# MECH 3140 SYSTEM DYNAMICS AND CONTROLS 

## EXAM NO. 1

September 28, 2023

## 1 hour and 15 minutes

## General Instructions

This exam is closed book and closed notes (and no calculators). You may only use the sheet of notes provided for the exam.

NOTE: If a system is non-linear you must provide the full non-linear equations of motion to receive full credit.

Solve each problem fully showing your work. Clearly indicate your answer(s) to each problem. State all assumptions used in your analyses. Since you are not allowed a calculator, you may make reasonable assumptions about constants such as $\pi \approx 3, \sqrt{2} \approx 1.4$, etc. Clearly state these assumptions if you use them!

Unless otherwise noted, assume all springs and dampers are ideal (i.e. massless and linear).

Total credit is 110 points, grade will be calculated out of 100 points.

Name: $\qquad$ Instructor: $\qquad$

Exam Time (5:30 or 7:00): $\qquad$

I certify that I have performed this exam according to the Auburn University honor code and have not given or received any external information during this exam.

Signature: $\qquad$

1. (20 points) Engineers are evaluating a sensor and notice that the sensor output amplitude and phase is different from the input (which was $1 \cdot \sin \left(\omega_{\text {in }} t\right)$ ) for all the tests performed. The sensor's measured amplitude and phase difference from the input for each frequency tests are given below.

| Input Frequency (rad/s) | 0.01 | 0.1 | 1 | 10 | 100 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Amplitude | 10.0 | 9.9 | 7.0 | 1.0 | 0.1 |
| Phase Difference (deg) | -1 | -6 | -45 | -84 | -89 |

a) Estimate the approximate amplitude and phase difference for a test performed at $5 \mathrm{rad} / \mathrm{s}$. Defend/explain your answer.
b) Calculate the actual amplitude and phase difference for a test performed at $5 \mathrm{rad} / \mathrm{s}$
2. (15 points) Given the following differential equation for the current $I$ of an electrical circuit with a nonlinear resistor: $L \dot{I}+R I^{4}=V_{S}$
a. Find the eigenvalue about the operating condition $I_{0}$.
b. For what operating conditions is the linearized system stable?
3. (20 points) Solve the following differential equation for $x(t)$. You may leave the initial condition coefficients unsolved.

$$
3 \dot{x}+4 x=5 \sin (t)
$$

4. (25 points). Engineers are trying to reverseengineer a controller used to control the speed of a motor. After running a simple test of the motor with no controller, the engineers develop an approximate model of the motor (the plant): $\dot{\omega}+$ $2 \omega=10 \cdot V$, where $V$ is the voltage input to the motor, and $\omega$ is the motor speed. The engineers then run a test with the controller, resulting in the plot on the right. Determine the controller used for the motor (control gain(s) and control law).

5. (30 points) Engineers wish to analyze a gear-pulley system which has gears with radii $R_{1}, R_{2}$, mass moment of inertia $J_{1}, J_{2}$, and rotational damping losses at the bearings $b_{1}, b_{2}$, respectively, and a block of mass $m$. You may assume no slip between the gears.
a. Determine the equation(s) of motion for the block.
b. How long does it take for the block to reach steady state velocity, starting from rest?

