Numerical Integration and Differentiation Techniques

Please don't memorize these formulas. Understand what they are doing – if you don't, come see me and I'll explain them with some simple examples. These numerical techniques are methods for ***approximating*** the integrals and derivatives!

Integration (area under the curve) $x(t) = \int_0^t \dot{x}(t)dt$

Euler Integration:

$$x_{k+1} = x_k + \dot{x}_k \times \Delta t$$

Trapezoidal Integration:

$$x_{k+1} = x_k + \left(\frac{\dot{x}_k + \dot{x}_{k+1}}{2}\right) \times \Delta t$$

Numerical Integration for solving differential equation (numerical simulation)

given: $a\ddot{x} + b\dot{x} + cx = F$

define = $v = \dot{x}$
$Ts = \Delta t$

$$\ddot{x} = \dot{v} = \frac{F - bv - cx}{a}$$

```
end_time=5;          % length of simulation in seconds
Ts=1/100;            % Sample rate (delta t) in seconds
ln=end_time/Ts;      % length of the simulation in samples

% Initialize vector size and set to zero:
v_dot(1:ln)=0;
v(1:ln)=0;
x(1:ln)=0;
t(1:ln)=0;

% Initial Conditions:
t(1)=0; v(1)=0; x(1)=0;

for index=1:ln
   v_dot(index+1)=(F(index)-b*v(index)-c*x(index))/a;      % calculate ẍ
   v(index+1)=v(index)+0.5*Ts*(v_dot(index+1)+v_dot(index));  % numer integrate ẍ
   x(index+1)=x(index)+0.5*Ts*(v(index+1)+v(index));     % numerically integrate ẋ
   t(index+1)=t(index)+Ts;                                  % record time
  end
```

Differentiation (slope of the curve) $\dot{y} = \frac{dy}{dt}$ $\ddot{y} = \frac{d\dot{y}}{dt} = \frac{d^2y}{dt^2}$

$$\frac{dy}{dt} = \frac{y_k - y_{k-1}}{\Delta t} \quad O(\Delta t)$$

$$\frac{d^2y}{dt^2} = \frac{y_{k+1} - 2y_k + y_{k-1}}{(\Delta t)^2} \quad O(\Delta t^2)$$

$$\frac{dy}{dt} = \frac{3y_k - 4y_{k-1} + y_{k-2}}{2 \times \Delta t} \quad O(\Delta t^2)$$

$$\frac{d^2y}{dt^2} = \frac{2y_k - 5y_{k-1} + 4y_{k-2} - y_{k-3}}{(\Delta t)^2} \quad O(\Delta t^2)$$

$$\frac{d^2y}{dt^2} = \frac{y_k - 2y_{k-1} + y_{k-2}}{(\Delta t)^2} \quad O(\Delta t)$$

$$\frac{dy}{dt} = \frac{y_k - y_{k-1}}{\Delta t} \quad O(\Delta t)$$

$$\frac{d^2y}{dt^2} = \frac{y_k - 2y_{k-1} + y_{k-2}}{(\Delta t)^2} \quad O(\Delta t)$$

$$\frac{dy}{dt} = \frac{3y_k - 4y_{k-1} + y_{k-2}}{2 \times \Delta t} \quad O(\Delta t^2)$$

$$\frac{d^2y}{dt^2} = \frac{y_{k+1} - 2y_k + y_{k-1}}{(\Delta t)^2} \quad O(\Delta t^2)$$

$$\frac{dy}{dt} = \frac{-y_{k+2} + 8y_{k+1} - 8y_{k-1} + y_{k-2}}{12 \times \Delta t} \quad O(\Delta t^4)$$

$$\frac{d^2y}{dt^2} = \frac{2y_k - 5y_{k-1} + 4y_{k-2} - y_{k-3}}{(\Delta t)^2} \quad O(\Delta t^2)$$