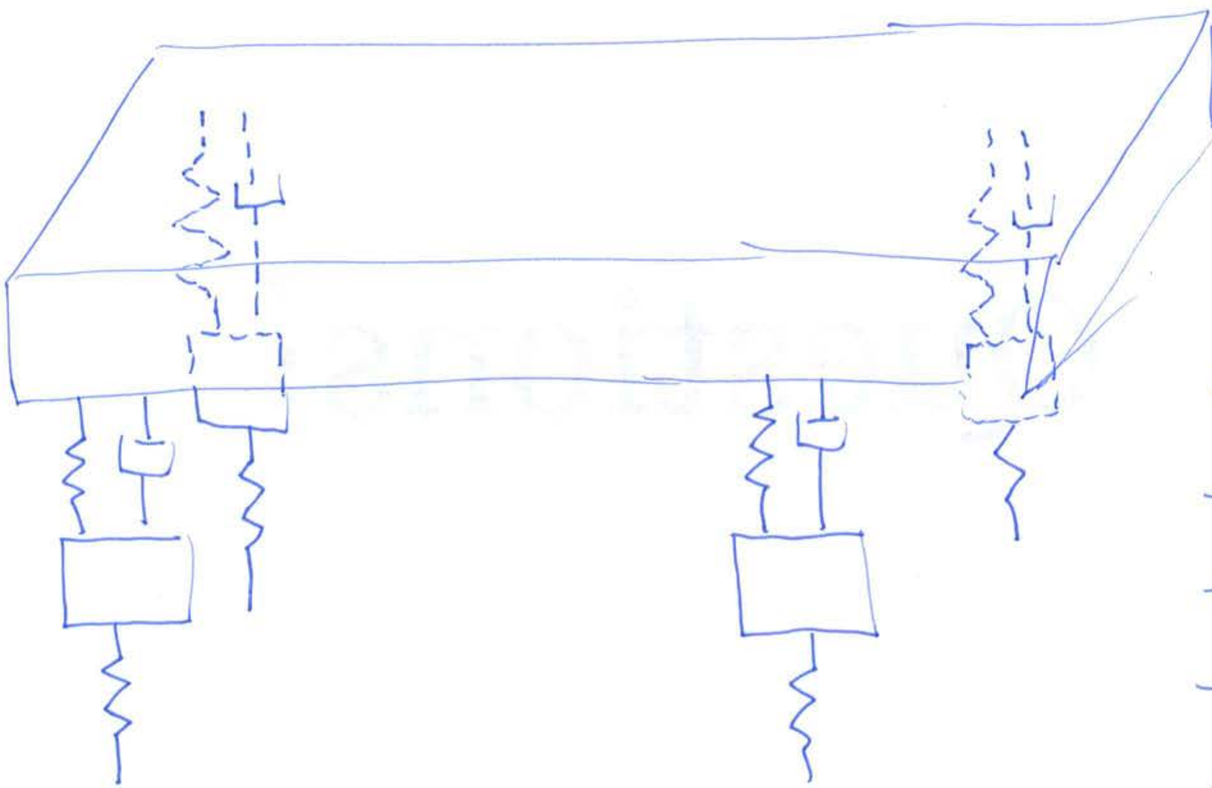
Ride Models

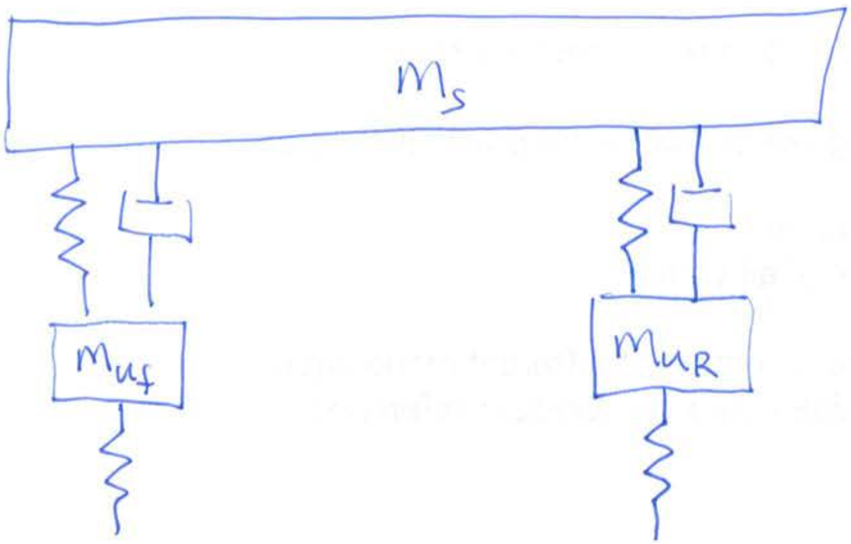
Full Car : Bounce, Pitch & Roll



- 14th order
- 7 DOF
  - 4 wheel (z_w^i)
  - Roll (ϕ)
  - pitch (θ)
  - Bounce (z_s)

Ride Models

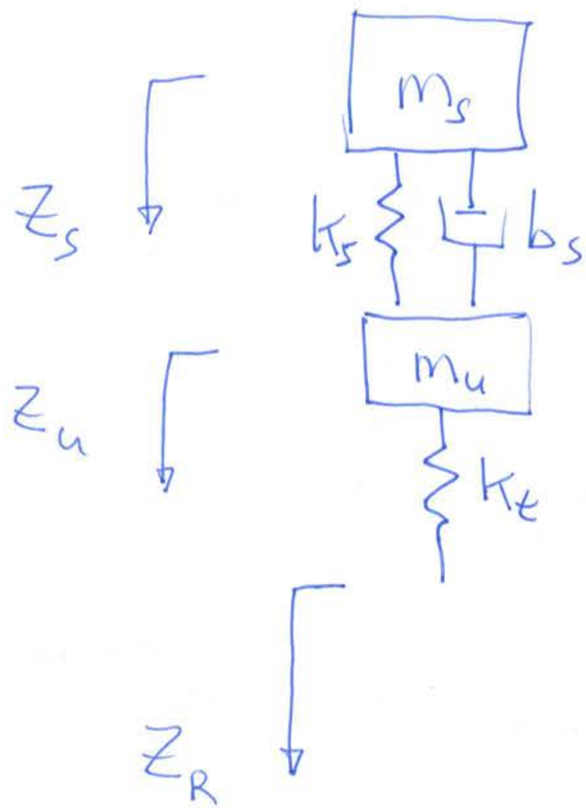
$\frac{1}{2}$ Car : Bounce & Pitch



- 8th order
- 4 DOF
  - 2 wheel (z_w^i)
  - Pitch (θ)
  - Bounce (z_s)

Ride Models

$\frac{1}{4}$ Car : Bounce



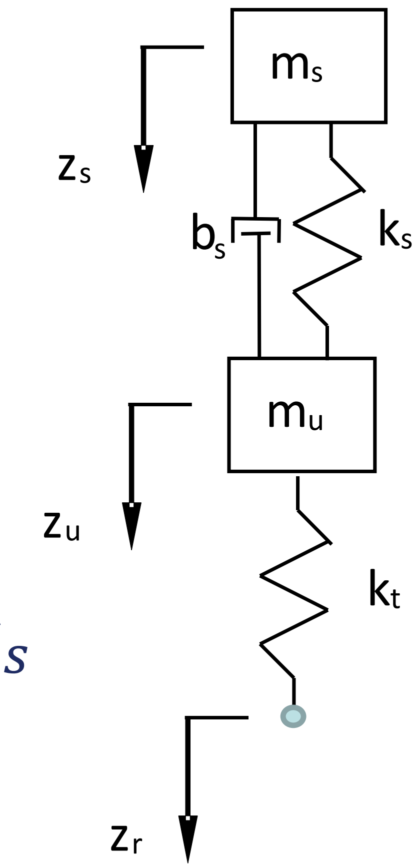
- 4^{th} Order
- 2 DOF
  - wheel (z_u)
  - Car (z_s)

Ride Model

- Assuming SEP:

[illegible]

$$m_u \approx 0.1 m_{total} \quad m_s \approx 0.9 m_{total} \quad k_t \approx 10 k_s$$



- Neglecting the tire damping:

$$\frac{\ddot{z}_s}{\ddot{z}_r} = \frac{z_s}{z_r} = \frac{k_t b_s s + k_s k_t}{m_s m_u s^4 + b_s(m_s + m_u)s^3 + \{k_s m_u + (k_s + k_t)m_s\}s^2 + b_s k_t s + k_s k_t}$$

Ride Model

- Note:

$$\omega_{n1} \neq \sqrt{\frac{k_s}{m_s}} \quad \text{and} \quad \omega_{n2} \neq \sqrt{\frac{k_t}{m_u}}$$

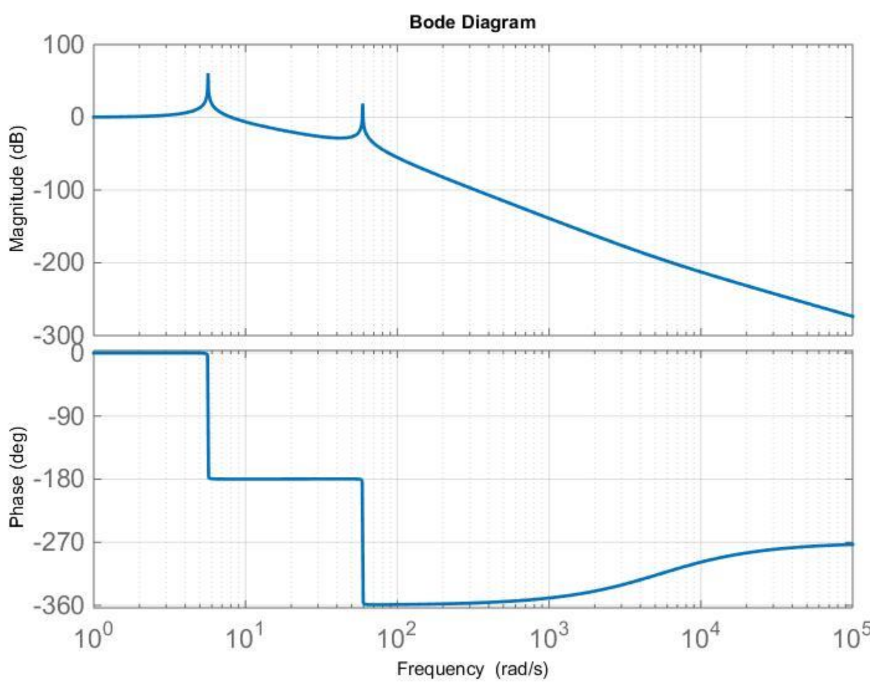
- However due to the values for a typical car:

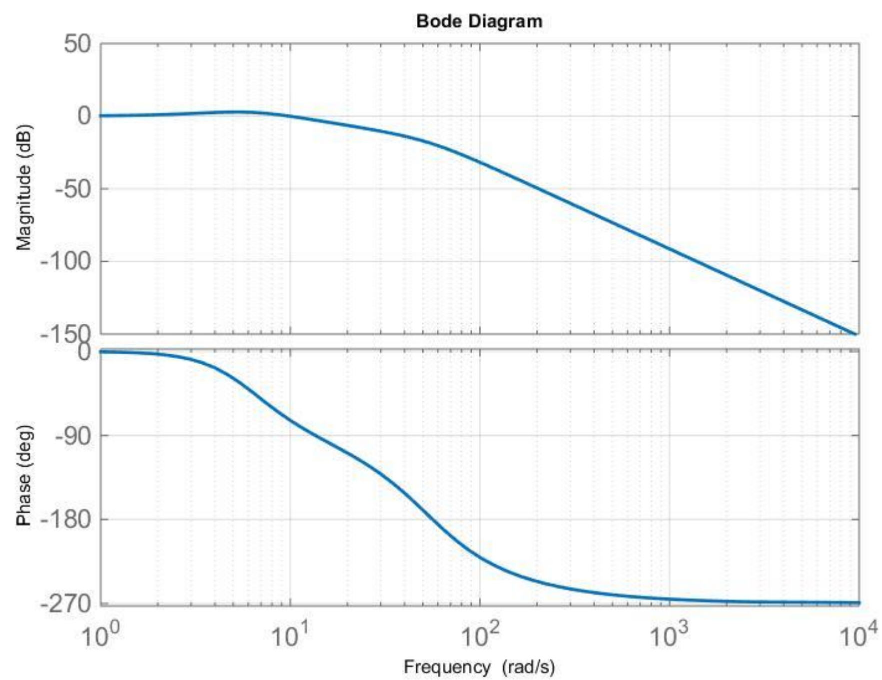
$$\omega_{n1} \approx \sqrt{\frac{k_s}{m_s}} \approx 6.2\,\frac{\text{rad}}{\text{s}}$$

$$\omega_{n2} \approx \sqrt{\frac{k_t}{m_u}} \approx 62\,\frac{rad}{s}$$

Bode Plot of z_s

- Small Suspension Damping
- Realistic Suspension Damping





- Note G_{DC}=1 (0 dB)

Simplified Ride Model

- Neglecting the sprung mass:

$$\frac{\ddot{z}_s}{\ddot{z}_r} = \frac{z_s}{z_r} = \frac{k_t b_s s + k_s k_t}{b_s m_s s^3 + (k_s + k_t) m_s s^2 + b_s k_t s + k_s k_t}$$

- Defining the effective stiffness (also called the ride rate) using springs in series:

$$RR = k_{eff} = \frac{k_s k_t}{k_s + k_t}$$

- Writing the above transfer function w/ the RR:

$$\frac{\ddot{z}_s}{\ddot{z}_r} = \frac{z_s}{z_r} = \frac{\frac{k_{eff}}{k_s} b_s s + k_{eff}}{\frac{k_{eff}}{k_s k_t} b_s m_s s^3 + m_s s^2 + \frac{k_{eff}}{k_s} b_s s + k_{eff}}$$

Simplified Ride Model

- Then for large k_t:

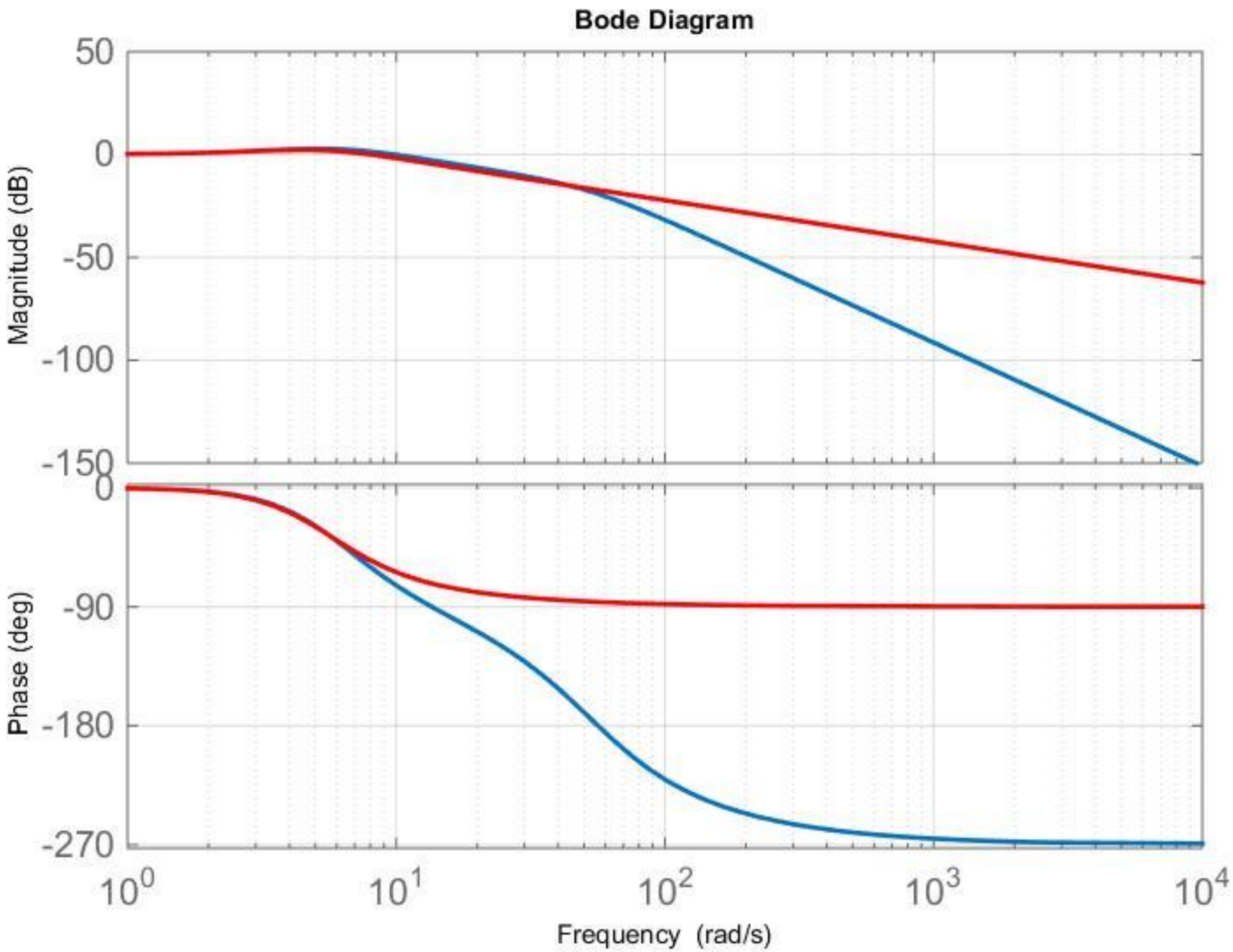
$$\frac{\ddot{z}_s}{\ddot{z}_r} = \frac{z_s}{z_r} = \frac{\frac{k_{eff}}{k_s} b_s s + k_{eff}}{m_s s^2 + \frac{k_{eff}}{k_s} b_s s + k_{eff}}$$

- This results in the dominate natural frequency (i.e. the suspension natural frequency) of:

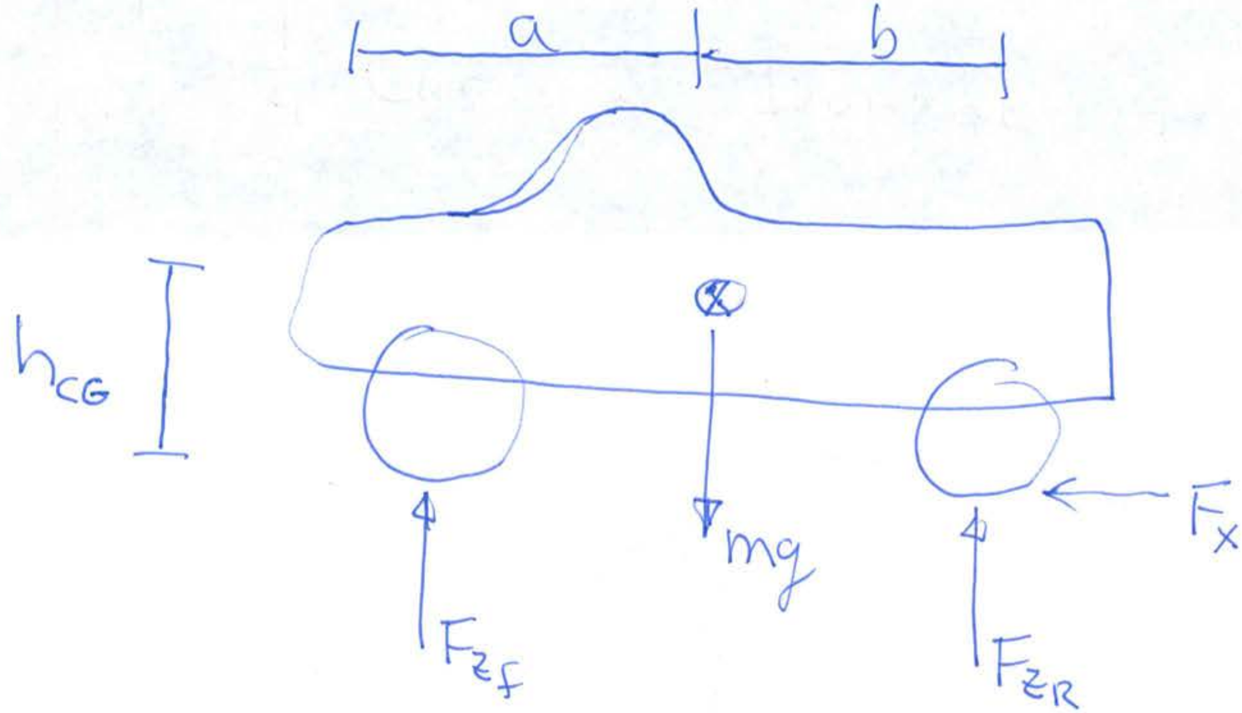
$$\omega_n = \sqrt{\frac{k_{eff}}{m_s}}$$

Bode Plot of z_s

- Comparison the 2nd and 4th order suspension models:



Pitch Dynamics



Recall:

$$F_{zf} = \frac{mgb}{L} - \frac{mh_{CG}}{L}\ddot{x}$$

$$F_{zR} = \frac{mga}{L} + \frac{mh_{CG}}{L}\ddot{x}$$

Given K_t & K_s, what will Δz_{tire} & Δz_{susp} equal ??

- $\Delta z_{tire} = \frac{m h_{CG}}{L K_{tire}} \ddot{x}$ (additional defection)

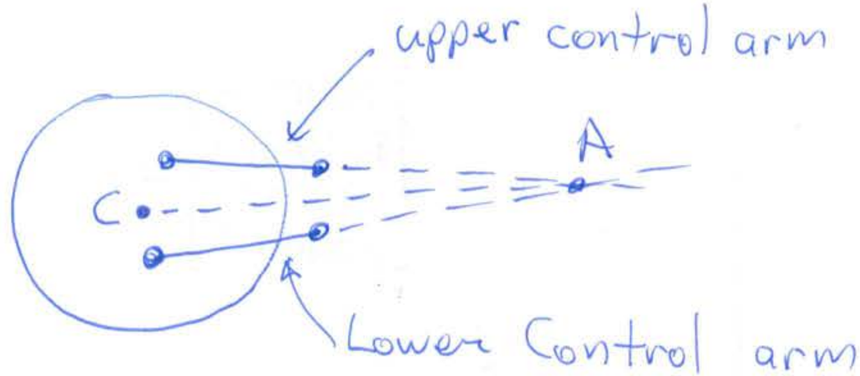
- $\Delta z_{susp} \neq \frac{m h_{CG}}{L K_{sus}} \ddot{x}$

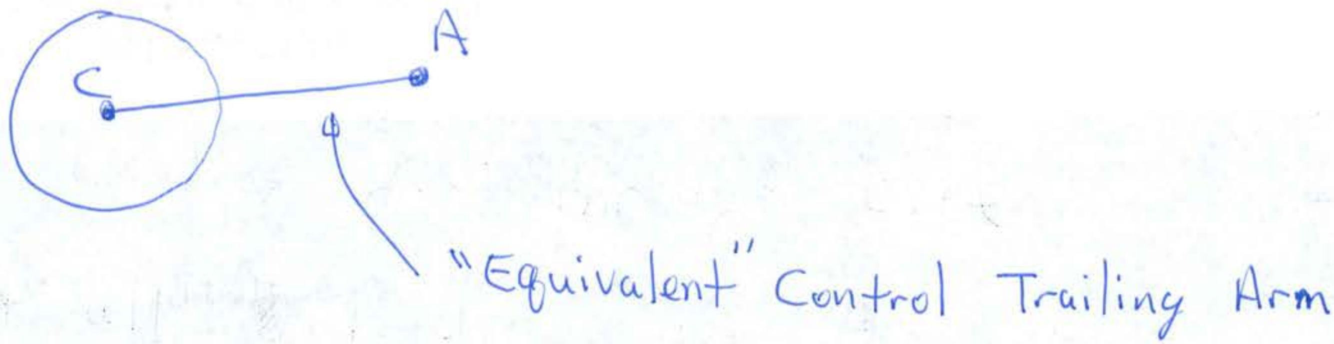
☆ All Loading goes through tire

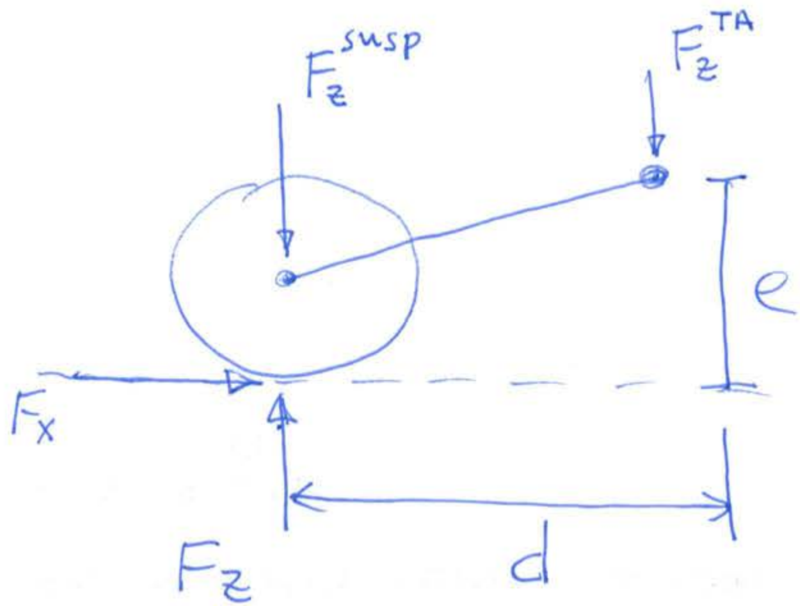
☆ Not all loading goes through suspension spring

⇒ Some goes through a-arms

Equivalent Trailing Arm







F_z^{TA} ⇒ Vertical load through control Arms

F_z^{susp} ⇒ Vertical load through suspension spring

$$F_{xf} = \xi m a_x = \xi m \ddot{x}$$

$$F_{xR} = (1-\xi) m a_x = (1-\xi) m \ddot{x}$$

$\xi \Rightarrow$ Defines % of Force on Front vs. Rear

$$\theta_{pitch} = \frac{1}{L} m \ddot{x} \left[\frac{h_{CG}}{K_R L} - \frac{(1-\xi)}{K_R} \left(\frac{e_R - R_w}{d_R} \right) + \frac{h_{CG}}{K_F L} + \frac{\xi}{K_F} \left(\frac{e_F - R_w}{d_F} \right) \right]$$

~~Known as~~

Solving for front + rear suspension deflection

$$\delta_R = \frac{1}{K_R}\left[\frac{mh}{L}\ddot{X} - F_{X_R}\left(\frac{e_R - R_w}{d_R}\right)\right]$$

$$\delta_F = \frac{1}{K_F}\left[\frac{-mh}{L}\ddot{X} - F_{X_F}\left(\frac{e_f - R_w}{d_F}\right)\right]$$

$$\theta_{pitch} \approx \frac{\delta_R - \delta_F}{L}$$

Can design ~~the~~ Control Arms to set how much the vehicle pitches under deceleration!!

Anti-Dive Geometry

$$\frac{e_f}{d_f} = \tan(\beta_f) = \frac{h}{\xi L}$$

$$\frac{e_R}{d_R} = \tan(\beta_R) = \frac{h}{(1-\xi)L}$$

($\xi \Rightarrow$ % Braking of front)