Find transfer functions and state space models for the multi-DOF modeling problems.

Multi-DOF Mechanical Modeling
Chapter 4, Problems 24, 25, 26, 56, 79, 80

Multi-DOF Electrical Modeling
Chapter 6, Problems 15, 16

1) Given the following differential equation:

\[
\begin{align*}
\dot{x} + 2x &= y \\
\dot{y} + 3y &= x + u
\end{align*}
\]

a) Find the two transfer functions \( \frac{X[s]}{U[s]} \) and \( \frac{Y[s]}{U[s]} \)

b) What is the characteristic equation for the system?

c) What are the eigenvalues?

2) Given the quarter car model below, find the transfer functions \( \frac{X_1[s]}{F[s]} \) and \( \frac{X_2[s]}{F[s]} \)
**Supplemental Problems**

Find transfer functions and state space models for the multi-DOF modeling problems.
Chapter 4: 29, 52, 81
Chapter 6: 18, 37

Derive EOM’s, Transfer Functions, and State Space Representation for the following systems:

Neglect friction between the ball and beam
Differential\n\[N = \frac{R_2}{R_1}\]
MATLAB Assignment (Due Monday 11/28/2022)

Develop a controller for an inverted pendulum in HW #7 \( l=1 \text{ m}, \ m=5 \text{ kg}, \ J_{pivot}=1.25 \text{ kgm}^2 \) (with no damping) controlled by a motor that provides a torque input to the system. Design the controller so that the controlled (closed-loop) system has a damping ratio of 0.707 and a bandwidth of 1 Hz.

a. What are the controller Gains?
b. Where are the closed-loop eigenvalues of the system?
c. Sketch the open loop roots of your system (poles and zeros)
d. Sketch the closed loop roots of your system (poles and zeros)
e. Simulate your controlled pendulum starting from rest at 20 degrees with a desired position of zero degrees. Plot the response of the nonlinear system.
f. Add a unit force disturbance horizontally at the top of the pendulum. Add integral control to remove the disturbance. Plot the angle of the pendulum (starting from 20 degrees) and the control input with the disturbance. Does the steady state error go to zero? What is the steady state value of the control input?

BONUS (can do as many of the following as you want):
g. Apply a constant torque to the pendulum that is large enough to “spin” the pendulum (~50-100 Nm). Compare this to the controlled system tracking a position that changes with the same average velocity as the constant torque input. Then add feed-forward to improve the rotational tracking.
h. Apply an impulse input (after it is inverted) to test your controller. Provide a plot of the pendulum position and motor torque. Discuss the results.