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Effects of LO Phase and Amplitude Imbalances and Phase Noise on M-QAM Transceiver Performance

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Abstract—**This paper presents a rigorous analytical model for analyzing the effects of local oscillator output imperfections such as phase/amplitude imbalances and phase noise on M-ary quadrature amplitude modulation (M-QAM) transceiver performances. A closed-form expression of the error vector magnitude (EVM) and an analytic expression of the symbol error rate (SER) are derived considering a single carrier, linear transceiver link with additive white Gaussian noise channel. The proposed analytical model achieves a good agreement with the simulation results based on Monte-Carlo method. The proposed QAM imperfection analysis model provides an efficient means for system and circuit designers to analyze the wireless transceiver performances and properly trade-off and specify the transceiver block specifications.**

I. Introduction

WITH the ever-increasing demand for high data rate in emerging communication systems, an M-ary quadrature amplitude modulation (QAM) with large M value, such as 32, 64 or 128 has received increasing interest for many wireless communications. On top of the complicated modulation schemes, the emerging high data-rate wireless systems are required to operate at higher and higher frequency bands, e. g., 2GHz for 3G systems, 2.4GHz/5.2GHz for WLAN networks, and so on. For a typical voltage controlled oscillator (VCO), the phase noise increases with the square of the center frequency [1]. Therefore, at high frequency band, the phase noise effect is rather significant for the M-ary QAM systems with large M value. Furthermore, the quadrature synthesizer will introduce non-negligible phase and amplitude imbalances for the LO frequency in the GHz range. Those imbalances will further degrade the QAM system performance [4].

It is thus desirable for both system and circuit level designs to have a rigorous analytical model for the imperfections of the transceiver such as phase and amplitude imbalances, phase noise, non-ideal synchronizations, and nonlinearity. There have been investigations related to this topic [2][3][4].

In this paper, a system model of a transceiver data link for an M-QAM system is built, which includes the phase and amplitude imbalances and phase noise in both transmitter and receiver LOs. With this model, a closed-form analytic expression of the EVM is derived. Moreover, an analytical expression of the SER is obtained with the presence of the imbalances and phase noise. This integration expression of the SER has no closed-form solution and can only be numerically calculated by the finite element method, which is still more efficient compared to the commonly used Monte-Carlo based simulation approaches.

II. System Model

This paper focuses on a one-directional physical link between the coder in the transmitter (Tx) and the decoder in the receiver (Rx) using QAM modulation. The effects of amplitude imbalance, phase imbalance, dc offset and phase noise will be discussed. The equivalent system model in vector presentation is shown in Fig. 1. The $\mathbf{s}(t) = [s_i(t),\ s_q(t)\ ]^{\mathrm{T}}$ presents the shaped QAM output, where the superscript T denotes the transpose operation. The quadrature modulator imperfection are modeled by a vector $\mathbf{a} = [a_i, a_q\ ]^{\mathrm{T}}$ which represents the DC offset and a matrix $\mathbf{T}(t) = 2[kcos(\omega t), sin(\omega t + \phi)\ ]$ which represents the imbalances where ω is carrier angular frequency, k presents the amplitude imbalance which has a value around 1, ϕ presents the phase imbalance. The constant, 2, is inserted to keep the gain of the transceiver link to be one. Therefore, the output signal from the quadrature modulator is a time-varying scalar expressed as

$$u(t) = \mathbf{T}(t)\left[\mathbf{s}(t) + \mathbf{a}\right]. \tag{1}$$

Similarly to the modulator, the imperfection of the quadrature demodulator is modeled by a vector $\mathbf{b} = [b_i, b_q\ ]^{\mathrm{T}}$ and $\mathbf{R}(t) = [lcos(\omega t + \alpha), sin(\omega t + \alpha + \gamma)\ ]^{\mathrm{T}}$ where α presents a constant phase difference between the transmitter LO and the receiver LO, l presents the amplitude imbalance, γ presents the phase imbalance. Let $n(t)$ denote AWGN with power single-side spectrum density (PSD) of N_0. Using all above expressions, the output signal $\mathbf{r}(t)$ can be expressed as

$$\mathbf{r}(t) = \mathbf{H}\mathbf{s}(t) + \mathbf{c} + \mathbf{n}_r(t), \tag{2}$$

with the output constant offset given as $\mathbf{c}=\mathbf{Ha}+\mathbf{b}$, the channel matrix given as

$$\mathbf{H} = LPF\langle \mathbf{R}(t)\mathbf{T}(t)\rangle = \begin{bmatrix} klcos(\alpha) & lsin(\phi - \alpha) \\ ksin(\alpha + \gamma) & cos(\alpha + \gamma - \phi) \end{bmatrix} \tag{3}$$

where $LPF\langle\ \ \rangle$ presents the function of low pass filter (LPF), and the received channel noise given as

$$\mathbf{n}_r(t) = LPF\left\langle \mathbf{R}(t)n(t)\right\rangle. \tag{4}$$

The received noise, $\mathbf{n}_r(t)$, is still a Gaussian noise, because it comes from a linear transform of the Gaussian channel noise. It has zero mean, and its covariance matrix is given by

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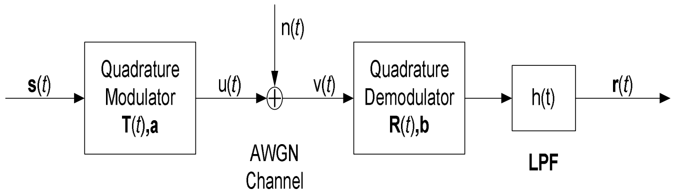

Fig. 1. Equivalent system model.

$$\mathbf{C}_{\mathbf{n}_r\mathbf{n}_r} = \frac{N_0 B}{4}\begin{bmatrix} l^2 & l\sin(\gamma) \\ l\sin(\gamma) & 1 \end{bmatrix}, \tag{5}$$

where B presents the bandwidth of LPF. Thus, the presence of the receiver phase imbalance results in a correlated noise in I and Q channels.

Assuming an ideal synchronization of the sampling time, the received signal samples then are expressed in discrete time domain as

$$\mathbf{r}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{c} + \mathbf{n}_r(n), \tag{6}$$

III. Error Vector Magnitude Calculation

The error vector is the difference between the transmitted and received signal vectors, namely,

$$\mathbf{e}(n) = \mathbf{r}(n) - \mathbf{s}(n) = (\mathbf{H}-\mathbf{I})\mathbf{s}(n) + \mathbf{c} + \mathbf{n}_r(n) = \mathbf{e}_s(n) + \mathbf{e}_n(n). \tag{7}$$

The above error vector consists of two components: one, denoted by $\mathbf{e}_s(n)=(\mathbf{H}-\mathbf{I})\mathbf{s}(n)+\mathbf{c}$, is related to the signal vector and is determined by a given symbol; another, denoted by $\mathbf{e}_n(n)=\mathbf{n}_r(n)$, comes from channel random noise with Gaussian distribution. Then the EVM can be found out as

$$\begin{aligned} EVM^2 &= \sigma_{\mathbf{e}}^2 = E\left[\mathbf{e}^{\mathrm{H}}(n)\mathbf{e}(n)\right] = E\left[\mathbf{e}^{\mathrm{T}}(n)\mathbf{e}(n)\right] \\ &= E\left[\left((\mathbf{H}-\mathbf{I})\mathbf{s}(n)+\mathbf{c}+\mathbf{n}_r(n)\right)^{\mathrm{T}}\left((\mathbf{H}-\mathbf{I})\mathbf{s}(n)+\mathbf{c}+\mathbf{n}_r(n)\right)\right], \end{aligned} \tag{8}$$

where the superscript H denotes Hermitian operator which is equivalent to transpose operator and used in the following derivations. Under the assumptions that the transmitted signal vectors have zero mean and are independent of noise, and that I and Q components of the signal vectors are independent, equation (8) can be simplified as

$$EVM^2 = E\left[\mathbf{s}^{\mathrm{T}}(n)(\mathbf{H}-\mathbf{I})^{\mathrm{T}}(\mathbf{H}-\mathbf{I})\mathbf{s}(n)\right] + \mathbf{c}^{\mathrm{T}}\mathbf{c} + E\left[\mathbf{n}_r(n)^{\mathrm{T}}\mathbf{n}_r(n)\right]. \tag{9}$$

Based on the above mentioned assumptions, the covariance matrix of signal is obtained as

$$\mathbf{C}_{\mathbf{s}(n)\mathbf{s}(n)} = \frac{E_s R}{2}\mathbf{I}, \tag{10}$$

where E_s presents average symbol energy, R presents symbol rate, and $\mathbf{I}$ presents a unit matrix. Therefore, with equation (3), the first term in the right hand of (9) can be expressed as

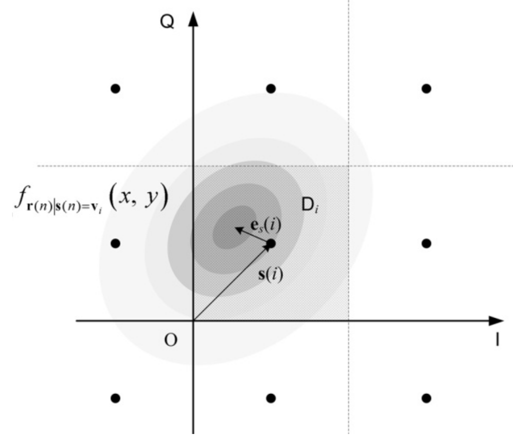

Fig. 2. PDF of the received vector and the decision zone.

$$\begin{aligned} &E\left[\mathbf{s}^{\mathrm{T}}(n)(\mathbf{H}-\mathbf{I})^{\mathrm{T}}(\mathbf{H}-\mathbf{I})\mathbf{s}(n)\right] \\ &= \frac{E_s R}{2}\left[\left(k^2 l^2\cos^2(\alpha) + k^2\sin^2(\phi-\alpha) + l^2\sin^2(\alpha+\gamma)\right.\right. \\ &\quad \left.\left. + \cos^2(\alpha+\gamma-\phi)\right) - 2\left(kl\cos(\alpha) + \cos(\alpha+\gamma-\phi)\right) + 2\right]. \end{aligned} \tag{11}$$

From equation (5), it can be obtained that

$$E\left[\mathbf{n}_r(n)^{\mathrm{T}}\mathbf{n}_r(n)\right] = \frac{N_0 B\left(l^2+1\right)}{4}. \tag{12}$$

And

$$\mathbf{c}^{\mathrm{T}}\mathbf{c} = \mathbf{a}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{H}\mathbf{a} + \mathbf{a}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{b} + \mathbf{b}^{\mathrm{T}}\mathbf{H}\mathbf{a} + \mathbf{b}^{\mathrm{T}}\mathbf{b}. \tag{13}$$

In practices, the EVM is usually normalized by signal power, $P=RE_s$. And by assuming that symbol rate equals to the pass bandwidth of the LPF, $R=B$, the normalized EVM is thus obtained as

$$EVM_{normalized} = \sqrt{\frac{1}{2}\mathrm{Tr}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right) - \mathrm{Tr}(\mathbf{H}) + 1 + \frac{\left(l^2+1\right)}{4E_s/N_0} + \frac{\mathbf{c}^{\mathrm{T}}\mathbf{c}}{P}}. \tag{14}$$

In the radical sign, $\mathrm{Tr}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)/2 - \mathrm{Tr}(\mathbf{H}) + 1$ represents the contribution of the phase and amplitude imbalances to EVM; $\left(l^2+1\right)/\left(4E_s/N_0\right)$ represents the contribution of the channel noise; and $\mathbf{c}^{\mathrm{T}}\mathbf{c}/P$ represents the contribution of the constant offsets. By using (14), the effects of the transceiver imbalances, channel noise and modulation offsets on EVM can be further analyzed.

IV. Symbol Error Rate Calculation

Considering an M-QAM, all possible symbols are defined as an alphabet of $\mathbb{S} = \{s_1, s_2, \ldots s_N\}$. And the corresponding modulated vectors are defined as a vector alphabet of $\mathbb{V} = \{v_1, v_2, \ldots v_N\}$. Recalling (6) and (7), for a given transmitted signal vector $\mathbf{s}(n) \in \mathbb{V}$, the received vector is expressed as

$$\mathbf{r}(n) = \mathbf{s}(n) + \mathbf{e}_s(n) + \mathbf{e}_n(n). \tag{15}$$

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As illustrated in Fig. 2, the received vector has a deterministic component, $\mathbf{s}(n)+\mathbf{e}_s(n)$, and a random component, $\mathbf{e}_n(n)$. With the covariance matrix obtained in (5), the Gaussian noise, $\mathbf{e}_n(n)$, has a joint normal probability density function (PDF) that is expressed as

$$f_{\mathbf{e}_n}(x,y)=N_2\left(0,0,N_0Bl^2/4,N_0B/4,\sin(\gamma)\right)$$
$$=A\exp\left\{-\frac{2}{N_0B\left(1-\sin^2(\gamma)\right)}\left[\frac{x^2}{l^2}-2\sin(\gamma)\frac{xy}{l}+y^2\right]\right\}, \quad (16)$$

and

$$A=\frac{2}{\pi N_0Bl\sqrt{1-\sin^2(\gamma)}}, \quad (17)$$

where $N_2(\,)$ denotes a 2-dimensional joint normal distribution PDF; x and y denote the I and Q components of received noise vector that are zero-mean random variables with their variances and correlation given by the covariance matrix in (5). Shown in Fig. 2, the concentric ellipses in different gray colors illustrate the PDF of the noise.

For the system model considered in this paper, the conditional detection probability is the integral of the conditional PDF of the received sample vector in the decision zone, $\mathbf{D}_i$, namely,

$$P_{D_i}=\iint_{\mathbf{D}_i} f_{\mathbf{r}(n)|\mathbf{s}(n)=\mathbf{v}_i}(x,y)\,dxdy=\iint_{\mathbf{D}_i} f_{\mathbf{e}_n}\left(x-\bar{r}_x(n),\,y-\bar{r}_y(n)\right)dxdy. \quad (18)$$

And the conditional error probability is the integration of the conditional PDF of the received sample vector in the area other than decision domain, $\mathbf{D}_i$, which is complement of the conditional detection probability as $P_{e_i}=1-P_{D_i}$.

The decision zone can be expressed in a same form as $D_i=\{x,\,y\,|\,x\in(a_i,\,b_i)\ \mathrm{y}\in(c_i,\,d_i)\}$ where the boundaries can be either finite or infinite. The detection probability then is expressed as

$$P_{D_i}=\int_{c_i}^{d_i}\int_{a_i}^{b_i}A\exp\left\{-\frac{4}{N_0B\left(1-\sin^2(\gamma)\right)}\left[\frac{\left(x-\bar{r}_x(n)\right)^2}{l^2}-2\sin(\gamma)\frac{\left(x-\bar{r}_x(n)\right)\left(y-\bar{r}_y(n)\right)}{l}+\left(y-\bar{r}_y(n)\right)^2\right]\right\}dxdy. \quad (19)$$

Based on a basic assumption each symbol in $\mathbb{S}$ has the same probability to be sent, the total symbol error rate (SER) is obtained by averaging the conditional detection error probability of each symbol as $P_e=\frac{1}{N}\sum_{i=0}^{N-1}P_{e_i}$.

V. PHASE NOISE CONSIDERATION

The phase noise results in both random phase imbalances in quadrature modulators and random phase difference between two LOs. We approximate the phase noise as a zero-mean Gaussian noise in phase representation whose variance is defined as the LO mean square phase error.

A. EVM Calculation

All approaches in above sections are valid formally except that the phase variables should be understood as random variables. Therefore, the phase difference, α, is updated as

$$\alpha=\alpha_d+\alpha_r,\quad \alpha_r\sim N\left(0,\ \alpha_{rms}^2\right) \quad (20)$$

where α_d denotes a constant phase difference and α_r is Gaussian random variable which represents the phase noise.

Substituting (20) to (3),

$$\mathbf{H}=\begin{bmatrix} kl\cos(\alpha_d+\alpha_r) & l\sin(\phi-\alpha_d-\alpha_r) \\ k\sin(\alpha_d+\alpha_r+\gamma) & \cos(\alpha_d+\alpha_r+\gamma-\phi)\end{bmatrix}. \quad (21)$$

Under the assumption that α_{rms}<<1, above equation can be simplified as

$$\mathbf{H}=\begin{bmatrix} kl\cos(\alpha_d) & l\sin(\phi-\alpha_d) \\ k\sin(\alpha_d+\gamma) & \cos(\alpha_d+\gamma-\phi)\end{bmatrix}+\alpha_r\begin{bmatrix} -kl\sin(\alpha_d) & -l\cos(\phi-\alpha_d) \\ k\cos(\alpha_d+\gamma) & -\sin(\alpha_d+\gamma-\phi)\end{bmatrix} \quad (22)$$
$$\triangleq \mathbf{H}_d+\alpha_r\mathbf{H}_r,$$

where $\mathbf{H}_d$ has the same form of $\mathbf{H}$ in (3), and α_r is factored out of the matrix and multiplied by a deterministic matrix $\mathbf{H}_r$. Then, (6) becomes

$$\mathbf{r}(n)=\mathbf{H}_d\mathbf{s}(n)+\alpha_r\mathbf{H}_r\left(\mathbf{s}(n)+\mathbf{a}\right)+\mathbf{c}+\mathbf{n}_r(n), \quad (23)$$

where $\mathbf{c}=\mathbf{H}_d\mathbf{a}+\mathbf{b}$.

Therefore, the normalized EVM in (14) becomes

$$EVM_{normalized}=$$
$$\sqrt{\frac{1}{2}\mathrm{Tr}\left(\mathbf{H}_d^{\mathrm{T}}\mathbf{H}_d\right)\text{-}\mathrm{Tr}\left(\mathbf{H}_d\right)+1+\frac{\alpha_{rms}^2}{2}\mathrm{Tr}\left(\mathbf{H}_r^{\mathrm{T}}\mathbf{H}_r\right)+\frac{\left(l^2+1\right)}{4E_s/N_0}+\frac{\alpha_{rms}^2\mathbf{a}^{\mathrm{T}}\mathbf{H}_r^{\mathrm{T}}\mathbf{H}_r\mathbf{a}+\mathbf{c}^{\mathrm{T}}\mathbf{c}}{P}}. \quad (24)$$

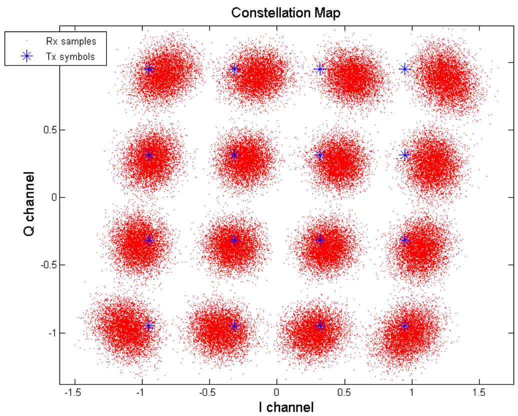
Comparing with (14), in the radical sign, $\mathrm{Tr}\left(\mathbf{H}_d^{\mathrm{T}}\mathbf{H}_d\right)/2\text{-}\mathrm{Tr}\left(\mathbf{H}_d\right)+1$ represents the contribution of the phase and amplitude imbalances to EVM; $\left(l^2+1\right)/\left(4E_s/N_0\right)$ represents the contribution of the channel noise; $\left(\alpha_{rms}^2\mathbf{a}^{\mathrm{T}}\mathbf{H}_r^{\mathrm{T}}\mathbf{H}_r\mathbf{a}+\mathbf{c}^{\mathrm{T}}\mathbf{c}\right)/P$ represents the contribution of the constant offsets which increased by a term related to phase noise; and a new component, $\alpha_{rms}^2\mathrm{Tr}\left(\mathbf{H}_r^{\mathrm{T}}\mathbf{H}_r\right)/2$, appears to represents the increased contribution from phase noise to EVM.

B. SER Calculation

With the presence of the phase noise, the expression of the error of the received signal becomes

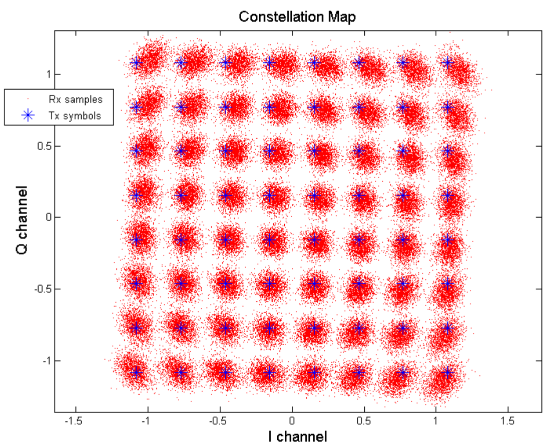
$$\mathbf{r}(n)=\mathbf{s}(n)+\mathbf{e}_s(n)+\mathbf{e}_p(n)+\mathbf{e}_n(n). \quad (25)$$

where $\mathbf{e}_s(n)$ becomes $\left(\mathbf{H}_d\text{-}\mathbf{I}\right)\mathbf{s}(n)+\mathbf{c}$, and $\mathbf{e}_p(n)=\alpha_r\mathbf{H}_r\left(\mathbf{s}(n)+\mathbf{a}\right)$ represents the phase noise effect. Since both the channel noise and phase noise are Gaussian, the sum of them, $\mathbf{e}(n)=\mathbf{e}_p(n)+\mathbf{e}_n(n)$, is still Gaussian.



Calculated EVM = 0.2067	Simulated EVM = 0.2063
Calculated SER = 2.963e-2	Simulated SER = 2.977e-2

Fig.3. 16-QAM sample situation: k = 1.05 = 0.42dB, φ=5°, $\mathbf{a} = [0.06,0]^T$, l = 1.05= 0.42dB, γ=2°, $\mathbf{b} = [0, -0.05]^T$, α_d=-3°, α_{rms}=3°, and N_0= -15dB with 1e5 symbols simulated.



Calculated EVM = 0.09679	Simulated EVM = 0.09668
Calculated SER = 4.615e-2	Simulated SER = 4.587e-2

Fig.4. 64-QAM sample situation: k = 1.02 = 0.17dB, φ=2°, $\mathbf{a} = [0.03,0]^T$, l = 0.97 = -0.26dB, γ=-1°, $\mathbf{b} = [0, -0.02]^T$, α_d=-1°, α_{rms}=2°, and N_0= -20dB with 1e5 symbols simulated.

Following the similar approach used for (18), and (19) in Section IV, the SER can be calculated with the presence of the phase noise as

$$P_e = \frac{1}{N}\sum_{i=0}^{N-1}\left(1-\iint_{\mathbf{D}_i} f_{\mathbf{r}(n)|\mathbf{s}(n)=\mathbf{v}_i}\left(x,\, y\right)dxdy\right). \qquad (26)$$

VI. Simulations and Calculations

Fig. 3 and Fig. 4 demonstrate two typical situations for 16-QAM and 64-QAM, respectively. As shown in the figures, the results from the analytical calculation are well matched with the results from Monte-Carlo simulation.

Fig. 5 shows the SER variation with E_s/N_0 in which only the transmitter phase imbalance is considered. Fig. 6 considers only the phase noise contribution. Several data points obtained from simulation are shown to verify the theoretical results.

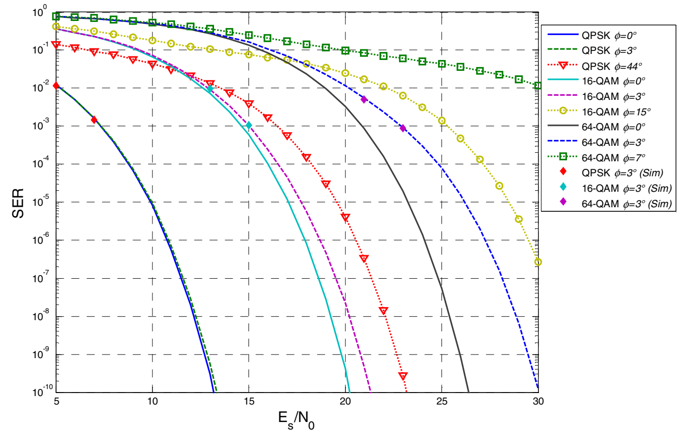


Fig. 5. SER~E_s/N_0 with different transmitter phase imbalance for QPSK, 16-QAM and 64-QAM.

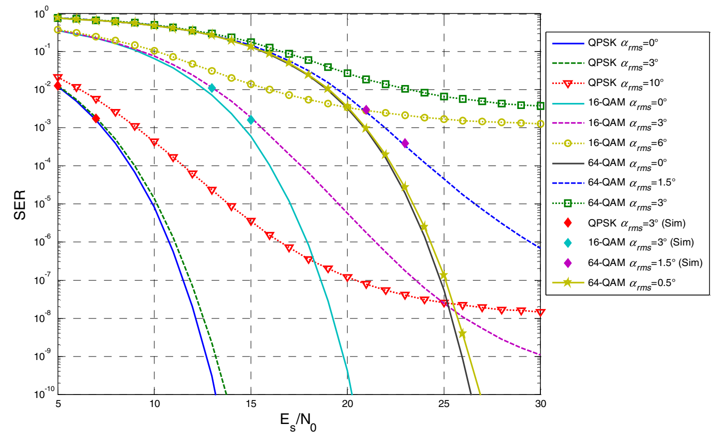


Fig. 6. SER~Es/N0 with phase noise for QPSK, 16-QAM and 64-QAM.

VII. Conclusions

In the paper, a theoretical analysis of the joint effects of the transmitter and receiver phase and amplitude imbalance, phase noise and channel noise on M-QAM systems has been presented. An analytic expression for EVM and an integral expression for SER have been derived in (24) and (26) , which provides an efficient means to specify the wireless system and block requirements considering the effects of those imperfections on the system performance. The analysis showed that the 64-QAM is very sensitive to phase noise and other system imperfections. The analytical results were also compared with time-consuming Monte-Carlo simulation results and showed a good agreement. For a more complete system analysis, the nonlinearity from RF amplifier and multi-path fading channel should be considered. This could be the subject of the future work.

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