Next Generation Vehicle Positioning Techniques for GPS-Degraded Environments to Support Vehicle Safety and Automation Systems

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EXPLORATORY ADVANCED RESEARCH PROGRAM

Auburn University

Sarnoff Corporation

The Pennsylvania State University

Kapsch TrafficCom Inc.

NAVTEQ North America LLC

Quarterly Report 1

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1. Scope

In an open environment, GPS provides a good estimation of vehicle position. Numerous improvements over the basic GPS framework have provided accuracies in the centimeter range. However, blockages of the GPS signal create significant problems for the positioning solution. In so-called “urban canyons”, GPS signals are blocked by the presence of tall buildings. Similarly, heavy foliage in forests can block line-of-sight to the satellites. Because of these problems, a broader approach is needed that does not rely exclusively on GPS. This project takes into account three key technology areas which have each been individually shown to improve positioning solutions where GPS is not available or is hampered in a shadowed environment. First, terrain-based localization can be readily used to find the vehicle’s absolute longitudinal position within a pre-mapped highway segment – compensating for drift which occurs in dead-reckoning system in long longitudinal stretches of road. Secondly, visual odometry keys upon visual landmarks at a detailed level to correlate position to a (visually) premapped road segment to find vehicle position along the roadway. Both of these preceding techniques rely on foreknowledge of road features – in essence, a feature-enhanced version of a digital map. This becomes feasible in the “connected vehicle” future, in which tomorrow’s vehicles have access to quantities of data orders of magnitude greater than today’s cars, as well as the ability to share data at high data rates. The third technology approach relies on radio frequency (RF) ranging based on DSRC radio technology. In addition to pure RF ranging with no GPS signals, information from RF ranging can be combined with GPS range measurements (which may be inadequate on their own) to generate a useful position. Based on testing and characterization of these technologies individually in a test track environment, Auburn will define a combined Integrated Positioning System (IPS) for degraded GPS environments, which will also incorporate ongoing FHWA EAR work at Auburn in fusion of GPS and on-board sensors. This integrated approach will blend the strengths of each technique for greater robustness and precision overall. This research is expected to be a major step forward towards exceptionally precise and reliable positioning by taking advantage of long-term trends in on-board computing, connected vehicles, and data sharing.

1.1 Sarnoff Corporation Contribution

The scope of Sarnoff’s work under Year One of this project is the evaluation of their Visual Aided Navigation System for providing highly accurate positioning for vehicles. As such there are 3 major tasks:

(1) Evaluate and provide a survey of Sarnoff’s existing Visual Navigation results

(2) Integrate Visual Navigation system on Auburn Engineering’s Sonata vehicle test platform and collect test data using the integrated system.

(3) Process and analyze the data from the tests and evaluate the performance and recommend any improvements and optimizations.

1.2 The Pennsylvania State University Contribution

Previous work at Penn State has shown that particle filters can be used for terrain-based localization, and the approach has proven to work offline for defense-grade sensors on specific road sections. The scope of Penn State's contribution to the project involves an extension of the above algorithms. Specifically, Penn State's contribution in the current project consists of the following three tasks:

(1) Developing the proven approach so that it can be used for localization with commercial-grade sensors, rather than defense-grade sensors,

(2) Modifying and optimizing the particle filter algorithm, and exploring alternative approaches, so that localization can take place online (in real-time) rather than offline, and

(3) Modifying and optimizing the algorithms as well as terrain map representation, so that the localization algorithms work over a large network of roads, rather than a small section of a single road alone.

1.3 Kapsch TrafficCom Inc. Contribution

Kapsch will investigate the accuracy of close proximity calculations available from the 5.9 GHz DSRC communications channel. A great deal of information related to positioning can be inferred from the DSRC communications channel. Basic calculations may provide a location region achieved through the channel ranging calculations to more precise lane based proximity determinations through advanced analysis of the communications channel. Kapsch will research a combination of both approaches through available data defined in the IEEE 802.11p standard for 5.9 GHz communication and through scientific Radio Frequency (RF) analysis.

Kapsch will support Auburn for the characterization of the ability to utilize the 5.9 GHz DSRC communication channel for next generation non-GPS localization services. The Received Signal Strength Indication (RSSI) in-conjunction with other aspects of the DSRC communications channel will be analyzed and a method developed for signal ranging. Kapsch does not believe RSSI ranging techniques will fully meet the desired localization needs. Year 2 will yield more advanced algorithms and DSRC equipment capable of providing lane level localization from the DSRC communications channel. This task includes the following sub-tasks:

(1) System Engineering and Deployment of DSRC Infrastructure at the Auburn Test Track

(2) Lab testing of DSRC signal ranging solution

(3) On-site testing of DSRC signal ranging solution

(4) Analysis of DSRC signal ranging test results

2. Current Progress

Auburn University has hosted a kick off meeting in which Sarnoff, Kapsch, Penn State, and Auburn gave an overview of their respective systems and technology. Since that time, Auburn students have been delegated to interface specifically with those partners. Chris Rose will be interfacing with Sarnoff, John Allen with Kapsch, and Jordan Britt with Penn State. Each partner has been contacted by their corresponding Auburn student lead, and contact information has been exchanged to facilitate in quick and effective communication. Additionally, Auburn has provided Penn State with Xbow IMU440 data so that Penn State can begin work on sensor characterization.

2.1 Sarnoff Progress

2.1.1 Distributed Aperture Visual Odometry

The localization software utilizes Sarnoff's Multi-Camera Visual Odometry technology to provide precise estimates of the vehicle's location and orientation in the world. The principle of operation is as follows. Distinctive image features are matched between the left and right images of a stereo pair at each frame time. Triangulation provides 3D range estimates to the corresponding points in the scene. These features are tracked over time, and their motion relative to one another is measured. From these 2D image features and corresponding 3D scene points the system calculates a precise estimate of the cameras' own location and pose, in 6 degrees of freedom. Sarnoff's system uses two or more stereo cameras to observe a wide extent of the scene in 3D. While some of the cameras may be occluded, the others maintain visual odometry. Pose estimation algorithms try to minimize the errors across all the distributed aperture camera views simultaneously.

2.1.2 Local Kalman Filter

A Kalman filter combines these solutions with rotation and acceleration readings from the IMU. Our approach uses MEMS grade low-cost IMUs for integration. Using the results of visual odometry Sarnoff is able to accurately model the bias of the accelerometers and the gyro readings enabling successful inclusion of this information within the localization system. When fused with GPS (if available), the result is a geo-located position and pose. Sarnoff has shown experimentally that the visual odometry has a drift rate of 0.1% of distance traveled in GPS denied environments.

2.1.3 Landmark Matching

Sarnoff's Landmark Matching technology is used for correcting long-term drift in the absence of GPS. As the vehicle navigates complex terrain, a set of distinctive image features, visual landmarks, and their 3D locations are stored in a database [7]. In this system, Sarnoff defines a landmark as a feature point in the scene. Specifically, it is extracted from the image using a Harris corner detector. For each landmark, it is associated with three elements: (1) a 3D-coordinates vector representing its 3D location, (2) a 2D-coordinates vector representing its 2D location in the image and (3) a feature descriptor that characterizes its appearance. Here, the Histogram of Oriented Gradients (HOG) descriptor is used. If the vehicle returns to the same area in the course of the exercise, the software attempts to match the stored feature set with a newly observed set by first matching the features and then verifying the 3D geometry of the scene. A successful match results in a global pose correction. This gives the system the ability to maintain the robot's absolute location for extended periods of time even without GPS.

The landmark database is built in real time and the system opportunistically provides matches for 6DOF pose correction.

Two images are shown in Figure 1. Both images show an overhead view of a road which enters into a patch of trees. The image on the left shows the red ground survey points on the exterior of the road. More sporadic yellow and blue lines from a GPS/INS solution follow the red survey points. Under the patch of trees, the GPS/INS solution becomes much more sporadic and indistinguishable as following the road due to GPS blockage. The image on the right shows the same red ground survey points. Blue and purple lines, which represent the visual odometry and IMU solution, follow the survey points much more closely on both the open road and under the canopy of the trees, which shows the benefit of the vision/INS solution in a difficult GPS environment.

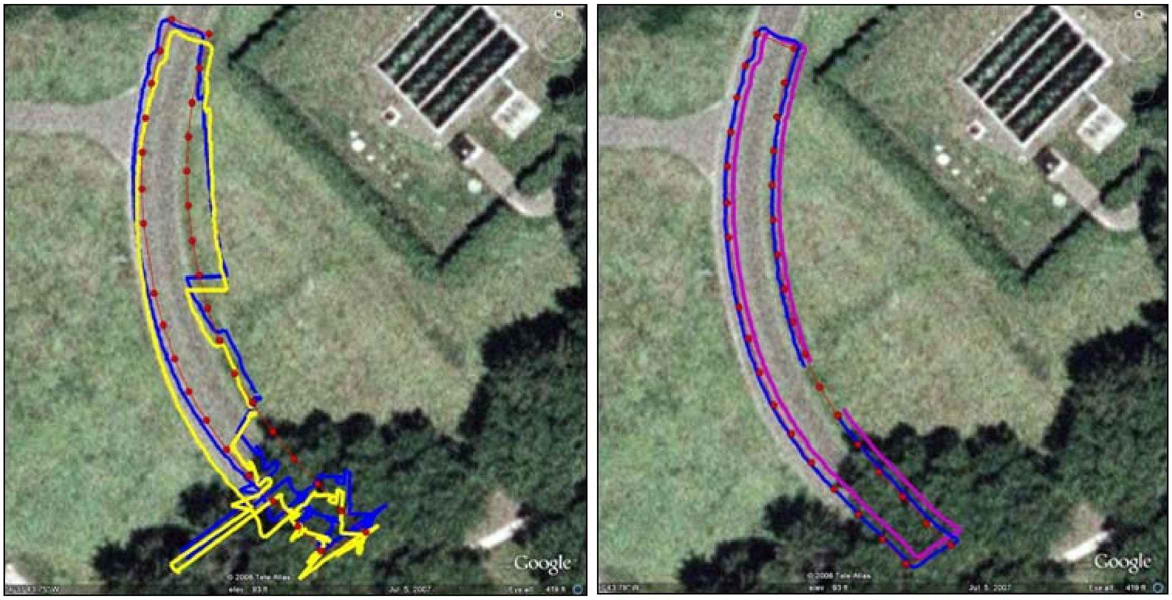

Figure 1. (Left) Track (yellow) estimated by fusing low cost MEMs IMU data with Satellite corrected GPS data shows unreliability of GPS near a tall tree cluster in contrast to ground survey points shown in red. (Right) Shows the results achievable when the same unreliable GPS information is fused with Visual Odometry and the low cost MEMs IMU (purple).

2.1.4 Distributed Multi-Vehicle Localization Filter

Additionally Sarnoff has the capability to use Inter-Site RF Ranging to further improve the results of our GPS-denied localization algorithms. The Inter-Site Ranging is performed by a micro-controller that obtains measurements to other radios in range to the system. The controller will simultaneously obtain ranging from other observers to the current observers and their locations. This forms the basis for a distributed Kalman filter for navigation. In simulations Sarnoff has shown that they can improve the localization accuracy by a factor of $\sqrt{N}$ where N is the number of ranging radios in the vicinity. Sarnoff has evaluated the use of UWB (ultra-wide-band) and Chirped RF radios for ranging in prior implementations.

The graph below in Figure 2 shows three plots – visual odometry drift in red and internode range measurements in blue and green. The x-axis of the graph is time in seconds ranging from 0-220 s, while the y-axis of the graph is the Averaged relative error in meters ranging from 0 to 0.5 meters. The internode range measurements start around 0.2 m, decrease to less than 0.1 m around 50 s, and increase slightly to 0.1 mand 0.2 m each at 220 s. The visual odometry drift in red is about 0.3 m at time 0 and gradually increases to 0.5m at time 220 seconds. The error and drift of the system is shown to be reduced when internode range measurements are available.

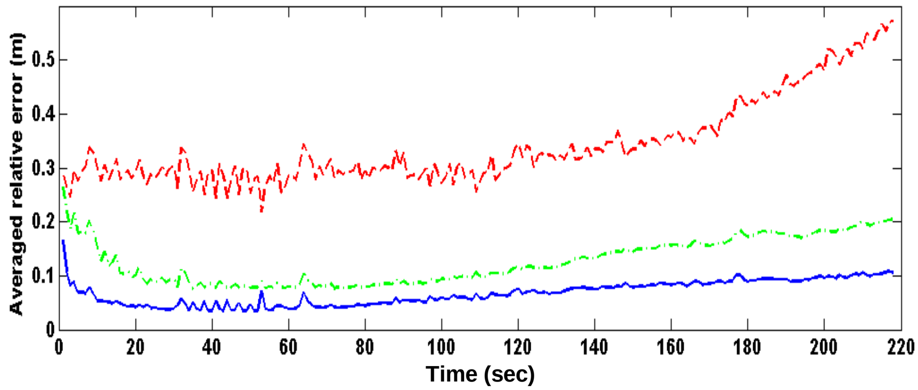

Figure 2. Distributed Navigation exploits the collaborative nature of team-oriented operations to provide substantial performance improvements. Error of the system is reduced when Visual Odometry drift (red curve) is constrained with inter-node range measurements (blue and green curves).

2.1.5 Visual Aided GPS-denied Systems

Sarnoff has implemented the Visual Aided GPS-denied system on a variety of platforms, including vehicles, robots and even body worn systems. With minimal tuning of parameters the same algorithms run on all platforms providing accurate localization for a variety of projects and circumstances. As shown in Figure 3, Sarnoff's large test-bed is an instrumented van which can be used to do long range tests as well as test performance at higher speeds.

Two images are shown in Figure 3. On the left image, a view of the multi-camera system is shown looking up towards the top of the van. Cameras are mounted on a roof rack that extends around the edge of the top of the vehicle. In the right image, the same van is shown but with a view looking from above the van down towards the front mounted cameras located above the driver's and passenger's seats.

Figure 3: Multi-camera system mounted on a van. The system include 2 stereo pairs, 4 monocular cameras, IMU and GPS

2.1.6 Evaluation

The van system allows Sarnoff to collect long distance sequence in urban environments. The sequence showcased in Figure 4 is a mile plus long loop through downtown Princeton, NJ, driven several times. This loop has several GPS challenging areas, including an overpass and large trees and buildings. Unfortunately since this a public area Sarnoff does not have this area instrumented and cannot provide accuracy measurements. In this experiment Sarnoff was looking for loop closure and performance in the urban environment as well as ensuring their van platform was functioning properly.

Figure 4 shows two images. The left image shows an aerial view of downtown Princeton. On the image are green and red lines representing vision odometry + GPS and GPS only, respectively. The green line forms a rectangular shape as the car is driven around a block. The red line follows a similar shape but is more sporadic than the green line. In the right image, a plot of the trajectories of the van is shown. The red line forms a rectangular shape with smoothed corners. The blue lines attempt a similar shape but with much greater offset from the red line and form a much more unlikely shape for a road path. Both images show the benefit of vision and odometry with GPS over GPS alone.

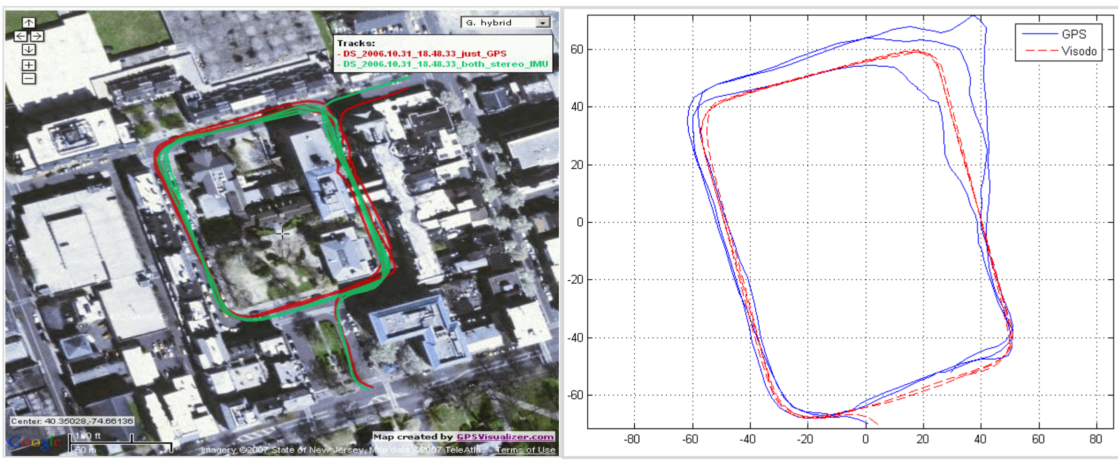

Figure 4. (Left) Google Earth map of the trajectory driven by the van. Red is GPS only and Green is Visodo + GPS (Right) Just the trajectories of the van. Blue is GPS only and Red is Visodo + GPS.

2.2 Penn State Progress

Currently, Penn State is working on Task (1), which involves sensor error characterization as well as determination of the relationship between localization accuracy and sensor models, with focus on commercial-grade sensors. Milestones for Q1, which included sensor characterization and terrain data collection, are well on their way to completion.

Auburn University has collected data from the Xbow IMU 440 as per the test protocol requirements given in Appendix 1 for sensor characterization.

Previous work at Penn State has shown that the limiting value of localization accuracy achievable with the developed particle filter algorithms is given by:

$$\sigma_{\infty}^{2} = P_{\infty} = \frac{Q}{2}\left(1 + \sqrt{1 + \frac{4R}{C^{2}Q}}\right)$$

where,

σ_{∞} = Localization accuracy (in meters),

Q = Process noise (i.e. variance in odometry)

R = Measurement noise (i.e. inertial sensor variance), and

C = Variation in inertial signal (i.e. variations in road pitch)

2.2.1 The Need for Sensor Characterization

One of the primary goals of the current work is to determine the minimum sensor accuracy required to obtain a certain level of localization accuracy. In order to do this, it is necessary to know the characteristics of the inertial sensors employed for measuring the road disturbances such as road pitch, roll and yaw. The sensor characteristics are indicative of the various noise components that corrupt the true measurement data. These noise components are errors arising from sources such as bias instability, angle random walk, rate ramp, quantization etc. The sensor characterization will allow the determination of these individual error components and their contribution towards the measurement noise (R). In this manner, sensor characterization enables the determination of inertial sensor variance (R), which further helps determine the localization accuracy (σ_{∞}). Working backwards, the same approach also enables one to use the desired localization accuracy to determine the necessary sensor accuracy, and hence the required sensor characteristics.

2.2.2 Methods for Sensor Characterization

There are numerous ways to perform sensor characterization, i.e. identify the various components that cause the measured value to be different from the true value of the variable. The sensor model can be characterized using the following steps, which are applicable for any generic sensor:

(1) **Literature review to identify potential error sources**: Measurements from any sensor may typically be affected by a subset of the entire population of possible error sources. Consequently, it may be necessary to include only a few specific error sources in their sensor noise models, and not others. Literature pertaining to noise modeling of inertial measurement units (IMUs) was reviewed [1],[2] and it was found that potential stochastic noise sources for IMUs include bias instability, angle random walk, rate random walk, and, in some commercial-grade sensors, quantization error.

(2) **Data collection to verify error sources and characterize sensors**: Data is collected across multiple runs (or alternatively, over long-range measurements) to obtain estimates for deterministic noise sources (scale factor, bias etc.) using the least-squares method, as well as for stochastic error sources using Allan variance analysis [3]. The Allan variance analysis is currently in progress using the data collected at Auburn.

(3) **Generating the sensor noise model using sensor characteristics**: It is known that various error sources have different power spectral densities. E.g. bias instability is essentially flicker noise. The sensor characteristics obtained from the previous step will be used to generate the sensor noise models for these error sources. In other words, the

sensor characteristics will be used to generate the differential equations that describe the behavior of each of the error sources.

In order to perform sensor characterization, a test protocol has been created at Penn State for collecting the requisite data. The test protocol has been utilized at Auburn University to collect data for the stochastic error source determination for the Xbow IMU 440 CA200. The test protocol is included in Appendix 1. Penn State is currently using the data to obtain the characteristics of the sensor.

To summarize, the data collected in these steps is being processed using Allan variance methods to determine individual error sources [4]. These steps help in generating the noise model for the sensors.

2.2.3 Noise modeling using sensor characteristics

Once the sensor characteristics (i.e. bias instability, angle random walk etc.) are known, they can be used to generate the noise model for the sensor. The power spectral densities (PSD) for the error sources are utilized to determine the differential equations that describe the behavior of the error sources. Differential equations for error sources with non-rational power spectrums are approximated by other processes [1]. For example, the bias instability is essentially flicker noise, so the PSD is a function of $\sqrt{j\omega}$. Thus, bias instability is modeled as a first order Gauss-Markov process. The differential equation describing the behavior of bias instability is given by:

$$\dot{b} = -\frac{b}{T_c} + \frac{B}{T_c}\eta$$

where,

b = Bias (as a function of time),

T_c = Correlation time

B = Sensor characteristic obtained from Allan variance analysis

η = White noise

Similarly, differential equations for other error sources have also been determined.

2.2.4 Determination of localization accuracy based on sensor characteristics

Further, these sensor characteristics can be used to determine the sensor variance (R), thus yielding a way of determining the localization accuracy. The various error sources and their corresponding differential equations are used to generate a single differential equation whose power spectral density is equivalent to the power spectral density of each of the error sources combined. For example the differential equations that describe the behavior of flicker noise (bias instability, b), angular rate random walk (arw) and rate ramp (rr) can be combined using a single variable as follows:

$$z(t) = b(t) + arw(t) + rr(t)$$

and the differential equation for $z(t)$ is generated using the individual differential equations of $b(t)$, $arw(t)$ and $rr(t)$ to yield:

$$z^{(4)}(t) + a_1 z^{(3)}(t) + a_2 z^{(2)}(t) + a_3 z^{(1)}(t) + a_4 z(t) = b_0 \eta^{(3)}(t) + b_1 \eta^{(2)}(t) + b_2 \eta^{(1)}(t) + b_3 \eta(t)$$

where $z^{(i)}$ denotes the ith derivative with respect to time, the coefficients a_i and b_i are obtained from the sensor characteristics, and η is white noise [1].

2.3 Kapsch TrafficCom Inc. Progress

This section summarizes the work activities accomplished by Kapsch TrafficCom Inc. in support of Year 1 Task 5 activities during the first quarter of the project.

(1) Finalizing sub-contract with Auburn University. Kapsch officer signatures should be in place no later than 25 Jan 2010.

(2) Preparing hardware for task 5 development and testing in anticipation of purchase requisition.

3. Future Work

Auburn University will continue to work with each partner to equip the test vehicle for testing.

3.1 Sarnoff Future Work

Future work for Sarnoff includes the completion of the remaining tasks in Year 1:

(1) Integrate Visual Navigation system on Auburn Engineering's Sonata vehicle test platform and collect test data using the integrated system.

(2) Process and analyze the data from the tests and evaluate the performance and recommend any improvements and optimizations.

3.2 Penn State Future Work

Future plans entail the scaling of the current particle filter algorithm to an entire city or county and the optimization for these algorithms. Alternative algorithmic approaches such as Kalman filter and feature-based localization techniques are currently being examined and compared with particle filter approaches. Analysis of the efficiency of various approaches will enable the development of algorithms which may be applicable for real-time vehicle localization.

To allow Penn State to continue advancing their localization algorithm, Auburn will interface their sensors to their test vehicle, provide a Kalman filter truth solution, as well as provide Penn State with the additional data requested in Appendix 1 .

As was mentioned in the scope of Penn State's contribution to the project, the overall objective of the project is to develop terrain-based localization algorithms which function over a large road network and are optimized for induction into real-time systems. Further, the project also entails the integration of the terrain-based localization algorithms with other alternative localization techniques. Penn State is simultaneously looking into techniques to realize these goals.

In view of the optimization problem, work is being planned to examine feature-based localization techniques [5], in addition to particle filter approaches, which may allow a more compact representation of terrain in terms of features. Feature-based localization involves preprocessing the road grade information from the terrain map to generate feature vectors, which are then grouped under a tree structure. A vehicle present at any location on the map can collect pitch data as a function of distance using onboard sensors and process it to obtain feature vectors. The collected feature vectors can then be matched efficiently with all the feature vectors in the existing tree data structure. Each feature vector match yields an estimate of the position of the vehicle and these position estimates are put together into a "position-estimate" histogram. The position in the histogram with the maximum number of votes is output as the best position estimate. This algorithm may enable faster convergence as compared to a particle filter-based approach.

In view of creating an algorithm that is applicable over a large road network, techniques such as "windowing" and buffering are being examined. "Windowing" involves localizing a vehicle within a large, coarse map and then a finely decimated map once the estimate converges [6]. Buffering involves loading a small segment of road that is expected to appear next in front of the vehicle before the vehicle actually arrives that location. The road network may be stored as a graph in order to facilitate this function. These techniques will help alleviate problems associated with branching roads and will also help decrease computational load.

3.3 Kapsch Future Work

This section summarizes the anticipated project tasks for the Kapsch team during the following quarter.

(1) Requisition of Kapsch 5.9 DSRC hardware by Auburn University

(2) Develop deployment plan for 5.9 DSRC hardware at the Auburn Test Track.

(3) Deploy 5.9 DSRC hardware and conduct on-site testing at the Auburn Test Track.

(4) Design, develop, integrate and test DSRC ranging software interface. Interface requirements based on data needs of Auburn system.

References

[1] Han, S., Wang, J., and Knight, N.; Using Allan Variance to determine the Calibration Model of Inertial Sensors for GPS/INS Integration, *Proceedings of the 6th International Symposium on Mobile Mapping Technology*, Sao Paulo, Brazil, 2009.

[2] Gupta, V.; Vehicle Localization Using Low-Accuracy GPS, IMU and Map-Aided Vision, Ph.D. dissertation, The Pennsylvania State University, 2009

[3] Allan, D; Statistics of Atomic Frequency Standards, *Proceedings of the IEEE*, vol. 52, no. 2, 1966.

[4] El-Sheimy, N., Hou, H., and Niu, X.; Analysis and Modeling of Inertial Sensors Using Allan Variance, *IEEE Transactions on Instrumentation and Measurement*, vol. 57, no. 1, 2008.

[5] Ferrari, V., Tuytelaars, T., and Van Gool, L.; Object Detection by Contour Segment Networks, *Proceedings of the European Conference on Computer Vision*, 2006

[6] Dean, A; Terrain-based Road Vehicle Localization using Attitude Measurements, Ph.D. Dissertation, The Pennsylvania State University, 2008

[7] Zhu, Zhiwei; Oskiper, Taragay; Samarasekera, Supun; Kumar, Rakesh; Sawhney, Harpreet S., "Ten-fold Improvement in Visual Odometry Using Landmark Matching," Computer Vision, 2007. ICCV 2007. IEEE 11th International Conference on , vol., no., pp.1-8, 14-21 Oct. 2007.

Appendix

A. Penn State Test Protocol

A.1. Test Protocol for characterizing deterministic error sources (or calibration)

The deterministic error sources under consideration are the scale factor error and the bias error. To determine the magnitude of these error sources, data for the variable of interest (in this case, the pitch of the road) has to be obtained across multiple runs. Since the variable of interest is the pitch, the sensors must ideally be co-located and otherwise be positioned along the y-axis (lateral axis) of the vehicle.

(1) Power up the apparatus (tactical-grade integrated IMU/GPS system, and other commercial-grade IMUs under consideration)
[NOTE: The tactical-grade integrated IMU/GPS system is necessary to establish the ground truth for position, velocity and attitude measurements.]
(2) Allow 5 minutes of warm-up time
(3) Begin driving the vehicle at approximately 30 mph and initiate data logging at a sampling rate of 100 Hz. Log position, velocity and attitude data (INSPVA) from the tactical-grade IMU/GPS system and from all the commercial-grade IMUs under consideration.
(4) Log data for 10 minutes on a highway.
[**NOTE**: Significant variations in pitch are observed at the scale of about 500 meters, so a dataset comprising between 5-10 km (or 3-6 miles) should provide an adequate set of measurements for calculating deterministic sources of error, such as scale factor. The time it takes to cover 3-6 miles at speeds of 30-60 mph is approximately 6 minutes. For ease of data collection and to guard against any erroneous data, it is recommended that the data be collected for 10 minutes.]
(5) Stop data logging and power down the system.
(6) Wait for 10 minutes and repeat steps (1) to (5) three more times, driving at approximately 40, 50 and 60 mph over the exact same stretch of highway and in the same direction.
(7) Repeat the procedure at slow speeds (say 20, 25, 30 and 35 mph) on city roads.
[**NOTE**: Collecting data across multiple road surfaces enables the characterization of the terrain, whose impact on localization accuracy needs to be studied.]
(8) Repeat the procedure at slow speeds (say 20, 25, 30 and 35 mph) on rural roads.

A.2. Test Protocol for characterizing stochastic error sources (or noise characterization)

The stochastic error sources can be characterized from a static test. The stochastic error sources include bias instability, angle random walk, rate random walk, rate ramp, quantization noise and sinusoidal noise. All or a subset of these error sources may be present depending on the type of sensor being investigated. The following steps may be performed for collecting the data:

(1) Place the sensing apparatus in a controlled environment or a closed lab at room temperature. It is desirable that extraneous sources of error such as temperature

fluctuations be minimized. However, if the sensing apparatus utilizes an integrated IMU/GPS system, place the apparatus in a stationary vehicle.

(2) Power up the apparatus (both the tactical-grade IMU/GPS system, and all other commercial-grade IMUs under consideration).

(3) Allow 5 minutes of warm-up time.

(4) Initiate data logging for all IMUs. Log position, velocity and attitude data for 5.5 hours at a sampling rate of 100 Hz. **Do not move the sensors in this duration.**

[**NOTE 1**: The lowest frequency random error source in an IMU is the rate ramp error. Thus the total duration of the test is limited by the time needed to estimate this error source. The rate ramp usually varies on the time scale of hours. Assuming that the characteristic time for rate ramp error is 1 hour and that one is willing to accept a 33% percentage error in determining this error component, the number of samples needed to estimate the rate ramp error is calculated using the formula:

$$\sigma = \frac{1}{\sqrt{2\left(\frac{N}{K} - 1\right)}}$$

where σ is the percentage error, N is the total number of data points required, and K is the number of data points per cluster. In the limiting case, there must be at least 1 unique data point per cluster. In terms of time, this translates to K = 1 hr for estimating rate ramp error. Substituting σ = 0.33, one obtains N = 5.5 hours. Thus the test should be conducted for the duration of 5.5 hours. (To provide a sense of scale, the test duration to obtain a 5% percentage error would be approximately 200 hours)]

[**NOTE 2**: The highest frequency random error source in a digital IMU is the quantization error (due to the inherent digital nature of the sensor). Since the error source is due to the digital nature of the signal itself, it has no bearing on the sampling rate. The random error source with the next highest frequency is the angle random walk. Thus, the sampling rate may be determined by:

(a) the frequency of the angle random walk error (of the order of 10 Hz), or

(b) the bandwidth of the sensor itself (in case it is an analog device).

In order to satisfy the Nyquist Theorem, the sampling rate must be at least twice as much as the highest frequency being measured, and preferably three to five times as much. A sampling rate of 100 Hz is adequate to capture the characteristics of angle random walk error. Similarly, considering a commercial analog IMU, MotionPak II, which has a bandwidth of 30 HZ, a sampling rate of 100 Hz is adequate.]

(5) Power down the system.