MECH 5970/6970 GPS
Homework #2
Due: 2/14/2024 (by noon)

1. Consider a simple estimation of a single parameter: $y = a$, where the measurement noise has unit variance.
   a. Determine the accuracy of the parameter estimate as a function of the number of measurements
   b. Verify the results through a monte-carlo simulation

2. Given the set of data in the table, perform a least squares fit to solve for the model coefficients Provide the estimate of the coefficients and the estimated/predicted estimation error (1-sigma) for the coefficient *a*? Assume the 1-sigma measurement noise on y is 0.4.

x	0	1	2	3	4
y	0.181	2.680	3.467	3.101	3.437

   a) $y = a + bx$
   b) $y = a + bx + cx^2$
   c) $y = a + bx + cx^2 + dx^3$
   d) Is the estimate for a consistent? Why or why not? Which is probably the correct prediction of the estimation error on coefficient a?

3. Given the following range equation $r^2 = (x - a)^2 + (y - b)^2$ with a range error of 0.5 meters (1-sigma) and the table below:

a	0	10	0	10
b	0	0	10	10
r^2	25	65	45	85

   a) Find Jacobian martrix for this System (i.e. the geometry matrix)
   b) What is the expected solution uncertainty (1-sigma)
   c) What is the solution for x and y
   d) Perform a monte-carlo simulation and verify part b

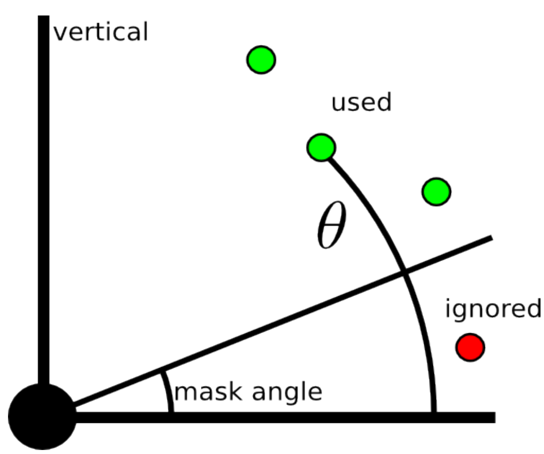
4. On the class website, download the data file for HW #2. This is a file of actual GPS satellites with simulated psuedoranges to a known location. The file also has simulated SOOP (signal of opportunity) stations with their simulated psuedoranges. The clocks on the SOOP are assumed to be synchronized with the GPS satellites (using GPS time). The receiver has a clock bias and measures the ranges with an 1 sigma accuracy of 0.5 meters.
   a. Calculate the position and expected horizontal and vertical error using the first 4 GPS satellites. Initialize your position to the center of the earth. How many iterations does it take to solve for your position.
   b. Repeat part a with all 9 GPS satellites
   c. Repeat part a assuming a perfect receiver clock (note you must correct the pseudo-ranges with the clock bias from part b).
   d. Calculate the position using 2 GPS satellites and 2 SOOPs. What happens and why?
   e. Repeat part d, but start with an initial guess of [423000, -5362000, 3417000]

   Note you should set up m-file functions that calculate your unit vectors and your observation (H) matrix and measurement vector. You will use these again.

5. (Undergrad Bonus) One of the sources of error in the GPS pseudoranges comes from the ionosphere (from 50km to 1000km height above the Earth). In effect, the amount of the ionospheric delay is a function of how much atmosphere the signal passes through, which is a function of the satellite elevation angle, θ. A simple model of the an ionosphere error (described in class) is:

$$I(\theta) = A_1 * \left(1 + 16 * \left(0.53 - \frac{\theta}{180}\right)^3\right)$$

   Where A_1 is a constant equal to 5×10^{-9}seconds and θ is in degrees. In order to combat the significant delays at low elevation angles, a mask angle is typically used. This is a constant angle below which all signals are ignored and not used in the position solution.



Using the 9 GPS satellite positions in the data file and the Ionospheric delay model, generate a plot of position accuracy vs mask angle (note that the mask angle can only be increased until less than four satellites are in view). Comment on this plot.