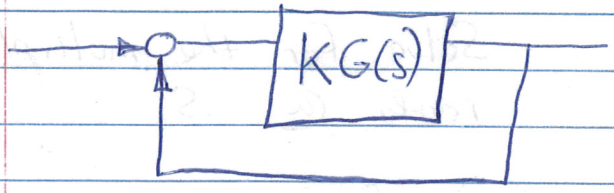
Rules for Sketching Root Locus



$$G(s) = \frac{b(s)}{a(s)} = \frac{b_0(s^m + \beta_1 s^{m-1} + \cdots + \beta_m)}{a_0(s^n + \alpha_1 s^{n-1} + \cdots + \alpha_n)}$$

1) Plot m zeros (roots of $b(s)$) w/ "o"
Plot n poles (roots of $a(s)$) w/ "x"

2) To the left of an odd # of real poles plus real zeroes is on the Locus.

3) Draw $(n-m)$ assymptotes centered @ α

$$\alpha = \frac{\sum p_i - \sum z_i}{n-m}$$

p = value of the ~~root~~ pole $(-a+jb)$
z = value of the zero $(-c+jd)$

$$\phi_\ell = \frac{180^\circ + 360^\circ(\ell-1)}{n-m} \qquad \ell = 1, 2, \cdots n-m$$

(ϕ_ℓ = $\angle$ of the assymptotes)

4) Departure & Arrival Angles

$$q\phi_{dep} = \sum \psi_i - \sum \phi_i - 180^\circ - 360^\circ \ell$$

$$q\psi_{Arr} = \sum \phi_i - \sum \psi_i + 180^\circ + 360^\circ \ell$$

ℓ is an integer such that $-180^\circ < \phi_{dep} < 180^\circ$
q = # of departing poles
ψ_i = angle from the remaining zeros
ϕ_i = angle from the remaining poles

5) Multiple Root Locations

$$\left(\frac{da}{ds}\right)b = \left(\frac{db}{ds}\right)a$$

Solve for the multiple roots @ S.

6) Calculate the Gain (K) for any point on the Locus

$$K = \left[\frac{\text{product of the lengths to the poles}}{\text{product of the lengths to the zeros}}\right] * \left(\frac{a_0}{b_0}\right)$$

$$K = \left|\frac{a(s)}{b(s)}\right|_{s = \text{pole location}}$$

7) Calculate Imaginary Axis Crossing (and corresponding K)

$$\left.\begin{aligned} \text{real}\{a(j\omega) + Kb(j\omega)\} &= 0 \\ \text{Imag}\{a(j\omega) + Kb(j\omega)\} &= 0 \end{aligned}\right\} \text{Solve 2 Eq., 2 unknowns } (K \& \omega)$$

8) 1 pole → 1 zero

Remaining poles → ∞