Report 930-490

Erosion Characteristics of Alabama Soils Obtained with the Erosion Function Apparatus and Correlations with Classification Properties

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ABSTRACT

This report presents measurements of soil erosion properties obtained with the Erosion Function Apparatus (EFA) and correlations between soil erosion properties and routine soil classification properties. Shelby tube soil samples from bridge sites in Alabama were tested in the EFA to determine their erosion function (scour rate vs. shear stress). Properties determined from the erosion functions were the critical shear stresses and the initial slopes of the erosion functions (initial erodibility). Erosion functions were measured for 20 soils. More cohesive behavior, quantified in the erosion functions, corresponded to sites of minimal historical scour. Classification properties considered were: soil description, particle size distribution, plasticity index, and blow counts from the standard penetration tests. Correlations indicated that critical shear stress decreases as particle size increases, initial erodibility decreases as critical shear stress increases, critical shear stress increases as plasticity index increases, and critical shear stress was not strongly related to blow counts.

ACKNOWLEDGMENTS

This project was supported by the Alabama Department of Transportation (Research Project No. 930-490) and administered by the Highway Research Center of Auburn University. The authors thank the Materials and Tests Bureau of the Alabama Department of Transportation for collecting the soil samples that were tested in this study. Thanks are also given to Ms. Priscilla Clark who helped with the hardcopy and web publication of the report.

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I. INTRODUCTION

Scour under bridges is a major concern. There are about 575,000 bridges in the National Bridge Inventory (NBI) and approximately 84 percent of these are over water (Lagasse et al., 1995). Scour which occurs during flooding is the most common cause for bridge failure (Richardson and Davis, 1995). An average cost of $50 million per year is spent on the federal aid system for flood damage repair (Lagasse et al., 1995).

Currently, scour prediction is done using methods described in HEC-18 and HEC-20 (Richardson and Davis, 1995; Lagasse et al., 1995). These are hydraulic engineering circulars that were published by the Federal Highway Administration (FWHA). These methods assume noncohesive behavior and predict the ultimate scour which would occur in a soil and do not consider the rate of scour development which is important in fine grained cohesive soils. To quantify the rate of scour in cohesive soils the Erosion Function Apparatus (EFA) was developed by Briaud et al. (1999, 2001a, 2001b).

The EFA can be used for any type of soil which can be sampled with a standard Shelby tube. It has been used for both coarse grained soils such as sands and for fine grained soils such as clays. In this particular study only cohesive soils are considered. The EFA is used to find the erosion function of a soil. The erosion function is the relation between the scour rate ($\dot{z}$) and the shear stress (τ) as shown in FIGURE 1. The critical shear stress (τ_c) is the shear stress below which no scour takes place. The initial erodibility (S_i) indicates how fast the soil scours at the critical shear stress and is the slope of a straight line tangent to the erosion function at the critical shear stress.

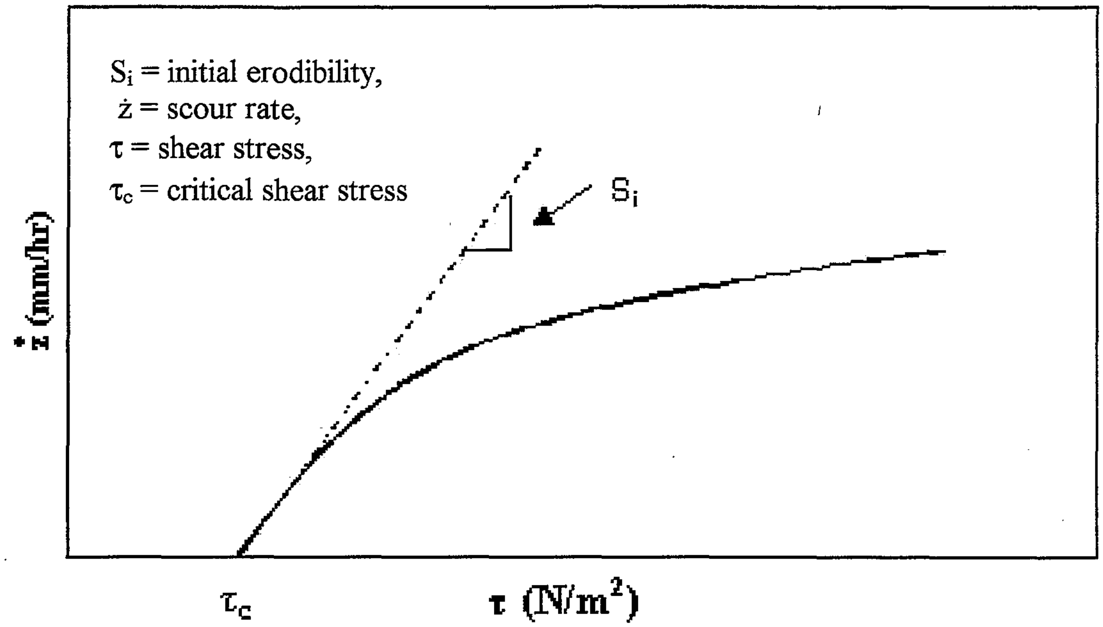


FIGURE 1. Erosion function obtained from running an EFA test.

Rate of scour at a bridge site can be predicted given the soil erosion function for a sample from the site. The method called SRICOS, Scour Rate In Cohesive Soils, is described in work by Briaud et al. (1999, 2001a, 2001b) in which the EFA data is used to predict the development of scour depth with time around a cylindrical bridge pier. The EFA data has also been used by Güven et al. (2002) to predict scour in cohesive soils at a bridge contraction. The results of a calculation are shown in FIGURE 2 where EFA data was used with a hydraulic model of flow through a contraction and a 400 day hydrograph to develop a prediction of the scour accumulation. The procedure used to obtain this figure can be found in Santamaria (2003). In general, knowledge of τ_c and S_i allows estimates for the rate of scour in cohesive soils which is an improvement over ultimate scour methods which are currently being used.

The purpose of this study was to perform EFA tests on soil samples from selected sites in Alabama and to determine if there are meaningful correlations between τ_c and S_i and basic soil classification properties. Such correlations can be used to provide preliminary estimates of scour when EFA testing cannot be justified. Correlations then also suggest possible variability of the values of τ_c and S_i. Correlations have been suggested in work done by Briaud et al. (1999, 2001a, 2001b) and Chen and Cotton (1984).

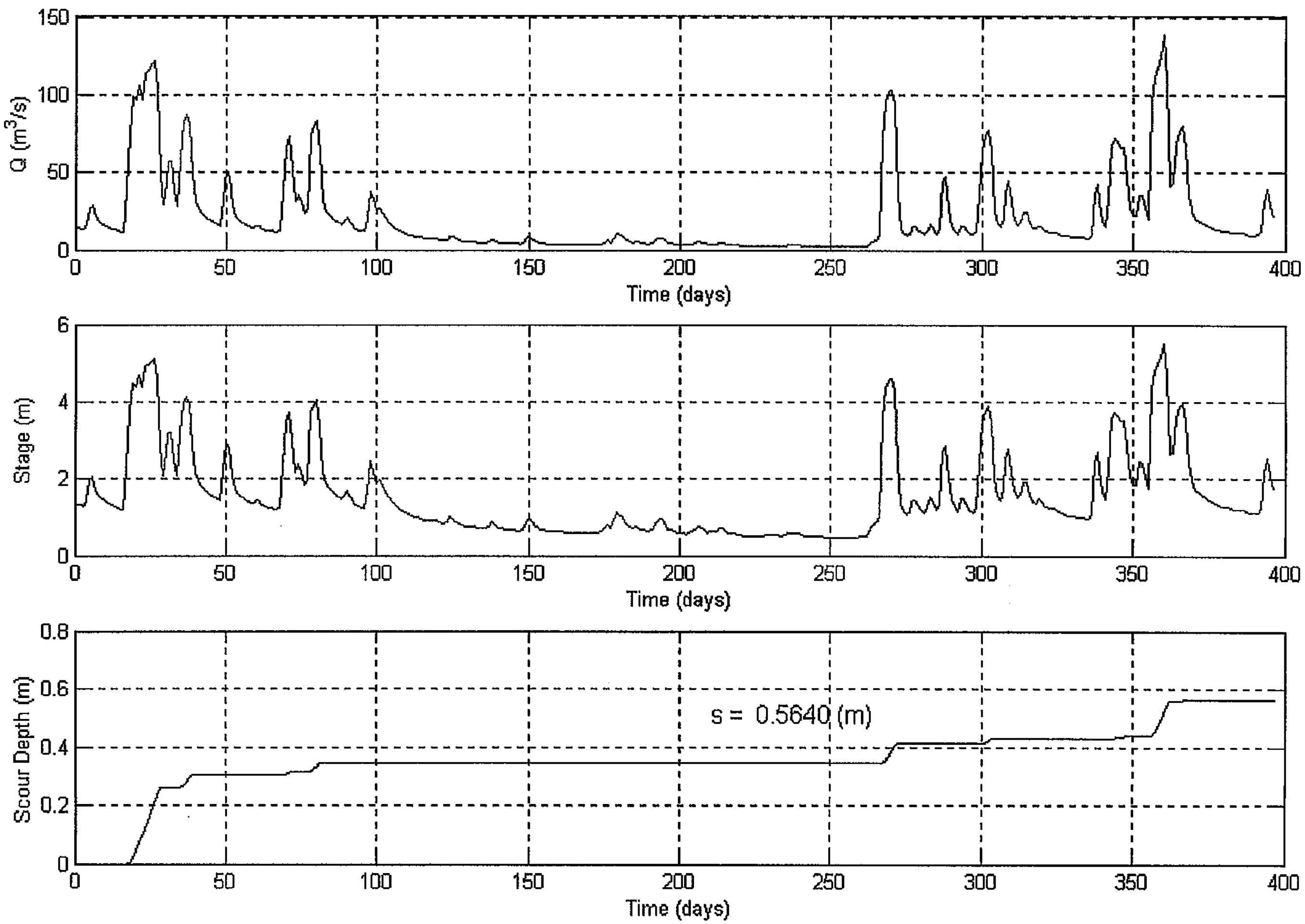


FIGURE 2. Hydrograph, stage, and scour depth vs. time plot from Santamaria (2003).

II. SAMPLING AND SOIL CLASSIFICATION TESTING

SAMPLING

The Alabama Department of Transportation (ALDOT) supplied the soil samples used in this study. The samples were obtained in the field by pushing or driving an ASTM standard Shelby tube with an outside diameter of 76.2 mm into the ground (ASTM-D1587). The samples came from various sites throughout the state of Alabama. Most of the soil samples that were tested were cohesive, but some had traces of sand. ALDOT also supplied the boring logs with the samples. These logs gave valuable information that was recorded during the actual sampling process. This information included the depth at which the sample was taken, soil descriptions, and blow counts (N). The blow counts were determined with standard penetration tests (ASTM-D1586).

Twenty samples were tested from six bridge locations. The samples came from various boreholes and depths at the sites. Samples were tested from a bridge site on Goose Creek in Wilcox County, from a culvert site on US 84 in Covington County, from a dual bridge on the Linden Bypass over the CSX and BNSF railroad in Marengo County, from a bridge on County Road 5 over Cheaha Creek in Talladega County, from a bridge on Alabama State Road 123 over the Choctawhatchee River in Dale County, and from a bridge over the Pea River in Elba. These samples are listed in TABLE 1 with depths, soil descriptions, and blow counts taken from boring logs. Based on EFA tests, soil erosion properties, τ_c, and S_i, for the twenty soils are also shown in TABLE 1.

SOIL CLASSIFICATION TESTING

Soils were tested to determine particle size and plasticity. Particle size analysis was done using procedures in ASTM-D422 to determine D_{50} and percent passing the number 200 sieve. Plastic limits, liquid limits, and plasticity indices of soils were determined using procedures in ASTM-D4318. Samples of material passing the number 200 sieve were tested with a laser based particle size analyzer to determine the percent clay, i.e., the percentage of particles smaller than 0.002mm. Results from these tests are contained in TABLE 1.

TABLE 1. Soils tested and properties measured.

Sample	Depth (ft)	Soil Description	N	D_{50} (mm)	% Passing #200	% clay*	PI	Activity (PI/%clay)	τ_c (N/m^2)	S_i ($mm/hr/N/m^2$)
Goose (TW1A)	15.5 - 17.5	Dark Brown Clay	4	-	-	-	8	-	3.00	0.50
Goose (TW5A)	8.0 - 10.0	Dark Brown Clay & Light Brown Silty Clay	2	-	-	-	15	-	0.40	5.60
Goose (TW6A)	40.0 - 43.0	Brown & Gray Clay	49	-	-	-	10	-	4.30	0.48
Goose (TW6B)	60.0 - 63.0	Clayey Sand & Silt	65	-	-	-	3	-	0.75	1.90
Goose (TW18A)	10.5 - 12.5	Brown Clay	10	-	-	-	14	-	4.50	0.18
Covington (1A)	6.0 - 8.0	Clay w/ Sand	22	0.21	20	0.67	6	9	1.10	6.06
Covington (1B)	11.0 - 13.0	Tan Clay w/ Soft Gray Clay	22	0.10	42	0.97	22	22	3.10	1.72
Marengo (2A)	1.0 - 3.0	Stiff Clay	10	0.03	100	4.07	32	8	4.70	0.38
Marengo (8A)	1.0 - 3.0	Stiff Clay w/ Small Amount Sand	13	0.17	31	0.91	12	13	0.90	11.42
Marengo (8B)	6.0 - 8.0	Stiff Clay w/ Sand	14	0.17	27	0.90	8	9	0.85	10.43
Talladega (4A)	1.0 - 3.0	Stiff Clay w/ Silt	6	0.04	66	3.51	7	2	2.82	0.42
Talladega (4B)	6.0 - 8.0	Soft Clay w/ Sand	9	0.15	26	1.36	3	2	0.53	-
Talladega (4C)	11.0 - 13.0	Soft Clay w/ Sand	12	0.15	26	1.36	3	2	0.60	-
Choctawhatchee (1A)	10.0 - 12.0	Gray Clay	45	0.03	100	3.40	24	7	2.50	0.96
Choctawhatchee (4C)	11.0 - 13.0	Gray Silt w/ Clay	26	0.03	100	3.30	14	4	1.25	1.20
Choctawhatchee (4B)	6.8 - 8.0	Sand w/Clay	16	0.32	21	0.56	NP	-	0.46	7.50
Choctawhatchee (3B)	5.0 - 7.0	Sand w/Clay	10	0.15	14	0.43	6	14	0.65	-
Pea (2B)	13.5 - 15.5	Gray Silt	30	0.03	100	2.77	11	4	1.40	-
Pea (2A)	10.0 -12.0	Tan Clay	8	0.04	74	2.85	13	5	2.70	0.70
Pea (3A)	10 - 11.5	Gray Silt	28	0.03	94	2.76	NP	-	1.50	3.70

* clay defined as particles smaller than 0.002mm
NP defined as non-plastic
- data that could not be determined

III. EFA TESTING

Briaud et al. (1999, 2001a, 2001b) developed the EFA and the basic operating procedures can be found in Briaud et al. (2001a). The EFA for this study (FIGURE 3) was essentially the same as the EFA described by Briaud et al. (2001a), but there were some differences that made the operating procedure a little different. A detailed description of the EFA testing procedure can be found in Crim (2003). FIGURE 4 shows a sketch of the important parts of the EFA.

To begin an EFA test the tank is filled with water and the prepared sample installed in the EFA. The Shelby tube is held vertically over the piston and slowly pushed down over the piston. Once the Shelby tube is in place over the piston it is secured by tightening a clamp.

After the Shelby tube is securely in place the soil sample is brought to the top of the tube by pushing the piston control on the EFA in the up position until the sample comes out of the top of the Shelby tube. Once this is done the soil is trimmed evenly with the top of the sampling tube.

The sample is then inserted into the rectangular conduit opening. The sample is raised into the opening by using the crank wheel, aligned flush with the bottom of the conduit, and the two screws on the platform tightened so that the Shelby tube cannot move during testing (FIGURE 5).

The pump is turned on and the valve to regulate water velocity is opened allowing flow through the conduit. The flow rate is measured by means of a propeller type flow

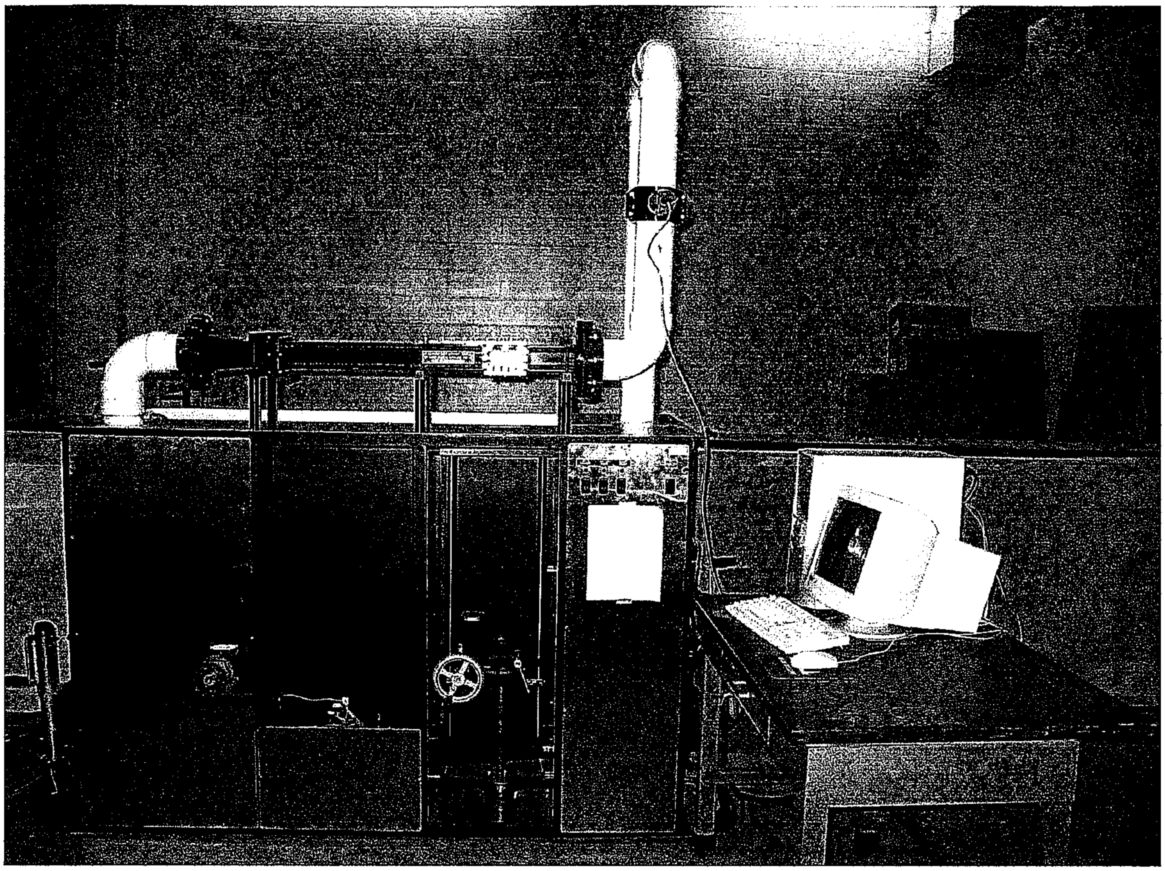


FIGURE 3. Auburn University's Erosion Function Apparatus (EFA).

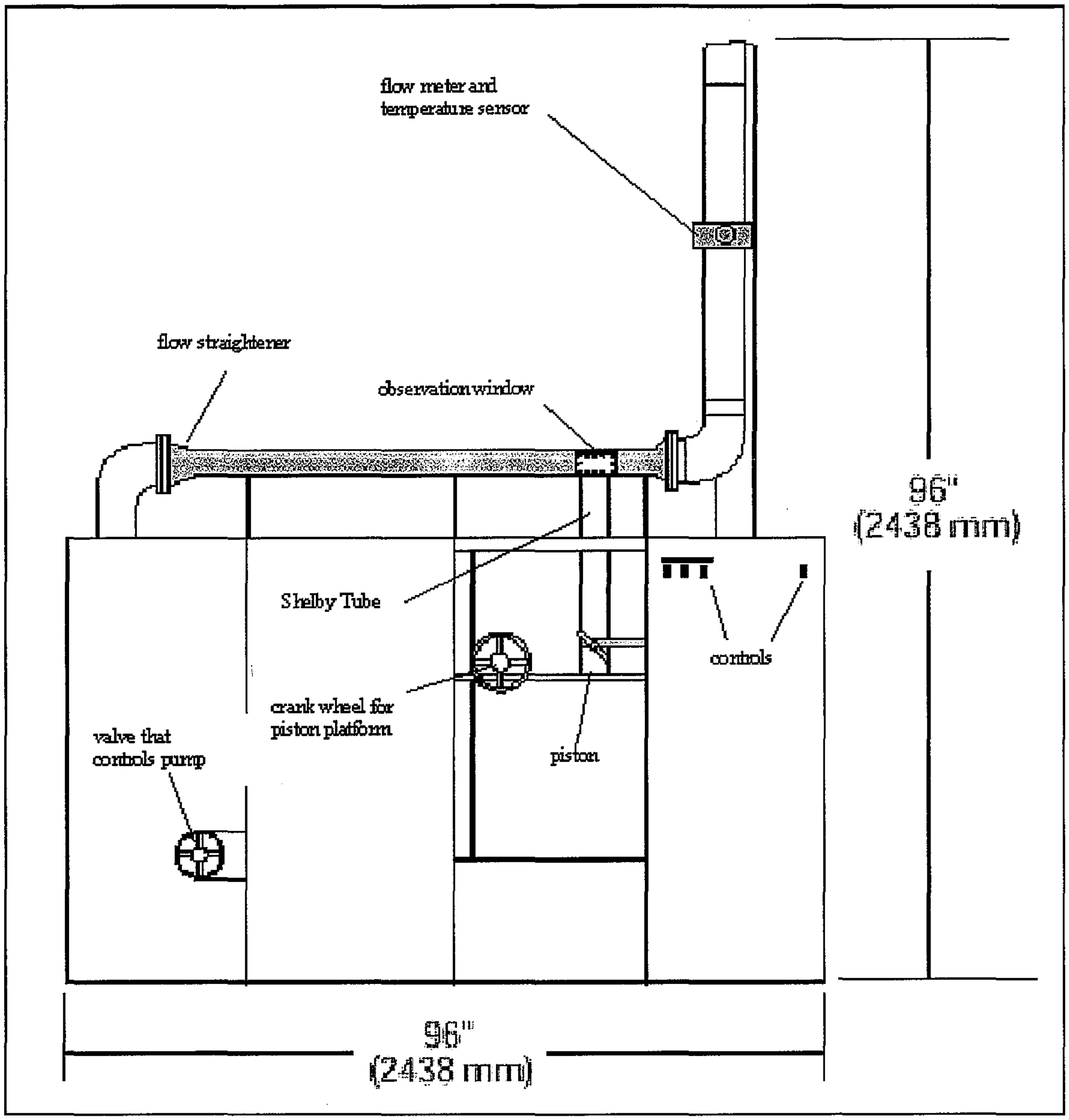


FIGURE 4. Schematic showing the important parts of the EFA.

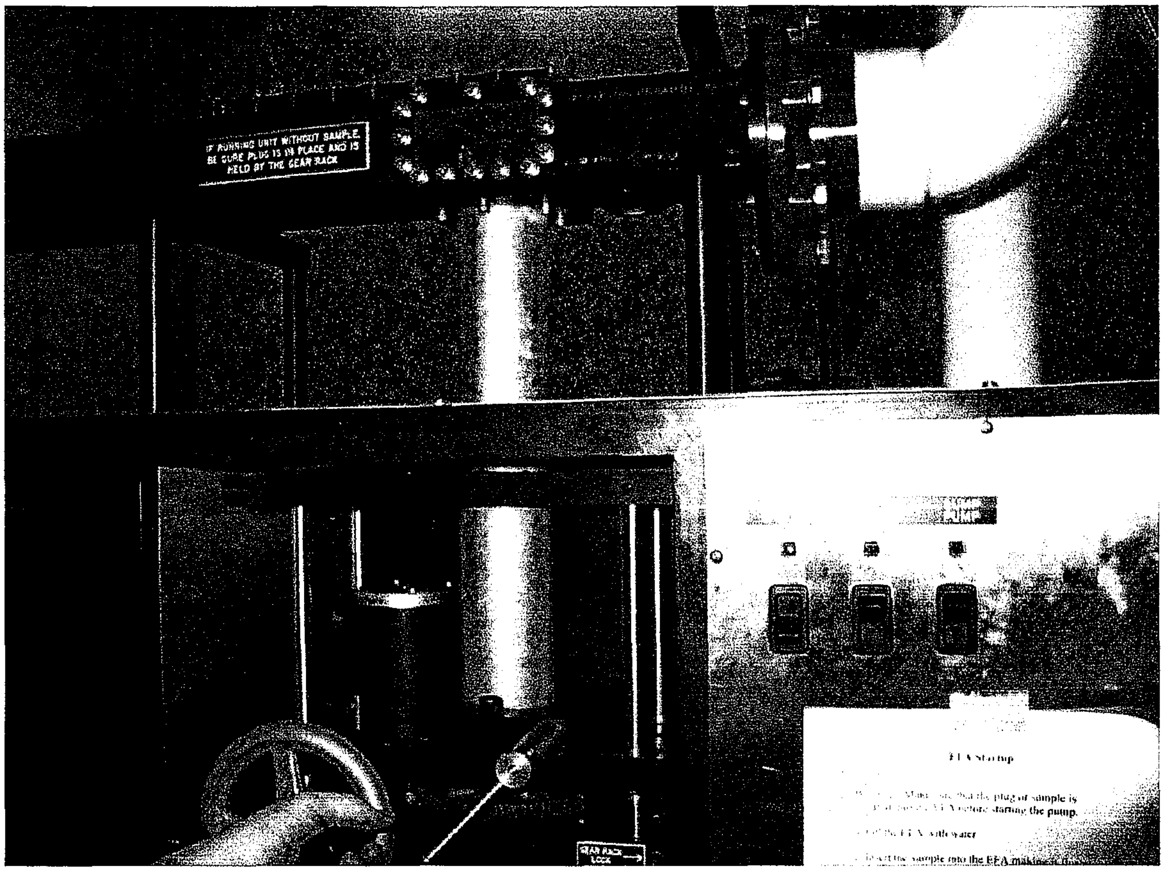


FIGURE 5. Raising the Shelby tube into the conduit opening and placing it flush with the bottom using the crank wheel.

meter. The flow rate is combined with the cross sectional area of the pipe to compute the average velocity of the flow.

After the velocity is set the soil is raised into the flow 1 mm in 0.5 mm increments, which is controlled by the EFA computer. FIGURE 6 shows the computer screen during testing. The computer records time, average velocity, temperature, the soil sample advance that is pushed, and elapsed time as shown in TABLE 2.

The flow is maintained until 1 mm of soil is completely scoured away. FIGURE 7 shows a sample that is in the EFA. The scour is usually not uniform and the surface of the soil sample usually becomes uneven through the duration of a test. Some of the exposed sample surface may have scoured more than 1 mm while some of it may have scoured less. When this happens the operator subjectively decides when scour is complete. The operator must decide when the scour is "on average" about 1 mm.

At the end of a test the pump is turned off and the water drains from the conduit. The soil sample can then be lowered out of the conduit opening and prepared for the next test. This is done by pushing some of the soil through the sampling tube and trimming it even with the top of the tube. The sample is again raised into the opening and the test at the next velocity is run. The test is repeated for between 5 and 8 velocities. By doing the test at several velocities, the scour rate (mm/hr) vs. velocity (m/s) data is obtained. This data is evaluated to give a scour rate (mm/hr) vs. shear stress (N/m^2) relationship, which is defined as the erosion function of the soil.

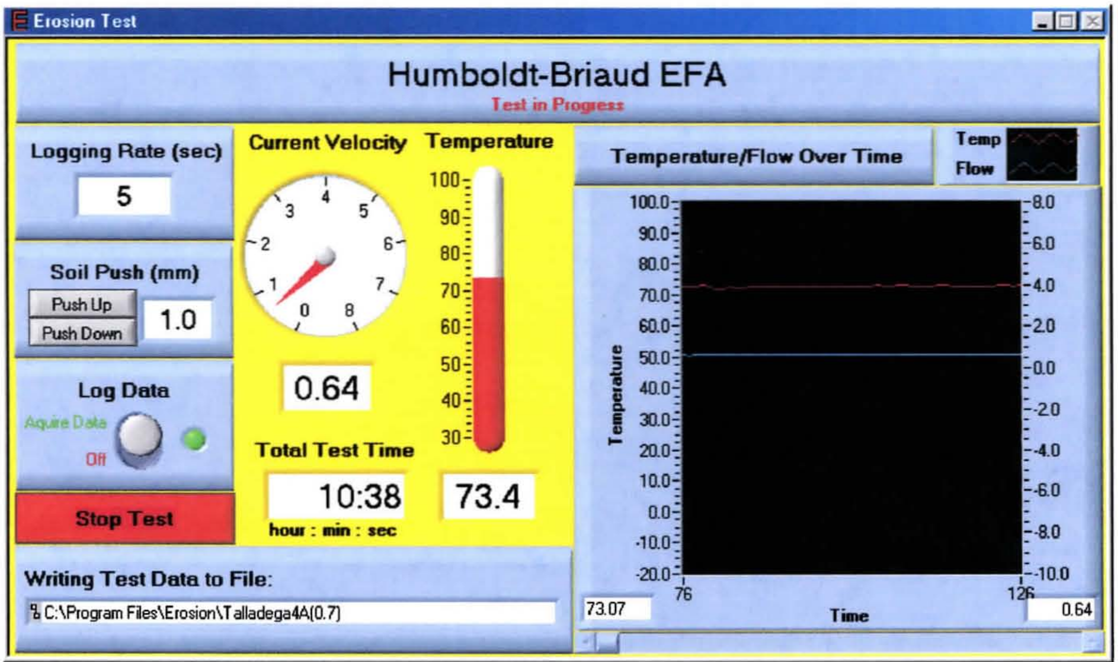


FIGURE 6. Example of the computer screen while running an EFA test.

TABLE 2. Computer data recorded during an EFA test (first minute shown).

Friday, November 22, 2002	10:37 AM			
INITIAL SOIL PUSH: 0.0				
Time	Temp (ºF)	Average Velocity (m/s)	Push (mm)	Elapsed Time (s)
10:38:18 AM	72.17	-0.007	0	0
10:38:23 AM	72.772	0.797	0	5
10:38:28 AM	72.471	0.701	0	10
10:38:33 AM	71.868	0.621	0	15
10:38:38 AM	71.868	0.609	0.5	20
10:38:43 AM	71.868	0.613	1	25
10:38:48 AM	72.17	0.6	1	30
10:38:53 AM	71.567	0.6	1	35
10:38:58 AM	71.868	0.605	1	40
10:39:03 AM	72.17	0.613	1	45
10:39:08 AM	72.17	0.617	1	50
10:39:13 AM	71.868	0.63	1	55
10:39:18 AM	71.567	0.621	1	60

FIGURE 7. View looking in as sample is being tested in the EFA.

IV. EFA TEST DATA REDUCTION AND PRESENTATION

The scour rate ($\dot{z}$) is the measure of how fast a particular soil erodes over time. The scour rate for a particular soil with a set water velocity flowing over it can be calculated from an EFA test. This scour rate is

$$\dot{z} = \frac{\Delta h}{\Delta t} \tag{1}$$

where Δh is the length of soil eroded in a time Δt. The length Δh that is eroded during an individual test at a specified velocity is 1 mm. The time Δt is how long it takes for the 1 mm of soil sample to be eroded.

The shear stress applied by the water to the soil at the soil water interface is generally considered to be the major parameter causing erosion (Briaud et al., 2001a). The EFA does not directly give the shear stress applied to the soil. It does however give velocity (V), which is related to the shear stress that the water imposes on the soil sample.

According to Briaud et al. (2001a) the best way to determine the shear stresses for the EFA is by using the Moody chart which gives the relation between the pipe friction factor f, the Reynolds Number Re, and the relative roughness ε/d where ε = roughness height and d = pipe diameter. The Reynolds number is calculated as

$$\mathrm{Re} = \frac{\mathrm{VD}}{\upsilon} \tag{2}$$

where V = average velocity in the pipe, D = $4R_h$ = hydraulic diameter of the pipe (R_h = hydraulic radius), and υ = kinematic viscosity of water (10^{-6} m^2/s at 20°C).

After the Reynolds number is calculated the friction factor can be determined. For the cohesive and fine-grained soils that were tested in this study the roughness was considered to be smooth in the EFA. The equation that is used to calculate the friction factor for smooth conditions is

$$\frac{1}{\sqrt{f}} = 2.0 \log \left(\mathrm{Re}\sqrt{f}\right) - 0.8 . \tag{3}$$

This equation was used for smooth conditions in Moody's original paper (Moody, 1944). An approximation of equation 3, called the Blasius equation (Henderson, 1966), that was used for $\mathrm{Re} < 10^5$ is

$$f = \frac{0.316}{\mathrm{Re}^{1/4}} . \tag{4}$$

This equation simplifies some of the calculations by making it possible to calculate f directly if the Reynolds number is known. If $\mathrm{Re} > 10^5$ then equation 3 is used to find the friction factor. These are the equations for the smooth line on the Moody diagram and make it possible to calculate the friction factor for a smooth surface without having to go to the actual Moody diagram (Henderson, 1966).

The shear stress in the EFA is calculated as

$$\tau = \frac{\rho f V^2}{8} \tag{5}$$

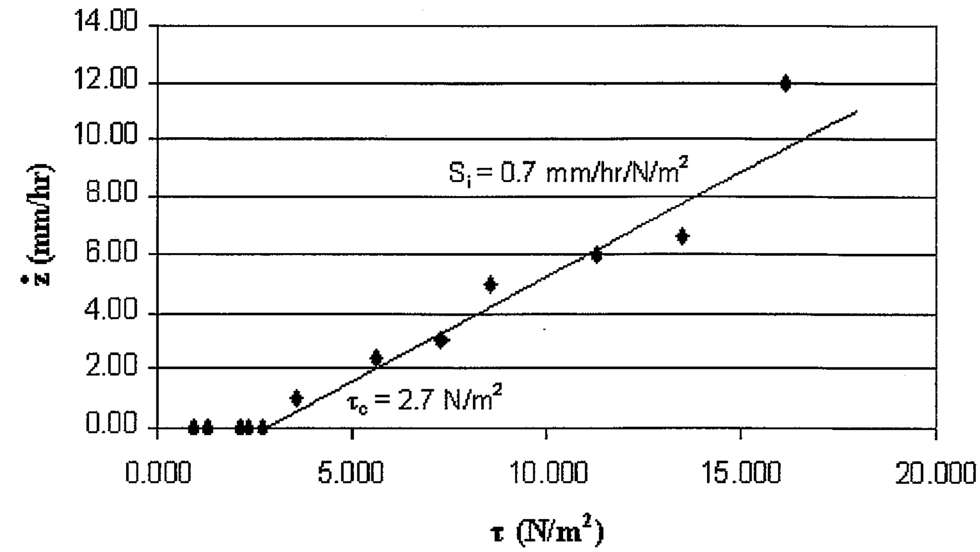
where τ = shear stress, f = friction factor from the Moody chart, ρ = mass density of water (1000 kg/m^3), and V = average velocity in the conduit.

A spreadsheet was set up to make the necessary calculations for an EFA test. Velocities, times, and measured scour come directly from the EFA test. This data is input and calculations are made for the Reynolds numbers, friction factors, and shear stresses. An example of these spreadsheet calculations is shown in TABLE 3.

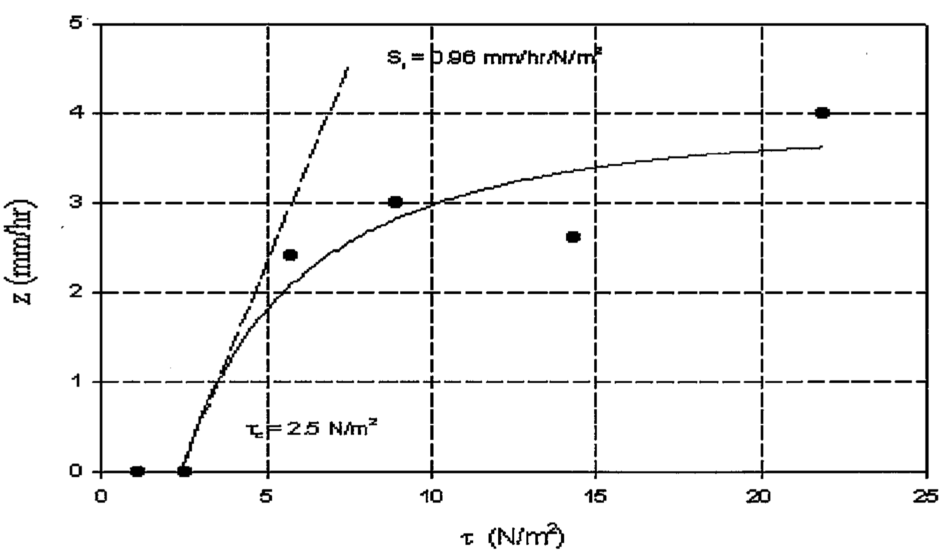
TABLE 3. Example of spreadsheet used for reducing data from an EFA test.

Velocity (m/sec)	Re	Friction Factor	Shear Stress (Pa)	Scour Reading (mm)	Test Time (min)	Scour Rate (mm/hr)
0.359	21796	0.0260	0.419	0	30	0.00
0.687	41711	0.0221	1.305	2.50	30	5.00
1.208	73343	0.0192	3.503	3.0	30	6.00
2.057	124889	0.0175	9.256	4.0	35	6.86
2.662	161621	0.0170	15.058	2.0	15	8.00
2.899	176011	0.0167	17.544	4.0	25	9.60

Scour rate and shear stress are plotted to develop erosion functions for soils, as illustrated in FIGURE 8. From these erosion functions, critical shear stress (τ_c), and initial erodibility (S_i) are determined. Best fit curves are visually determined for shear stresses greater than τ_c. For the example shown in FIGURE 8a, a straight line best fits all the data points and, therefore, defines S_i. For the example shown in FIGURE 8b, the erosion function is nonlinear for $\tau > \tau_c = 2.5$ N/m^2.



a. Sample Pea 2A.



b. Sample Choctawhatchee 1A

FIGURE 8. Example erosion functions.

V. CORRELATIONS

The first step in determining meaningful correlations between soil classification properties and critical shear stress and initial erodibility was to develop linear regressions between classification properties and $\log_{10}\tau_c$ and $\log_{10}S_i$. An example regression shown for D_{50} and $\log_{10}\tau_c$ is shown in FIGURE 9.

TABLE 4 summarizes R and p values for all the regression equations. Higher absolute R values indicate stronger correlations and lower p values indicate higher confidence in the significance of the correlations; p = 0.05 corresponding to 95% confidence level. The soil classification property that provides the best correlation with the log of critical shear stress is D_{50} with R = -0.73 and p = 0.001. As shown in FIGURE 9, the regression equation is

$$\log \tau_c = 0.41 - 2.69\, D_{50}. \tag{6}$$

The soils classification property that provide the best correlation with log of initial erodibility is % clay with R = -0.85 and p = 0.001. The regression equation is

$$\log S_i = 1.10 - 0.35\ \%\ \text{clay}. \tag{7}$$

Percent clay and D_{50} are particle size properties and indicate, as expected, that initial erodibility decreases and critical shear stress increases as a cohesive soil becomes finer grained.

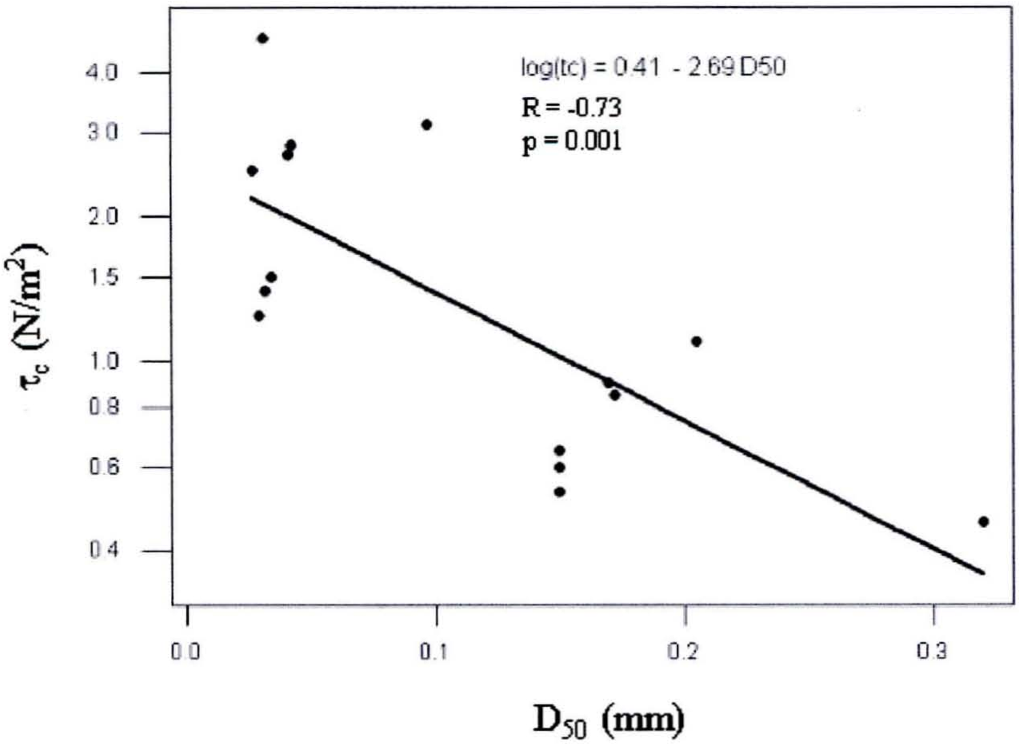


FIGURE 9. Example linear regression.

TABLE 4. Correlation data from linear regression.

Independent / Dependent		D_{50}	% Clay	%<#200	PI	Activity	N
τ_c	**R**	-0.73	0.7	0.66	0.51	0	0
	P	0.001	0.002	0.005	0.018	0.617	0.688
S_i	**R**	0.79	-0.85	-0.7	0	0.19	0
	P	0.002	0.001	0.01	0.358	0.289	0.918

Multiple linear regressions indicate the correlation with critical shear stress is most improved when PI is combined with D_{50} as independent variables. The resulting equation

$$\log \tau_c = 0.08 - 1.77\ D_{50} + 0.02\ PI \quad (8)$$

has a $R^2 = 0.68$ and $p = 0.001$. This increases R^2 from 0.53 with no change in p. The addition of Activity to % clay as an independent variable provides the most improvement to the correlation with initial erodibility. The resulting equation

$$\log S_i = 1.49 - 0.44\ \%\ \text{clay} - 0.03\ \text{Activity} \quad (9)$$

has a $R^2 = 0.79$ and $p = 0.004$. This increases R^2 from 0.72. The addition of the second variable, Activity, also increases p from 0.001. This increase is, however, considered inconsequential since $p = 0.004$ remains well below the commonly accepted threshold for significance of 0.05. The addition of the second variables indicates, as expected, critical shear stress increases and initial erodibility decreases as soil plasticity increases.

FIGURE 10 shows a correlation between critical shear stress and initial erodibility. This figure includes the data from Briaud et al. (2001a) as well as the data that were obtained from the soils tested in this study. The two sets of data seem to agree with each other. It can be seen that at high critical shear stresses the initial erodibility is small and at low critical shear stresses the initial erodibility is high. In general soils with low critical shear stresses and high initial erodibility are sands and soils with high critical shear stresses and low initial erodibility are cohesive soils such as clay.

FIGURE 11 shows correlations between critical shear stress and plasticity index for cohesive soils. The three lines are from Chen and Cotton (1988) and are for compact (N = 30-50), medium compact (N = 10-30), and loose soils (N = 4-10). The soils that were tested in this study are also plotted on FIGURE 11. The blow counts are shown in the figure next to the points. The critical shear stresses were converted from N/m^2 to lb/ft^2 for the figure. A comparison can be made between the original lines and points that came from the soils that were tested in this study. The data confirms that in general if the plasticity index is higher then the critical shear stress will also be higher. Therefore the more cohesive a soil is then the higher its critical shear stress will be. Soils with little cohesion, such as soils with a high sand content, would have smaller critical shear stresses. It cannot be confirmed from the data of this study that there is a refinement of the critical shear stress vs. plasticity index relationship that includes the influence of soil compactness. The data does not show that increased compactness means increased critical shear stress.

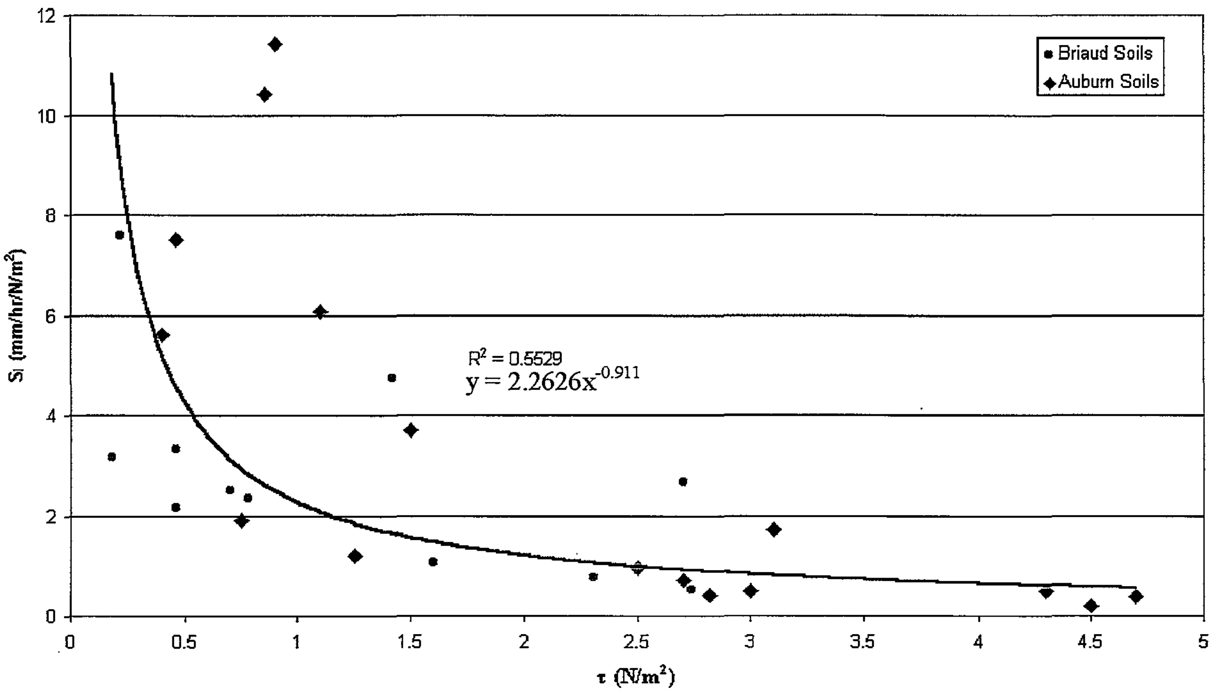


FIGURE 10. Correlation between critical shear stress and initial erodibility with Briaud et al. soils (2001a) and Auburn soils plotted.

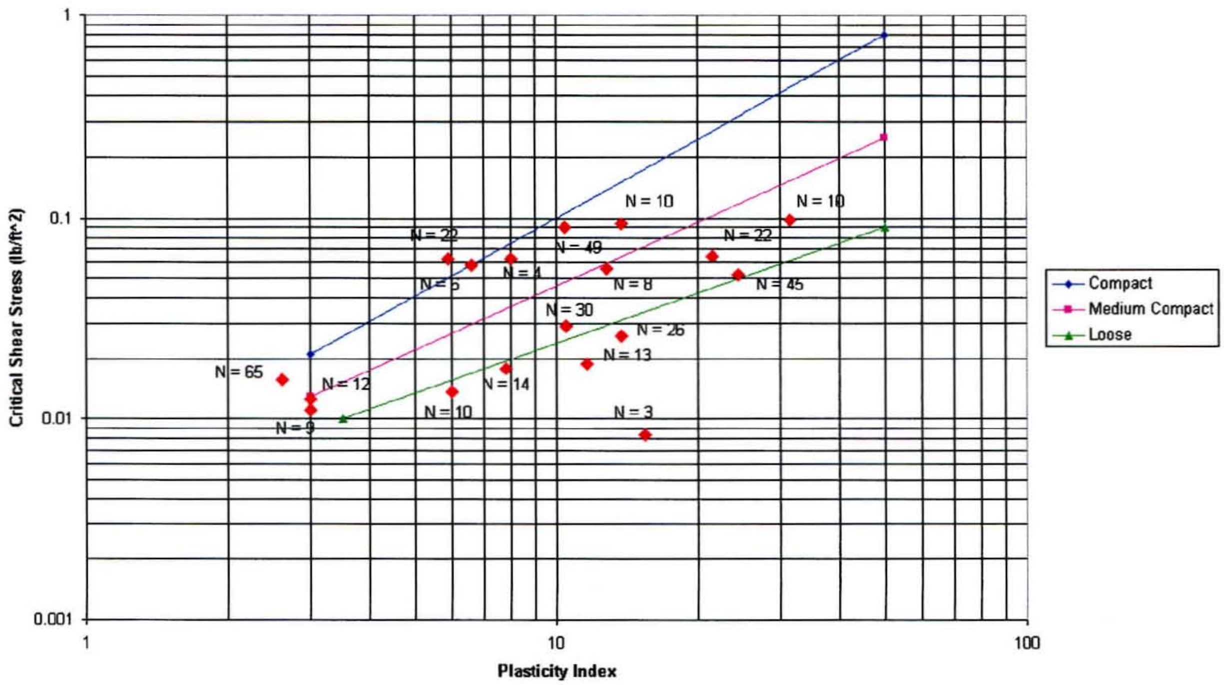


FIGURE 11. Correlation between critical shear stress and plasticity index for cohesive soils (Chen and Cotton, 1988) with Auburn soils plotted.

VI. CONCLUSIONS

The Erosion Function Apparatus (EFA) is a very useful tool to determine soil erosion properties. The erosion function, critical shear stress, and the initial erodibility of a particular soil can be determined with EFA tests. These results can be used to predict scour rates at a bridge site during a flood. More cohesive behavior, quantified in the erosion functions, corresponded to sites of minimal historical scour based on samples tested from 6 different bridge sites in Alabama. The tests showed that more cohesive soils generally had an erosion function with a higher critical shear stress and a lower initial erodibility than cohesionless soils. These scour rate data are also useful in the application of predictive hydraulic models.

This study has contributed to, what appear to be reasonable, correlations between τ_c and S_i and basic soil classification properties. These correlations have the potential to provide valuable information about a site without an EFA test. These correlations will make it possible to do preliminary scour calculations without EFA data. All that would be needed is basic soil classification tests. The correlations also provide some quantification of the expected variability of τ_c and S_i. It should, however, be emphasized that running extensive EFA tests on samples from a bridge site provides the best data for scour calculations.

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