}$

$$\frac{dy}{dt} = \frac{y_k - y_{k-1}}{\Delta t} \qquad O(\Delta t)$$

$$\frac{dy}{dt} = \frac{3y_k - 4y_{k-1} + y_{k-2}}{2 \times \Delta t} \qquad O(\Delta t^2)$$

$$\frac{d^2y}{dt^2} = \frac{y_k - 2y_{k-1} + y_{k-2}}{(\Delta t)^2} \qquad O(\Delta t)$$

$$\frac{d^2y}{dt^2} = \frac{y_{k+1} - 2y_k + y_{k-1}}{(\Delta t)^2} \qquad O(\Delta t^2)$$

$$\frac{d^2y}{dt^2} = \frac{2y_k - 5y_{k-1} + 4y_{k-2} - y_{k-3}}{(\Delta t)^2} \qquad O(\Delta t^2)$$