Mechanical Modeling Steps	Mechanical Elements
1) DOF and System Order 2) Constitutive coordinates and equations 3) FBD 4) Sum Forces and Moments as needed 5) Repeat 2 through 4 as needed 6) Simplify to one equation of motion per degree of freedom 7) Solve EOM if needed	Mass in translation $m\ddot{x} = \sum F_x$ Inertia in rotation $J\ddot{\theta} = \sum M$ Translational Damper $F_b = b\dot{x}$ Rotational Damper $\tau_b = b\dot{\theta}$ Parallel Axis Theorem $J_p = J_{cg} + ml^2$
Electrical Modeling Steps	Electrical Elements
1) DOF and System Order 2) Constitutive coordinates and equations 3) KCL and KVL as needed 4) Repeat 2 and 3 as needed 5) Simplify to one equation of motion per degree of freedom 6) Solve EOM if needed	Resistors $v_r = IR$ Inductor $v_L = L\dot{I}$ Capacitor $v_c = \frac{1}{c}\int I dt$

Linearization

Small Angles $\cos(\theta) \approx 1 \quad \sin(\theta) \approx \theta$	Taylor Series $f(x) \approx f(x_0) + \frac{\delta f}{\delta x}\|_{x_0}(x - x_0)$ $f(x,y) = f(x_0) + f(y_0) + \frac{\delta f}{\delta x}\|_{x_0}(x - x_0) + \frac{\delta f}{\delta y}\|_{y_0}(y - y_0)$

Solving Differential Equations

Eigenvalue = s

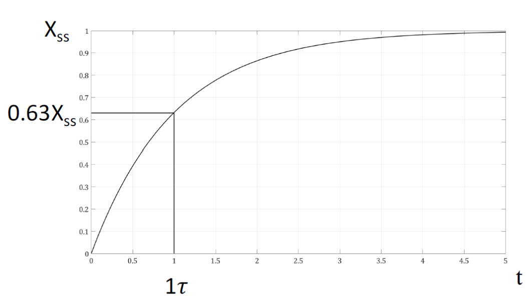
Characteristic Equation = 0

General Solution $x(t) = x_h(t) + x_p(t)$

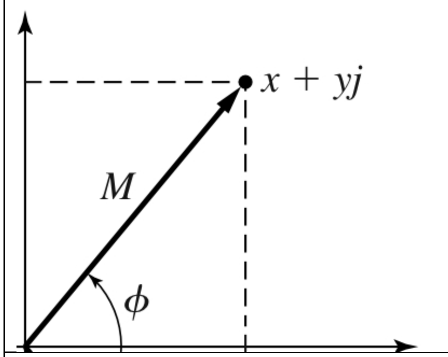
Specific Solution - Given initial conditions solve for unknown homogeneous constant (i.e. C_1)

Real Eigenvalue Homogeneous Solution $x_h(t) = C_1 e^{st}$
Particular Solution Follows form of input $F(t) = C$ => $x_p(t) = C * G_{DC}$ $F(t) = A\sin(\omega t)$ => $x_p(t) = A(gain)\sin(\omega t + \phi)$

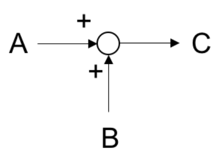
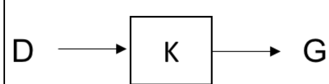
1st Order Step Response

Time Constant		$1\,\tau$	$2\,\tau$	$3\,\tau$	$4\,\tau$	$4.6\,\tau$	$5\,\tau$
$\tau = \frac{1}{\|s\|}$		63%	86%	95%	98%	99%	99.3%

Frequency Response

	$M = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}\frac{y}{x}$ $\text{Gain} = \lvert H\rvert = \left\lvert\frac{B}{A}\right\rvert = \frac{\lvert B\rvert}{\lvert A\rvert} = \frac{\text{Output Amplitude}}{\text{Input Amplitude}}$ $\text{Phase} = \angle H = \angle\frac{B}{A} = \angle B - \angle A$
Real Eigenvalue Corner Frequency: $\omega_c = \lvert s\rvert \quad G_{\omega_c} = \frac{\sqrt{2}}{2}G_{DC} = 0.707G_{DC}$ Real Eigenvalue Bandwidth: $\omega_{bw} = \lvert s\rvert$ (1^{st} order only) High Frequency gain asymptote slope -1 High Frequency phase asymptote -90 deg	

Controls

 A + B =C NOT REQUIRED for EXAM 1	 G=K*D NOT REQUIRED for EXAM 1	Proportional Control $F = K_p E \qquad E = X_{des} - X$ Remove Steady State Error 1) Pre-reference Scaling 2) Feedforward - Also can be used to remove known disturbance