Quick Review of Systems and Introduction to GPS Signal Structure

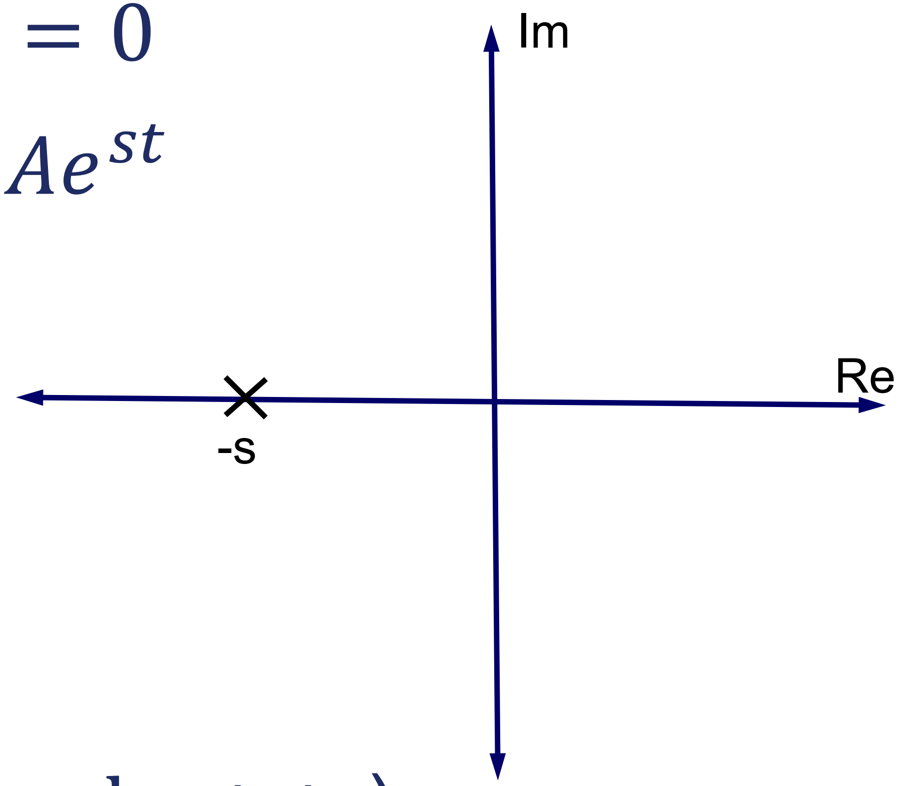
MECH 5970/6970
Fundamentals of GPS

System Response Review

- 1st order Response:

$$\dot{y} + sy = 0$$

$$y(t) = Ae^{st}$$

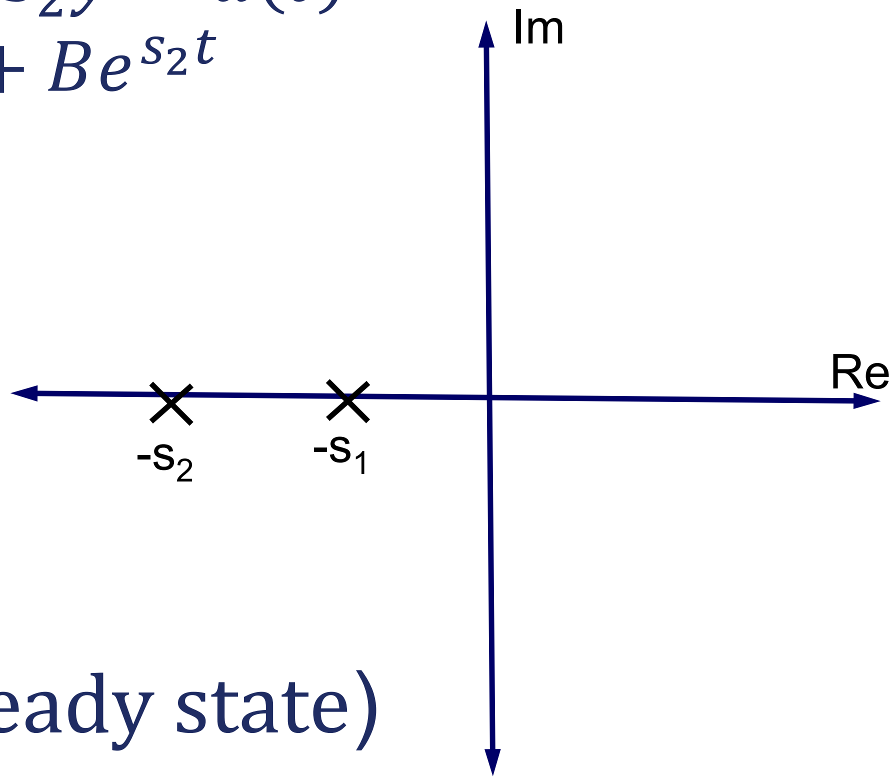


- $\omega_{BW} = s$
- $\tau_c = \frac{1}{s} \quad (4\tau_c = 98\%\ \text{steady state})$

System Response Review

- 2^{nd} order (overdamped) Response:

$$\ddot{y} + (s_1 + s_2)\dot{y} + s_1 s_2 y = u(t)$$
$$y_h(t) = Ae^{s_1 t} + Be^{s_2 t}$$



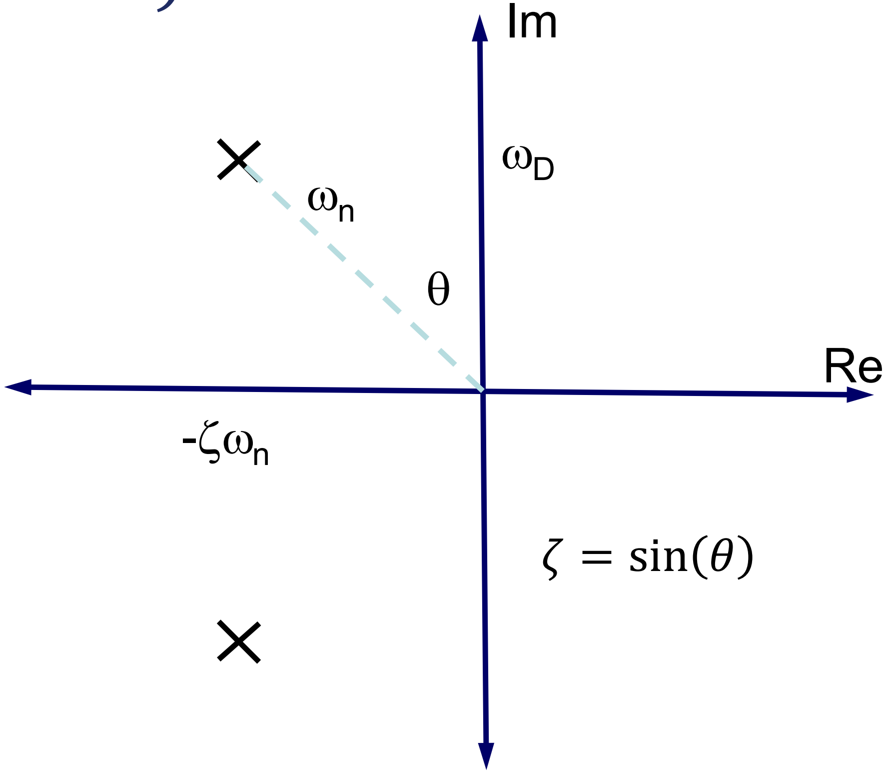
- $\omega_{BW} = s_1$
- $\tau_c = \frac{1}{s_1}$ $\quad (4\tau_c = 98\% \text{ steady state})$

System Response Review

- 2nd order (underdamped) Response:

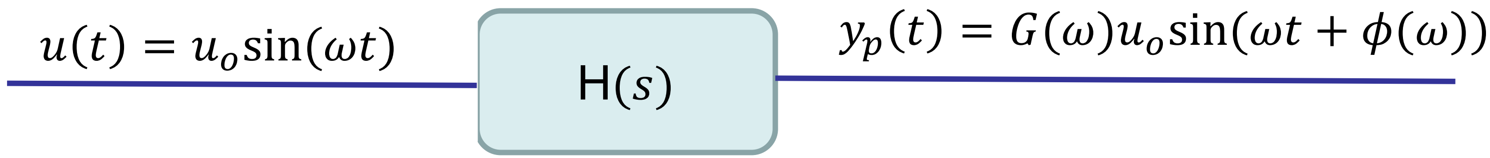
$$\ddot{y} + 2\zeta\omega_n\dot{y} + y = u(t)$$

$$y_h(t) = Ae^{-\zeta\omega_n t}\cos(\omega_D t + B)$$



- $t_{settle} = \dfrac{4.6}{\zeta\omega_n}$
- $t_{peaks} = \dfrac{\pi}{\omega_D}$
- $t_{rise} = \dfrac{1.8}{\omega_n}$
- $\%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$
- $\omega_{BW} \approx \omega_n \; (\omega_{BW} = \omega_n \, , if \; \zeta = 0.707)$

System Frequency Response



- where

$$G(\omega) = |H(j\omega)| = \frac{|num(j\omega)|}{|den(j\omega)|}$$

$$\emptyset(\omega) = \angle H(j\omega) = \angle num(j\omega) - \angle den(j\omega)$$

Control Loops

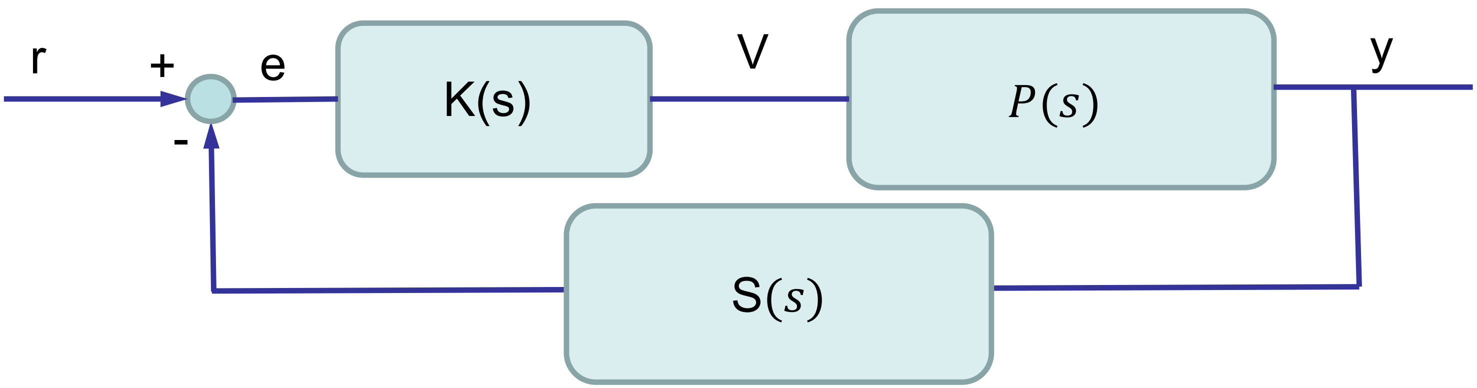
- Closed Loop Transfer Functions:

$$\frac{Y(s)}{R(s)} = \frac{KP}{1+KPS}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+KPS}$$

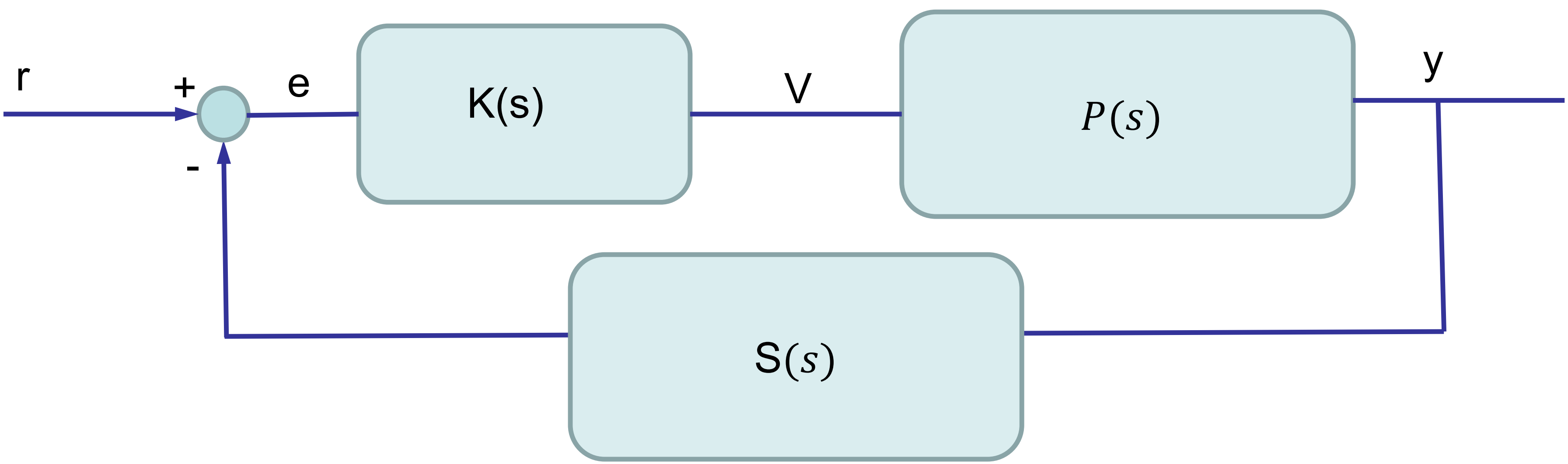
- Final Value Theorem (FVT):

$$y_{ss} = \lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s)$$



System Type

- Consider the following feedback loop
- System type is defined as the number of pure integrators (eigenvalues at zero) of the open loop L(s)=K(s)P(s)S(s)



System Type

- System Type
  - Defined by the number of integrators in the (open) loop transmission
    - Dictates steady state error to various reference input types
- Type 0 (no integrators)
  - Constant steady state error to a step (constant) input.
  - ***Infinite*** steady state error to ramp or parabolic input

System Type

- Type 1 (one integrator)
  - zero steady state error to a step (constant) input.
  - Constant steady state error to ramp input
    - Error depends on controller gain and slope of the ramp
  - Infinite steady state error to a parabolic input
- Type 2 (two integrators)
  - Zero steady state error to a step or ramp input
  - Constant steady state error to a parabolic input
    - Error depends on the controller gain and coefficient in front of the parabolic

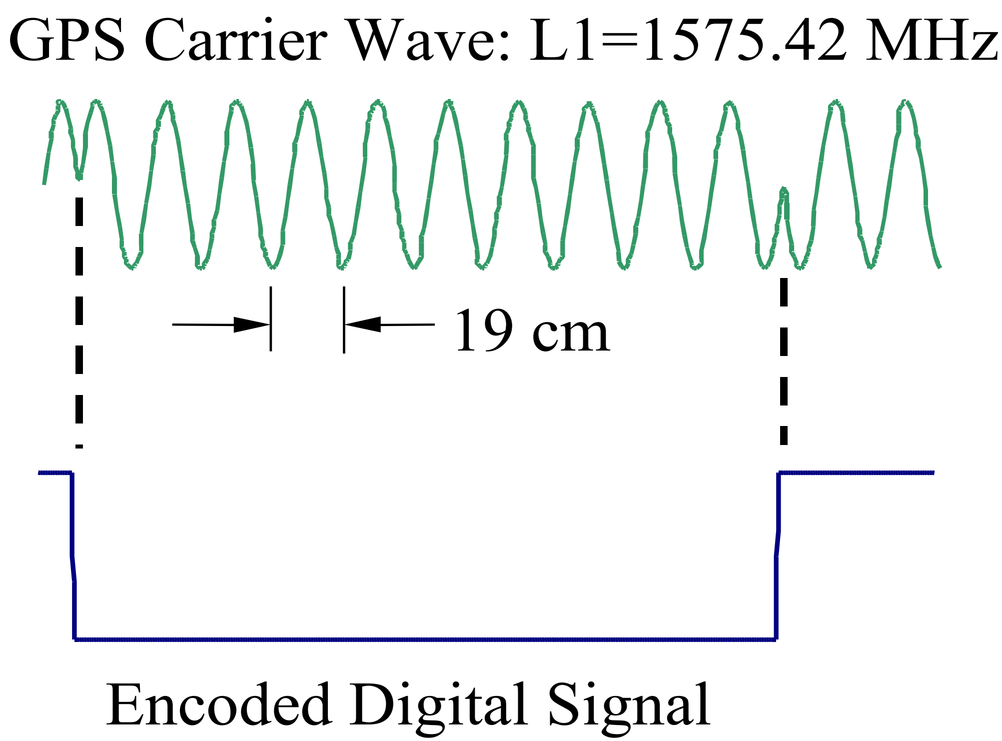
System Type

- Why is system type important in GPS?
  - Tracking of the incoming signal is a feedback loop
  - The reference input is a function of the line of sight signal
  - Two types of signal tracking:
    - Tracking Phase
    - Tracking Frequency

System Type

- Tracking Phase
  - Constant range results in a ramp reference
    - Phase is changing over time
  - Constant range rate rate results in a parabolic reference
- Tracking Frequency
  - Constant range rate results in a constant reference
  - Constant range acceleration results in a ramp reference input

GPS Signal (-160 dBw ~ 10^{-16} watts)



digital code:

(satellite info)

- satellite #
- time
- location
- velocity

Roughly equivalent to viewing a 25-watt light bulb from a distance of 10,000 miles.

GPS Broadcast Signal Structure

- Each satellite transmits the precise time (UTC-USNO), the complete parameters of its orbit, and the major parameters of all other satellite's orbits
  - These parameters are collectively known as *ephemeris data.*
- The Navigation message which includes the ephemeris data from the satellite is 30 secs. in duration and is transmitted in digital form at a rate of 50 bps.
- This data transmission modulates the GPS carrier wave using *binary phase-shift keying* (BPSK)

Gold Codes and (Spread-Spectrum Transmission)

- Gold Codes are a family of unique binary sequences which have very low cross-correlation with other sequences in the family and low auto-correlation as well.
- Modulating each GPS satellite’s signal by a unique Gold Code, known as the PRN number, spreads the signal over a wider bandwidth, which provides noise rejection and enables multiple access (CDMA).
  - Allows satellites transmit on the same frequency at the same time without interfering with each other

Carrier Wave

- L1 at 1575.42 MHz (154 x 10.23 MHz)
- L2 at 1227.60 MHz (120 x 10.23 MHz)
- L5 at 1176.45 MHz (115 x 10.23 MHz)
- Modulated with Code and Navigation Data using Binary Phase-Shift Keying (BPSK)
- C/A and P(Y) are transmitted orthogonally on L1
  - And now on L2 (called L2C) with the newer satellites

Code Signal

- Code Division Multiple Access (CDMA)
- Course Acquisition - C/A
  - Gold Codes
  - Code Period of 1 ms
- Precision Code - P(Y)
  - Anti-Spoofing Mode
  - Code reset each week
  - Encrypted (authorized users only)

Received GPS Signal (of jth Satellite)

$$s_{L1}^{j}(t) = \sqrt{2P_{C1}^{j}}X^{j}(t)D^{j}(t)\cos\left(2\pi f_{L1}^{j}t + \theta_{L1}\right) + \sqrt{2P_{Y1}^{j}}Y^{j}(t)D^{j}(t)\sin\left(2\pi f_{L1}^{j}t + \theta_{L1}\right)$$

$$s_{L2}^{j}(t) = \sqrt{2P_{Y2}^{j}}Y^{j}(t)D^{j}(t)\sin\left(2\pi f_{L2}^{j}t + \theta_{L2}\right)$$

Note: Received frequency includes Doppler: $f_{L1}^{j} = f_{L1} + f_{doppler}^{j}$

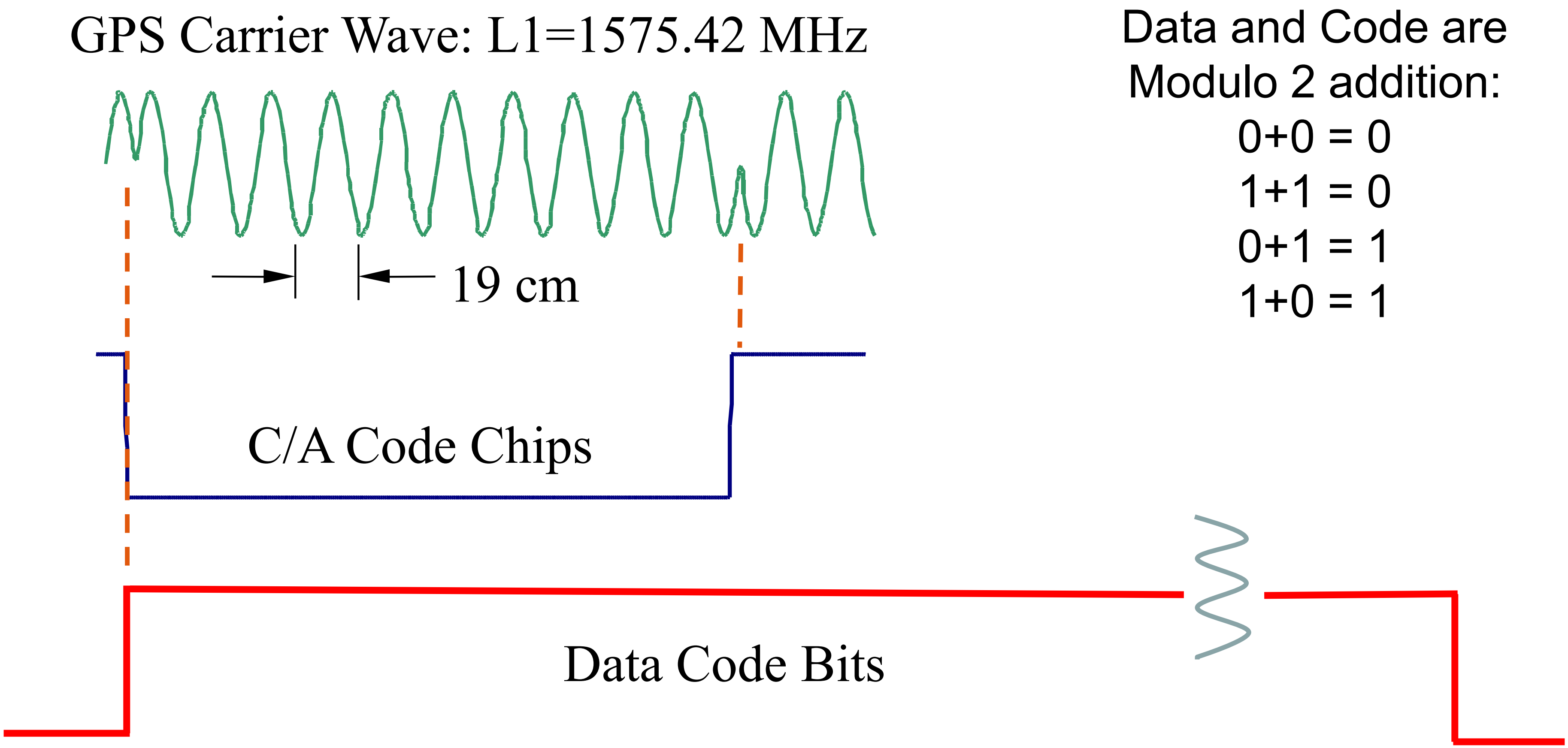
- P: Received signal power
- X: C/A Code, or PRN/Gold Code (-1 or +1)
- D: Data Bit (-1 or +1)
- Y: P-Y *encrypted* Code (-1 or +1)

Codes

- C/A Code Chip=$\frac{1}{1023} ms$ (≈ 1 µs or 300 m)
  - C/A code is 1023 chips long (1 ms)
- Data Bit = 20 ms
  - Recall data bits are transmitted at 50 bits/sec (i.e. 50 Hz)
  - 20 C/A code repetitions in a single data bit
  - Data is 12.5 minutes long
    - But ephemeris and clock parameters repeats every 30 seconds
- P Code is 10^{14} chips (repeats every week)!
  - Chip rate for P code is 10.23 Mchip/sec
    - Each chip is approximately 30 meters long
  - Requires C/A code to find place in P code (or precise time)
  - P code has been encrypted since 1994: P(Y) Code
    - For authorized users only

GPS Signal

$$s = PRN * Data * carrier$$



Data and Code are Modulo 2 addition:
0+0 = 0
1+1 = 0
0+1 = 1
1+0 = 1