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A Two-Dimensional Numerical Model Study of Clear-Water Scour at a Bridge Contraction with a Cohesive Bed

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ABSTRACT

Recently, Güven et al. (2001, 2002) presented a one-dimensional approach for modeling time-dependent clear-water contraction scour in a cohesive soil, where the scour rate for the soil is described by an erosion function. This report extends that method by using a two-dimensional model to calculate scour. The output from the two-dimensional Finite Element Surface Water Modeling System (FESWMS) model in the Surfacewater Modeling System (SMS) software provides sufficient information to calculate scour over time at desired time steps for the entire two-dimensional domain.

For this study, a 4:1 bridge contraction with a beveled abutment geometry was investigated. The model output (velocity, depth, and water surface) and the SMS data calculator, which allows the user to calculate new data sets from existing ones, are used to calculate time-dependent scour based on the erosion function at each coordinate in the model. The erosion function describes the erosion rate for a given soil. For the time-dependent method, the bed elevation is then updated at each node in the model, and the streambed is set at a new elevation so that the model can be run again to obtain new output. While it is possible to change the flow rate for each time step, the flow rate was kept constant for this study.

An ultimate scour depth (referred to in Güven et al. (2002) as "maximum scour depth") can also be calculated based on the unit flow rate and other model parameters at a particular time step.

This report covers the methodology and results for calculating time-dependent as well as ultimate scour using a two-dimensional finite element computer model.

ACKNOWLEDGEMENTS

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1. INTRODUCTION

1.1 Purpose

Two-dimensional hydraulic modeling can provide a more comprehensive and realistic view of riverine hydraulics at bridge sites to help account for the threat of scour in contracted bridge reaches. The Finite Element Surface-Water Modeling System: Two-Dimensional Flow in a Horizontal Plane (FESWMS or FESWMS-2DH) was developed especially for modeling "complex hydraulic conditions" such as those existing at bridge sites where traditional one-dimensional techniques "cannot provide the needed level of solution detail" (Froehlich, 1996, page 1-1 of the original reference). Whereas one-dimensional research by Güven et al. (2002) assumed a uniform flow distribution in the contraction, two-dimensional modeling accounts for the nonuniform nature of the flow in the horizontal plane. The FESWMS program within the Surface-Water Modeling System (SMS) 7.0 software package developed at Brigham Young University is being used in bridge design and research. Hydrologic Engineering Center Research Document No. 42 (RD-42), titled "Flow Transitions in Bridge Backwater Analysis," has shown with the RMA-2 finite element model, a model very similar to FESWMS and also available in the SMS 7.0 software package, that two-dimensional modeling can provide a high degree of correlation with actual field conditions (Hunt and Brunner, 1995). Aside from RD-42, research in the area of two-dimensional hydraulic computer modeling of contractions has been limited. The purpose of this study is to extend the clear-water contraction scour research presented by Güven et al. (2002) based on one-dimensional modeling.

1.2 Choice of Base Case

A contraction with vertical abutments similar to those in RD-42 (Hunt and Brunner, 1995) was chosen as a starting point for the present study. A Manning's roughness coefficient (n) of 0.02, characteristic of a channel with a relatively smooth boundary, was chosen for the base case. The kinematic eddy viscosity (Vo), which accounts for the effects of lateral momentum transfer in the model, was set at 10 ft^2/sec; SMS documentation suggests a value of 5-50 ft^2/sec. Further discussion of these choices for the material properties is given in the first author's Masters thesis (McLean, 2002).

Figure 1 depicts the base case geometry and boundary conditions. A floodplain width (B) of 1000 ft was consistently used for the transition sections upstream and downstream of the contracted reach. The lengths of the upstream and downstream reaches were always set at five times the floodplain width (5B). For the base case, abutments had a 45-degree bevel that cut off 40 ft in length (a) and width (a) from each abutment corner creating a more gradual entrance and exit geometry. The length of the contracted section for the base case was determined by the abutment length (l), which was set at 140 ft. The midpoint of the contraction is set as the origin, x = 0 ft, y = 0 ft. The contraction width (b) was set at 250 ft for the base case, resulting in a contraction ratio (b/B) of 1:4. The two boundary conditions, flow rate (Q) and downstream water depth (Ho), were set at 20,000 cfs and 8 ft respectively. For this study, the flow rate and downstream water depth were kept constant for all time steps. In general, however, it is possible to change these boundary conditions in the model at each time step. The channel bed starts out flat but changes over time based on the chosen soil's erosion characteristics.

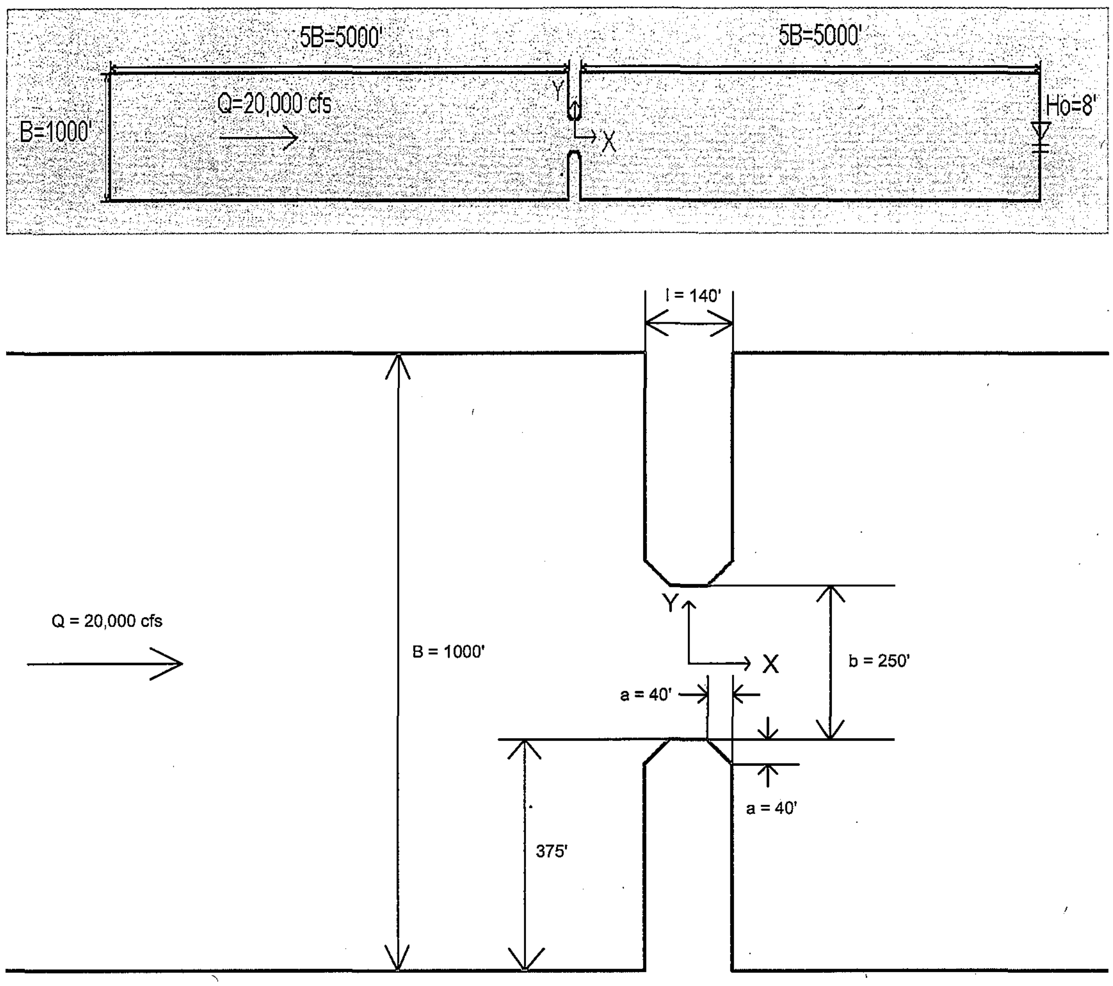

Figure 1. Base case SMS model geometry and boundary conditions

The properties of the cohesive soil used in this report correspond to those of Soil No. 1 used in Güven et al. (2002), based on the original Erosion Function Apparatus (EFA) data of Briaud et al. (2001). For Soil No. 1, a low plasticity clay soil:

Critical shear stress (τ_c) = 0.0572 lb/ft^2 = 2.74 N/m^2

The scour rate (S_i) = 1.936 (ft/day)/(lb/ft^2) = 0.51 (mm/hr)/(N/m^2)

1.3 Model Behavior

The distribution of velocity, depth, and flow rate per unit width in the channel were of particular interest because of their effects on scour patterns at bridge sites. Distributions of the velocity, depth, and flow rate per unit width (unit flow rate), calculated for the base case with a flat bed, are depicted in Figure 2. Relatively little change in velocity or depth occurs in the region upstream of the contraction. As the flow enters the contraction, velocity increases, while a corresponding decrease in depth occurs. The unit flow rate (the product of velocity and water depth) is high along the abutments at the entrance to the contraction, and then in the center of the channel further downstream as the flow is forced towards the center along the beveled entrance. The actual depth is mainly dependent upon the boundary conditions Q and Ho, and the velocity retarding parameters n and Vo. The jet-like behavior at the exit of the contraction creates a distinct separation zone on each side of the channel between the main flow and the eddy zones behind the abutments. The effect of lateral momentum transfer and bed shear stresses results in a reduction of the velocity, expansion of the main flow jet, and eventually reattachment of the flow to the side boundaries.

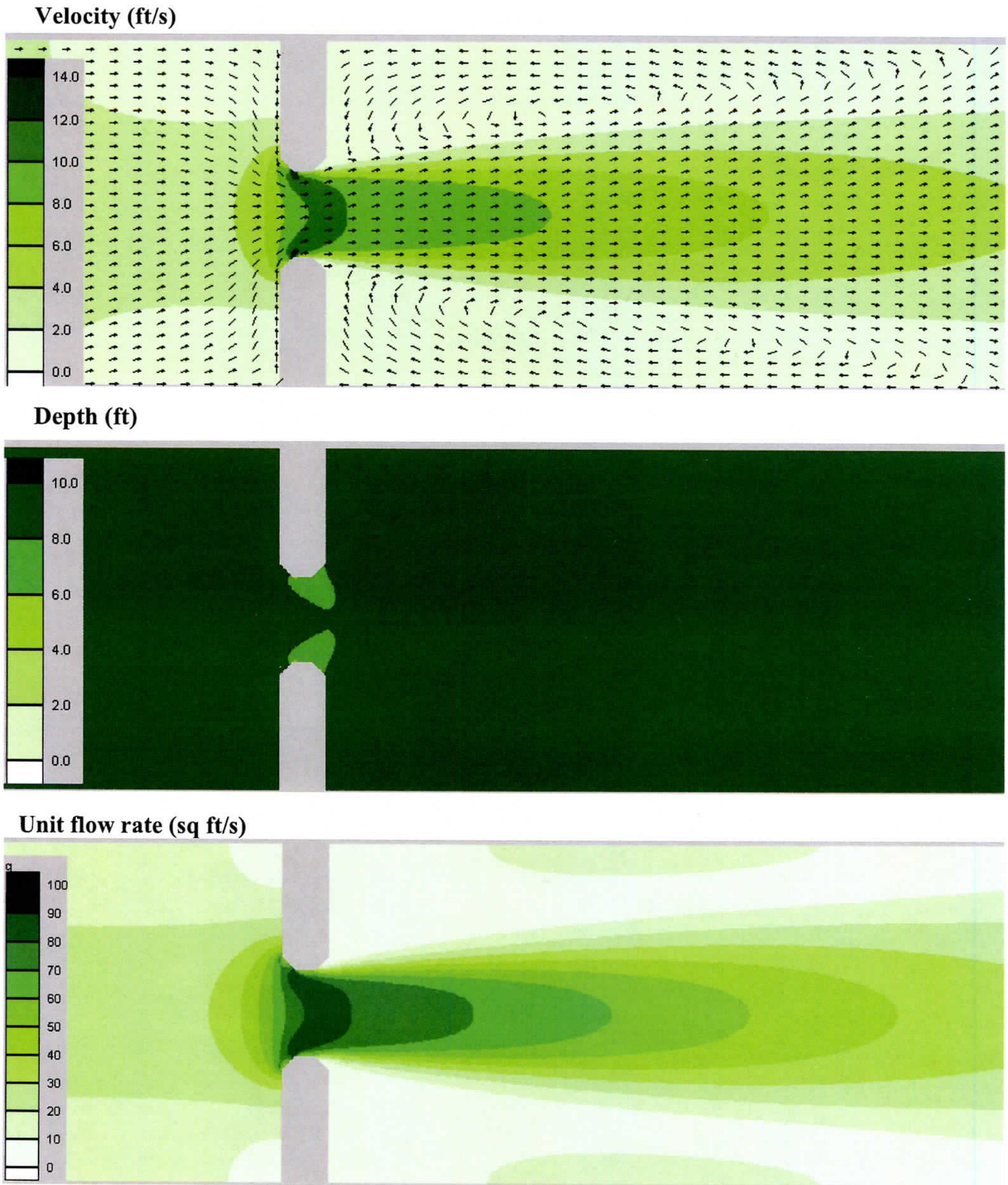

Figure 2. Velocity, depth, and unit flow rate distribution for the base case

The effects of the contraction geometry on the hydraulic behavior for the base case are visible in the transverse velocity, depth, and unit flow rate profiles. Figures 3, 4, and 5 depict these profiles at what will be referred to as the contraction entrance (x = -30 ft) and Figures 6, 7 and 8 depict them at the midpoint of the contraction (x = 0 ft). Velocity peaks are noticeable near the side boundaries as a result of the large amount of flow that is diverted along the 45-degree beveled entrances and forced towards the center of the contraction. Further upstream in the contraction, these peaks occur closer to the boundary, and further downstream, they occur closer to the center of the channel. The transverse unit flow rate profiles show the flow concentrating towards the center of the channel after entering the contraction.

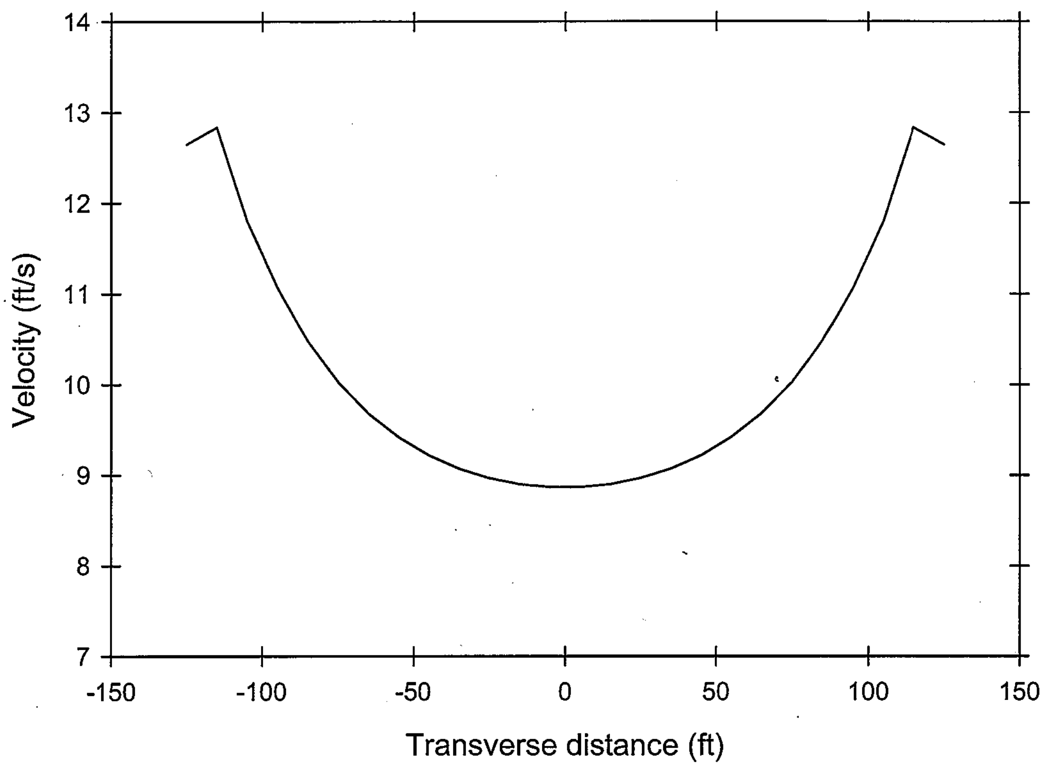

Figure 3. Transverse velocity profile for the base case at entrance to contraction (x = -30 ft)

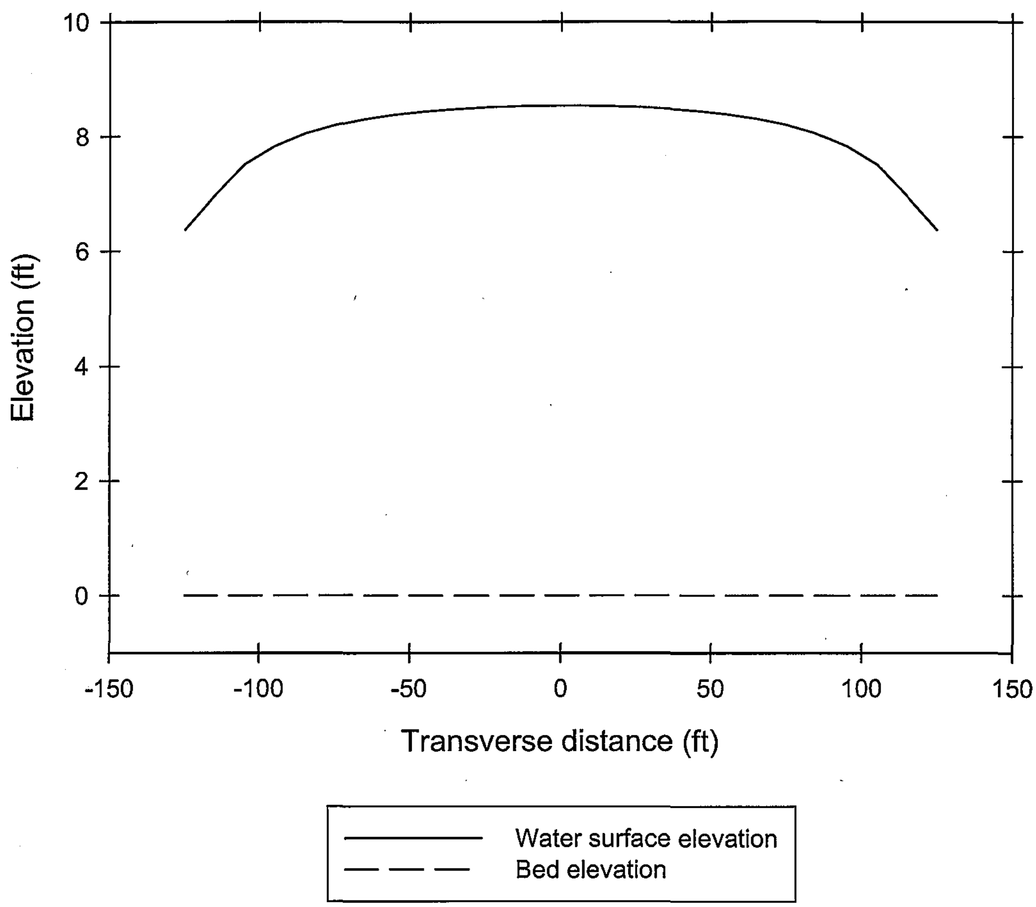

Figure 4. Transverse bed and water surface profiles for the base case at entrance to contraction (x = -30 ft)

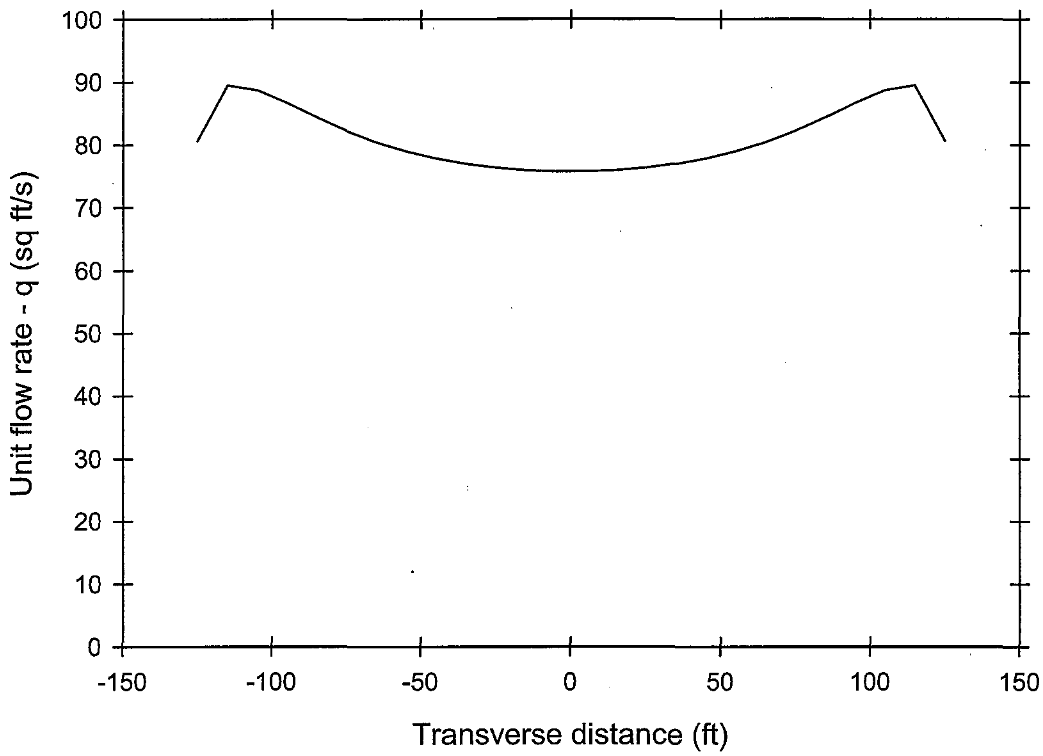

Figure 5. Transverse unit flow rate (q) profile for the base case at entrance to contraction (x = -30 ft)

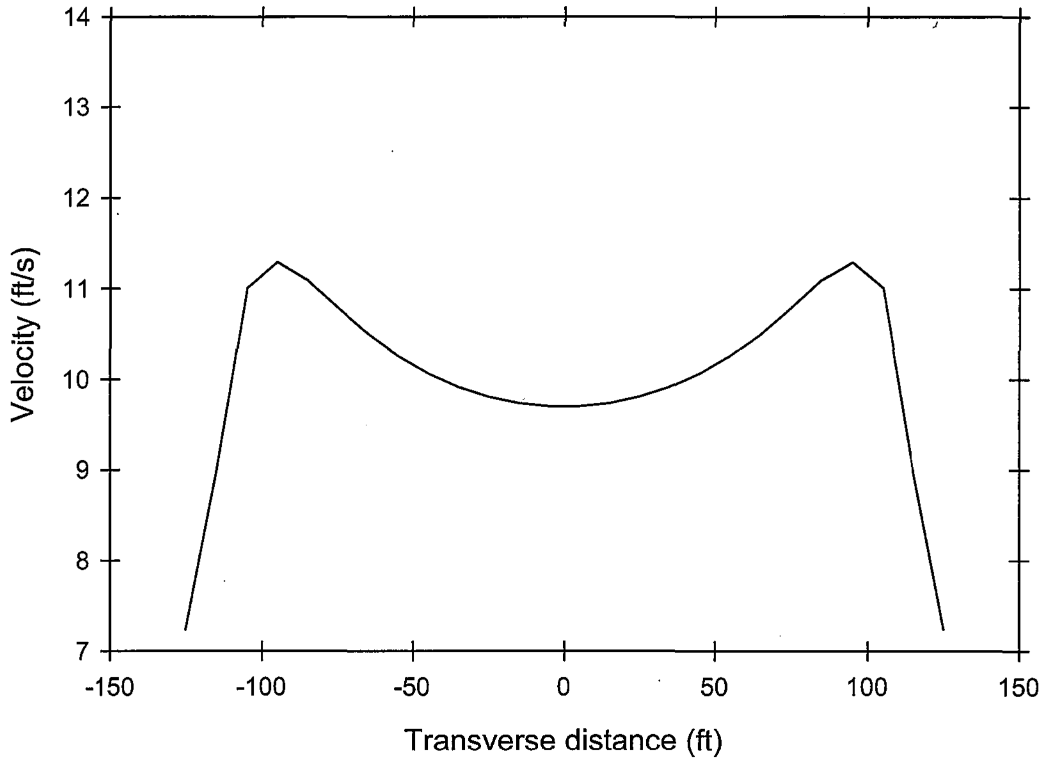

Figure 6. Transverse velocity profile for the base case at midpoint of contraction (x = 0 ft)

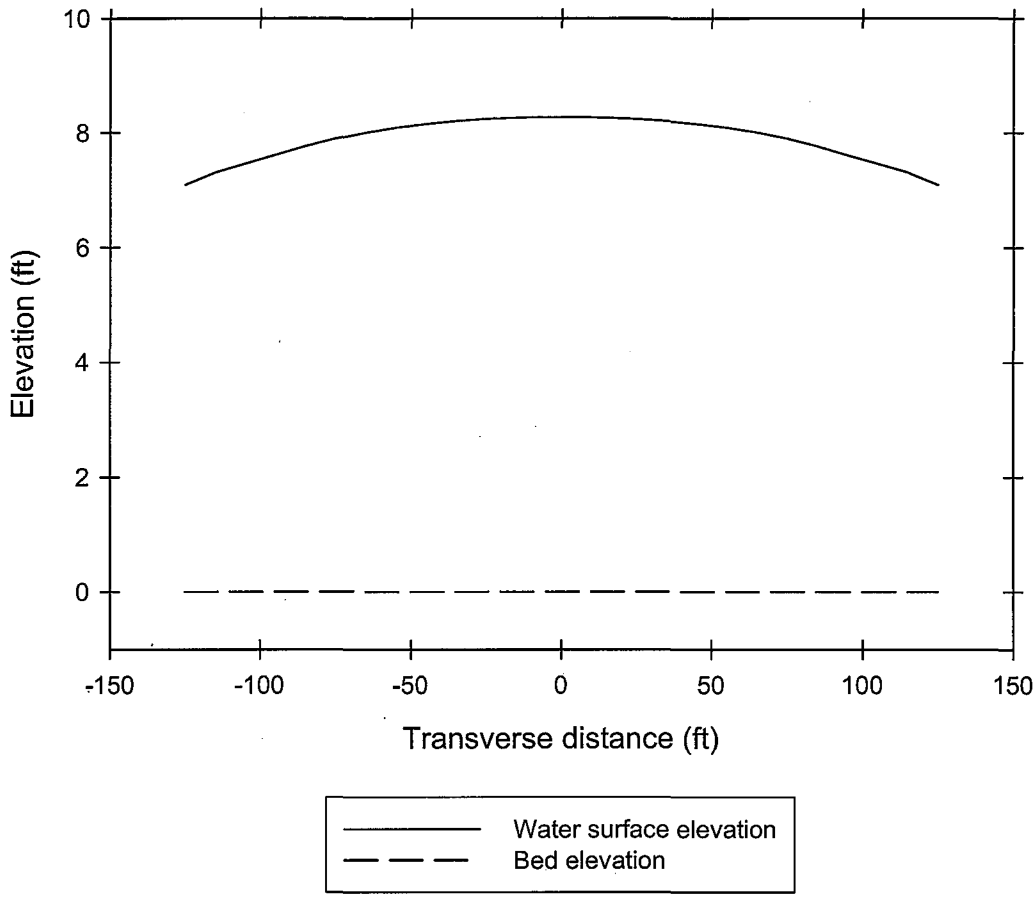

Figure 7. Transverse bed and water surface profiles for the base case at midpoint of contraction (x = 0 ft)

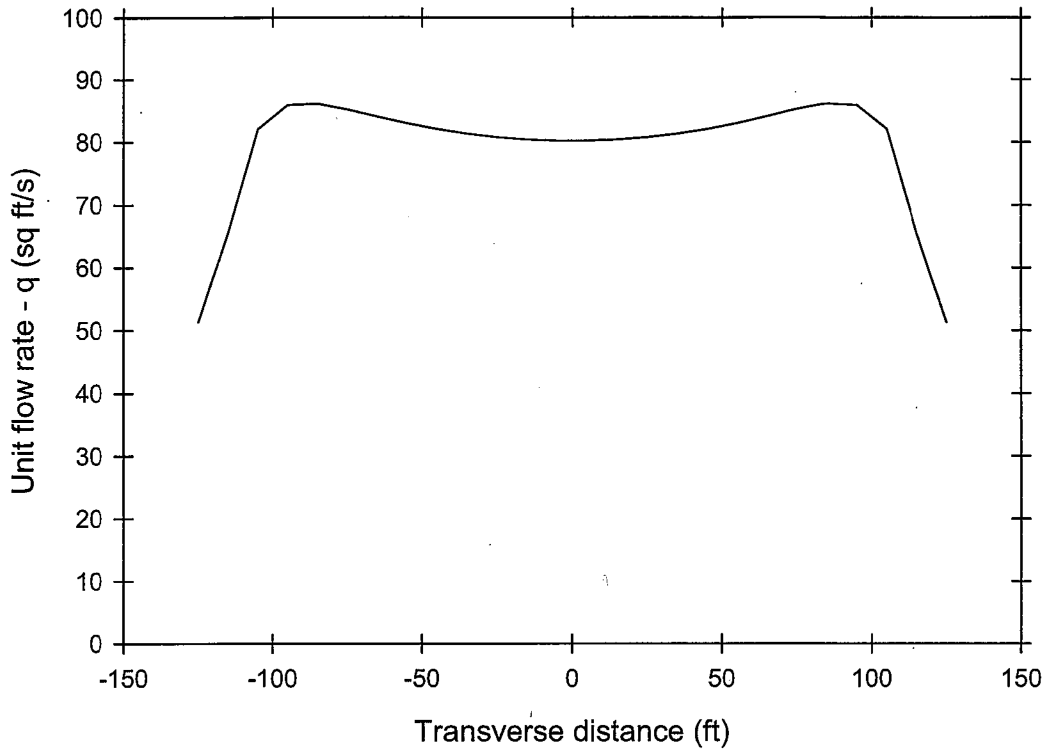

Figure 8. Transverse unit flow rate (q) profile for the base case at midpoint of contraction (x = 0 ft)

2. SCOUR CALCULATIONS METHODOLOGY

2.1 Time Dependent Scour Calculation Method

The time dependent nature of scour is important to consider when calculating scour, especially in cohesive soils where scour may take much longer to develop than in non-cohesive settings. The "scour rate in cohesive soils" (SRICOS) method, introduced by Briaud and his colleagues at Texas A & M University for bridge pier scour, was extended by Güven et al. (2002) to account for contraction scour. Güven et al. (2002) presented a one-dimensional approach to modeling time dependent clear-water contraction scour in a cohesive soil based on the SRICOS method. While Güven et al. (2002) investigated a contraction with a uniform flow distribution using a one-dimensional model, the present study employs two-dimensional modeling to investigate the effects of nonuniform flow at a contraction. At each time step, the two-dimensional procedure was used to calculate scour for each node, or coordinate, in the model. Then the scour was subtracted from the starting bed elevation to yield an updated streambed. The procedure is to run the FESWMS model in SMS with the starting bed elevation, flow rate, and contraction geometry. The SMS output file gives you velocity (V), water depth (H), and water surface elevation at each node. In the SMS data calculator, these parameters can be used to compute the time dependent scour based on the method described below.

The Henderson implicit formula can be used to calculate f in an iterative fashion. The formula is

$$f = \frac{0.25}{[\log(\frac{k_s}{12H} + \frac{2.5}{Re\sqrt{f}})]^2} \quad (1)$$

f = friction factor

k_s = roughness height

H = flow depth

$$Re = \text{Reynolds number} = \frac{4q}{\nu} \quad (2)$$

q = unit flow rate = VH

V = magnitude of velocity

ν = kinematic viscosity of water = 1.06×10^{-5} ft^2/s at 70° F

Iteratively solving for the friction factor requires an initial value of f for the first iteration. The Swamee-Jain explicit formula, originally for pipe flow, can be used to obtain an initial estimate for the value of f:

$$f = \frac{0.25}{[\log(\frac{k_r}{3.7} + \frac{5.74}{Re^{0.9}})]^2} \quad (3)$$

$$k_r = k_s/(4H) \quad (4)$$

Subsequent values of f are calculated using the Henderson formula. For this model, the percent change between the 2^{nd} and the 5^{th} iterations was less than 1% for each node, so five iterations was chosen as a more than sufficient approximation of f.

The friction factor is then used to calculate the shear stress at each node in the model:

$$\tau = \frac{\rho V^2}{8} f = \frac{\rho q^2}{8H^2} f\left(\mathrm{Re}, k_s, H\right) \quad (5)$$

τ = bed shear stress

ρ = density of water = 1.94 slugs/ft^3 at 70°F

For Soil No. 1, which was found to have a linear erosion function, the rate of scour (R) in units of length/time can then be calculated by multiplying the slope of the erosion function (S_i) by the difference in the bed and critical shear stress values. The soil erodability information (S_i and τ_c) can be obtained from Erosion Function Apparatus (EFA) tests. (For more information, see Briaud et al. (2001) and web pages: http://tti.tamu.edu/geotech/scour/ & http://tti.tamu.edu/geotech/scour/efa_ overview.pdf). The rate of scour is calculated as

$$R = S_i * \max(0,(\tau - \tau_c)) \quad (6)$$

τ = bed shear stress

τ_c = critical shear stress

The "max" function chooses the maximum value from the values inside the parenthesis set apart by a comma, 0 and (τ-τ_c). If the bed shear stress is less than critical then the rate of scour is zero. Using the rate of scour, the incremental scour (ΔS) can be calculated for the desired time step.

$$\Delta S = R * \text{Time Step} \quad (7)$$

The new bed elevation can be calculated by subtracting the scour that occurs during the specified time interval from the bed elevation at the beginning of the interval:

$$\text{New Bed Elevation} = \text{Bed Elevation} - \Delta S \qquad (8)$$

Next, use the "save as" option to save the model with a new name. For the new file you have created, select the New Bed Elevation as your Bed Elevation for the new time step and give it a name. To do this, use the "Map Elevation..." option under the "Data" pull-down menu in the mesh module. Now, you can delete your old data files except for the current elevation file and restart the process by running the model again. This procedure can be performed in a sequential fashion to arrive to any time using any time step size.

2.2 Ultimate Scour Calculation Method

Scour, or erosion of bed material, occurs when the shear stress acting on the bed is greater than the critical shear stress for the bed material. For clear-water scour, the increase in flow area results in a decrease in the velocity and a coinciding decrease in bed shear stress. Eventually, the depth of the scour hole will increase until the bed shear stress is no longer sufficient for scouring to continue; this depth will be referred to as the ultimate scour depth (referred to as "maximum scour depth" in Güven et al. (2002)). The ultimate scour depth, or ultimate scour, corresponding to a particular and constant unit flow rate ($q_i=V_iH_i$) can be calculated at each node in the model for a particular time (t_i), where V_i and H_i are the magnitude of the flow velocity and the flow depth at that particular time.

The following procedure was used to calculate the ultimate scour based on the output from SMS modeling. The friction factor was calculated in the manner described in section 2.1 using the Swamee-Jain formula as an initial estimate, and the Henderson formula to iteratively come to a good approximation of f.

Equation (5) presented in section 2.1 for the calculation of shear stress can be rearranged to solve for H. If the shear stress is set to be the critical shear stress, then the depth that is solved for is the ultimate water depth that will occur before scouring ceases. This depth will be called H_{ult}.

$$H_{ult}^2 = \frac{\rho q_i^2}{8\tau_c} f\left(Re, k_s, H_{ult}\right) \tag{9}$$

For equation (9), H_{ult} was originally solved in an iterative fashion since the friction factor is a function of the ultimate depth. H_{ult} was initially solved based on the friction factor calculated from the initial depth (H_i) at the chosen time step.

$$H_{ult}^2 = \frac{\rho q_i^2}{8\tau_c} f\left(Re, k_s, H_i\right) \tag{10}$$

Once an initial value for H_{ult} has been calculated, the friction factor is then recalculated iteratively as previously described. The Reynolds number is not recalculated, in other words, q is assumed to stay constant. Five iterations were performed to arrive at the final value of H_{ult}. After the iterations were performed, a check comparing the fifth iteration to the initial calculation of H_{ult} based on the initial depth (H_i) indicated that there was essentially no difference in the ultimate water depths (the differences were less than 0.01 ft). Accordingly, H_{ult} was plotted based on that initial calculation, which used H_i to calculate f, since iterating did not yield an increase in accuracy of H_{ult} but did take a substantial amount of time.

The scour depth ($S_{ult,i}$) corresponding to the ultimate water depth can be realistically estimated by applying an energy equation to take into account the change in velocity head that occurs during the development of the scour. The assumption of a constant total head outlined in Güven et al. yields the following equation (Güven et al., 2001, page 3-4 of the original reference):

$$S_{ult,i} = H_{ult} + \frac{q_i^2}{2gH_{ult}^2} - \left(H_i + \frac{q_i^2}{2gH_i^2} \right) \tag{11}$$

H_i = flow depth at time t_i

g = gravitational constant (32.2 ft/s^2)

Positive $S_{ult,i}$ (scour depth) values resulting from Equation (7) correspond to areas where scouring occurs, while negative values correspond with low shear stress areas where no scouring occurs and the bottom depth remains unchanged. As a result, negative $S_{ult,i}$ values had to be reassigned a value of zero in order to indicate that no scouring occurred at these locations. Reassigning a scour depth of zero to these locations was done using the maximum value function in the data calculator. The remaining positive $S_{ult,i}$ values represent the additional scour to the existing bed (at time t_i) from which q_i, Re, and H_i were obtained. The ultimately scoured bed elevation is calculated as follows:

$$\text{Ultimately Scoured Bed Elevation} = \text{Bed Elevation at time } t_i - S_{ult,i} \tag{12}$$

3. RESULTS AND DISCUSSION OF SCOUR CALCULATIONS

3.1 Time Dependent Scour Results

The base procedure for the calculation of time dependent scour was to use 1-day time steps up to a time of 50 days and 10-day time steps from 50 up to 150 days, to use the Henderson implicit formula for open channel flow to calculate the friction factor (f), and to assume a smooth boundary with a roughness height equal to zero ($k_s = 0$ ft). A contour plot of the bed shear stress for the base case model at time zero is depicted in Figure 9. Figure 10 depicts the bed elevations at times 10, 50, and 150 days. Water surface and bed elevation plots along the longitudinal centerline are depicted for times of 50, 100, and 150 days in Figure 11; also shown is the ultimate scour based on the q_i, Re, and H_i for $t_i = 150$ days. Plots of the unit flow rates (q) along the longitudinal centerline for those times, as well as the unit flow corresponding to the ultimate scour based on the conditions at $t_i = 150$ days, are given in Figure 12. Transverse plots of the same parameters are given in Figures 13 and 14, respectively, for the cross-section at the midpoint of the contraction (x = 0 ft) and Figures 15 and 16 for the entrance of the contraction (x = -30 ft). In each figure, the base procedure was used to calculate the scour over time. The ultimate scour was calculated based on the procedure in section 2.1. Figure 17 shows the maximum scour along the centerline versus time with data points taken at 10-day intervals out of the available data (1-day intervals for days 1-50, and 10-day intervals for days 50-150). The maximum scour along the centerline is the lowest bed elevation along the centerline of the long axis for a particular time. The ultimate scour depicted is the lowest bed elevation along the centerline for the ultimately scoured bed based on the q_i, Re, and H_i for $t_i = 150$ days.

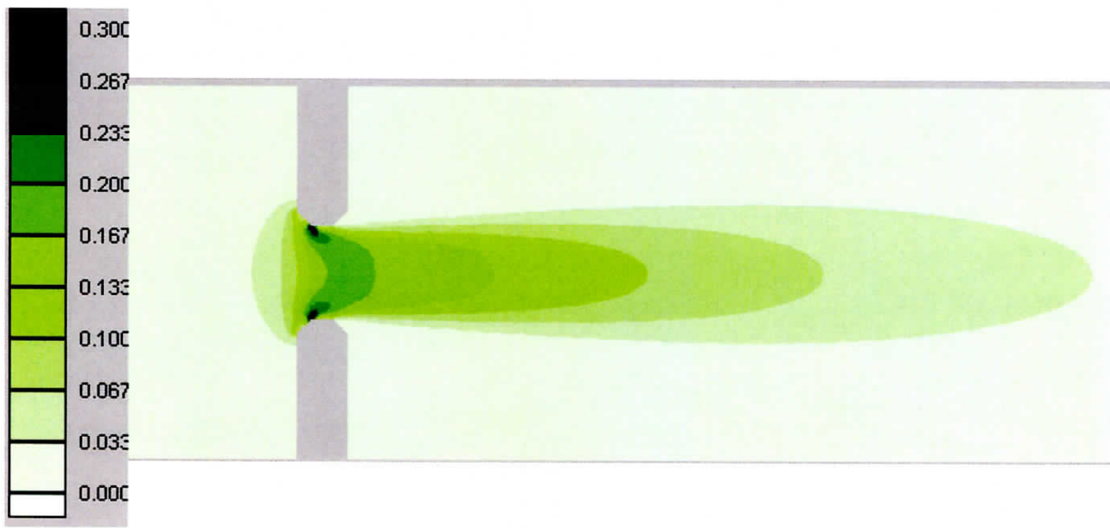

Figure 9. Distribution of bed shear stress (lb/ft^2) for the base case at time t = 0

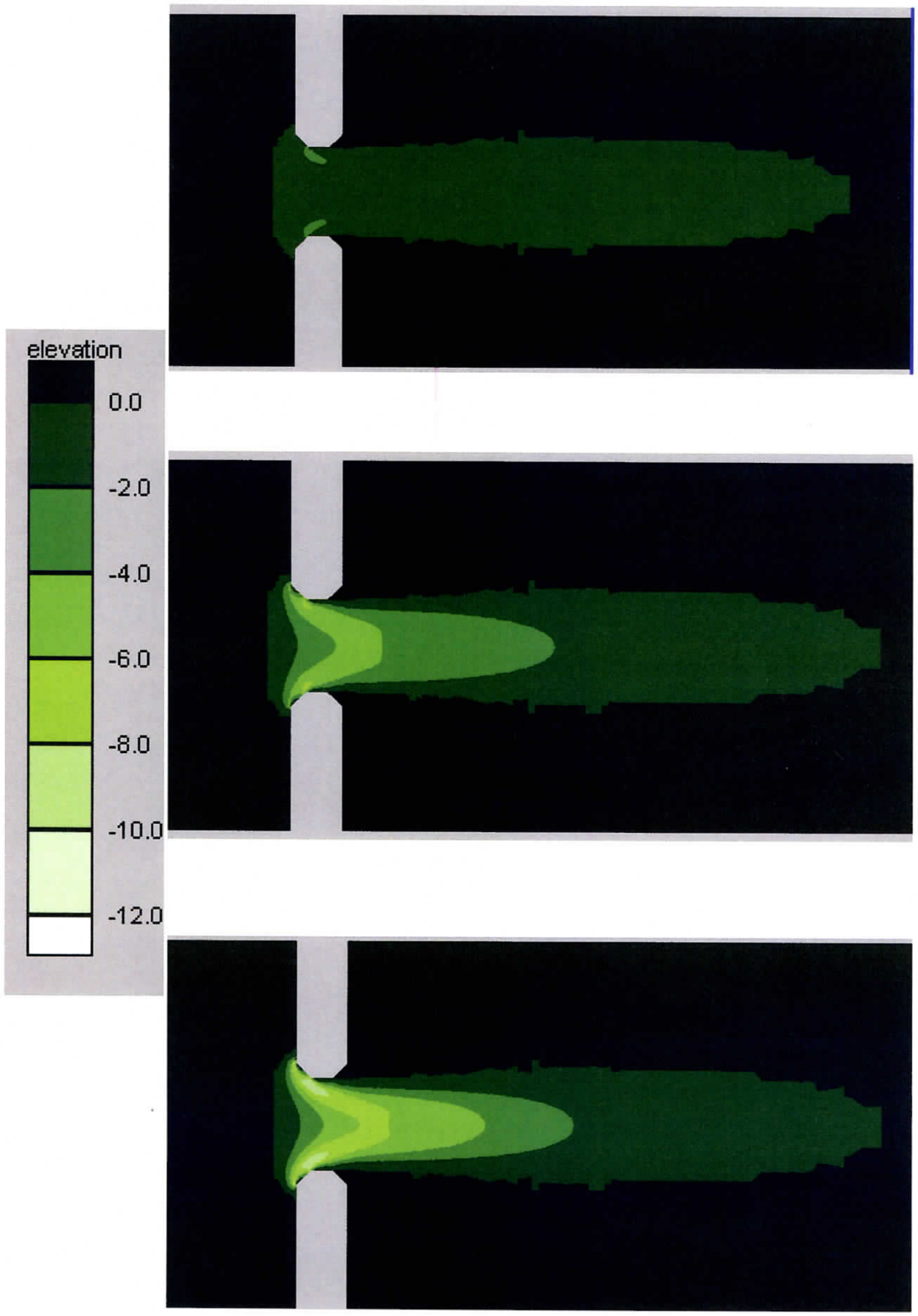

Figure 10. Bed elevation (ft) contours at times t = 10, 50, and 150 days (top to bottom)

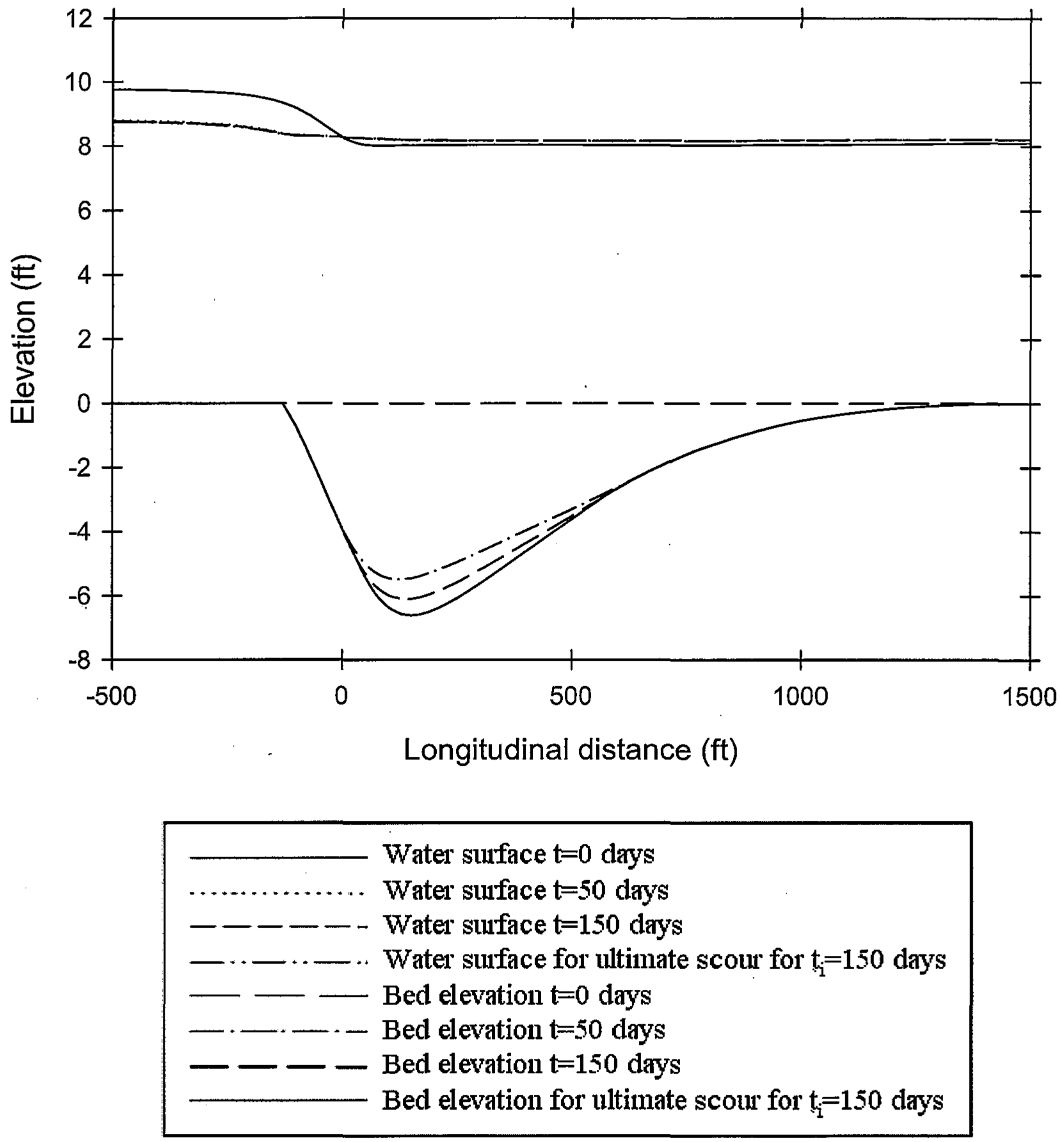

Figure 11. Longitudinal plot of water surface and bed elevation along centerline from x = -500 to 1500 ft at times 0, 50, and 150 days and for ultimate scour based on conditions for t_i = 150 days

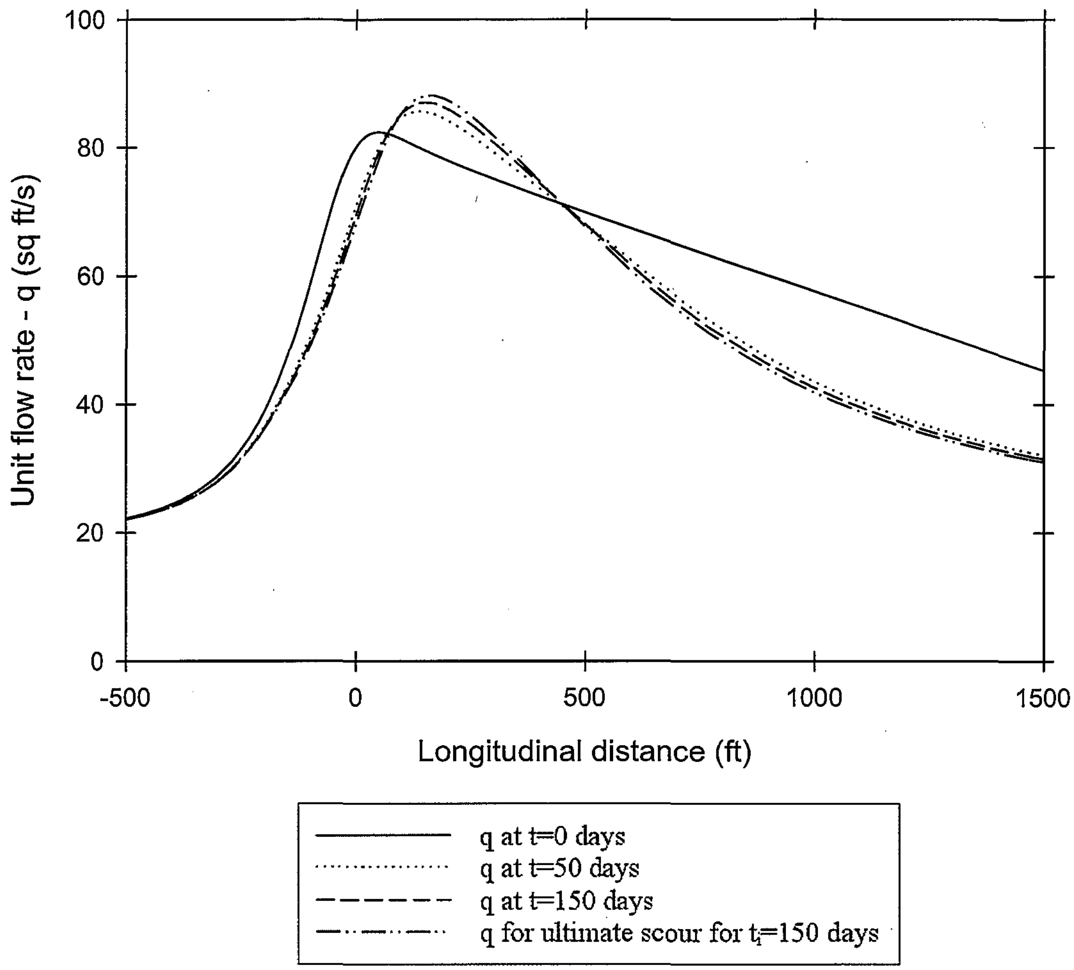

Figure 12. Longitudinal plot of unit flow rate (q) along centerline from x = -500 to 1500 ft at times 0, 50, and 150 days and for ultimate scour based on conditions for t_i = 150 days

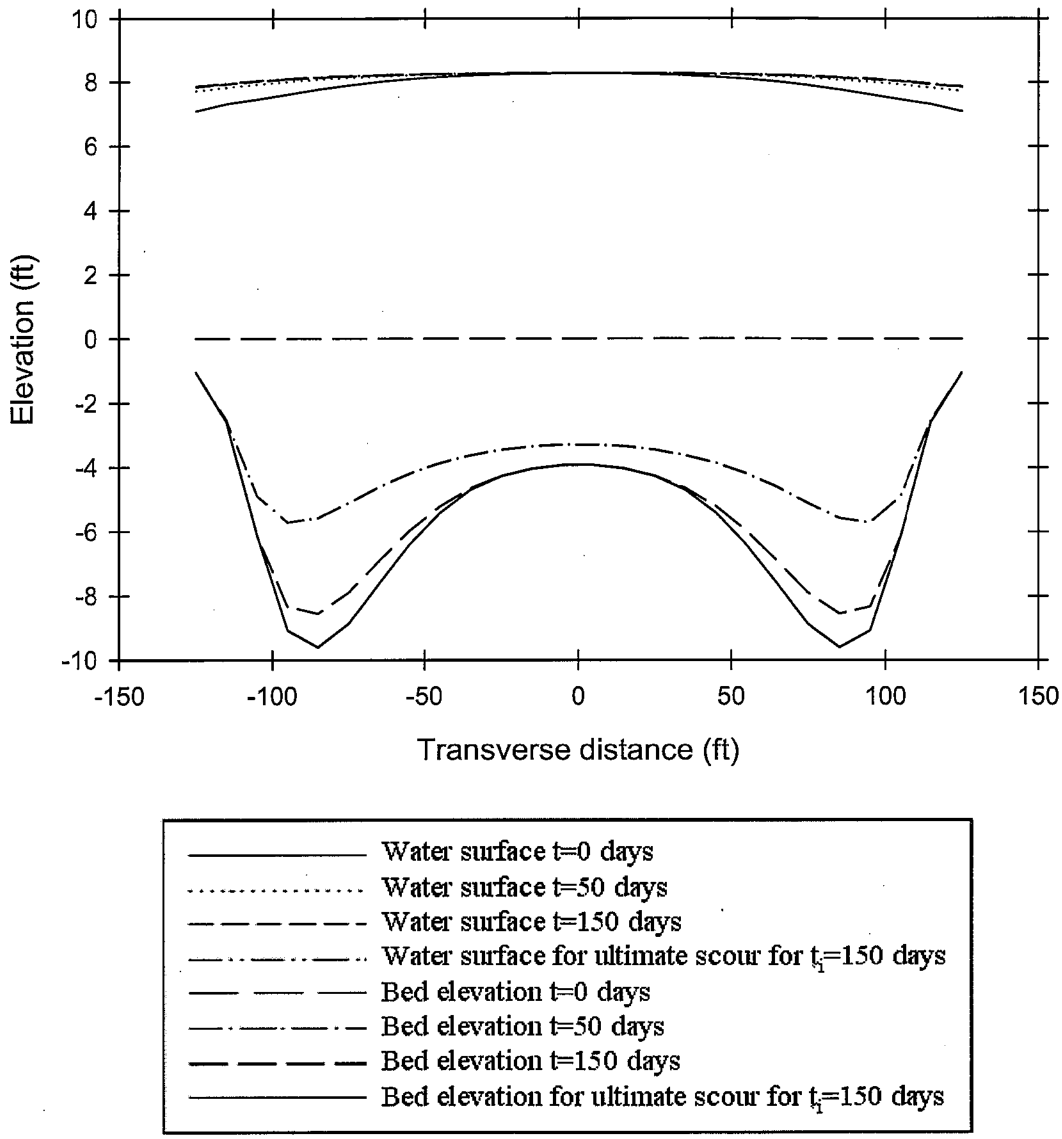

Figure 13. Transverse bed and water surface profiles at midpoint of contraction (x = 0 ft) at times 0, 50, and 150 days and for ultimate scour based on conditions for t_i = 150 days

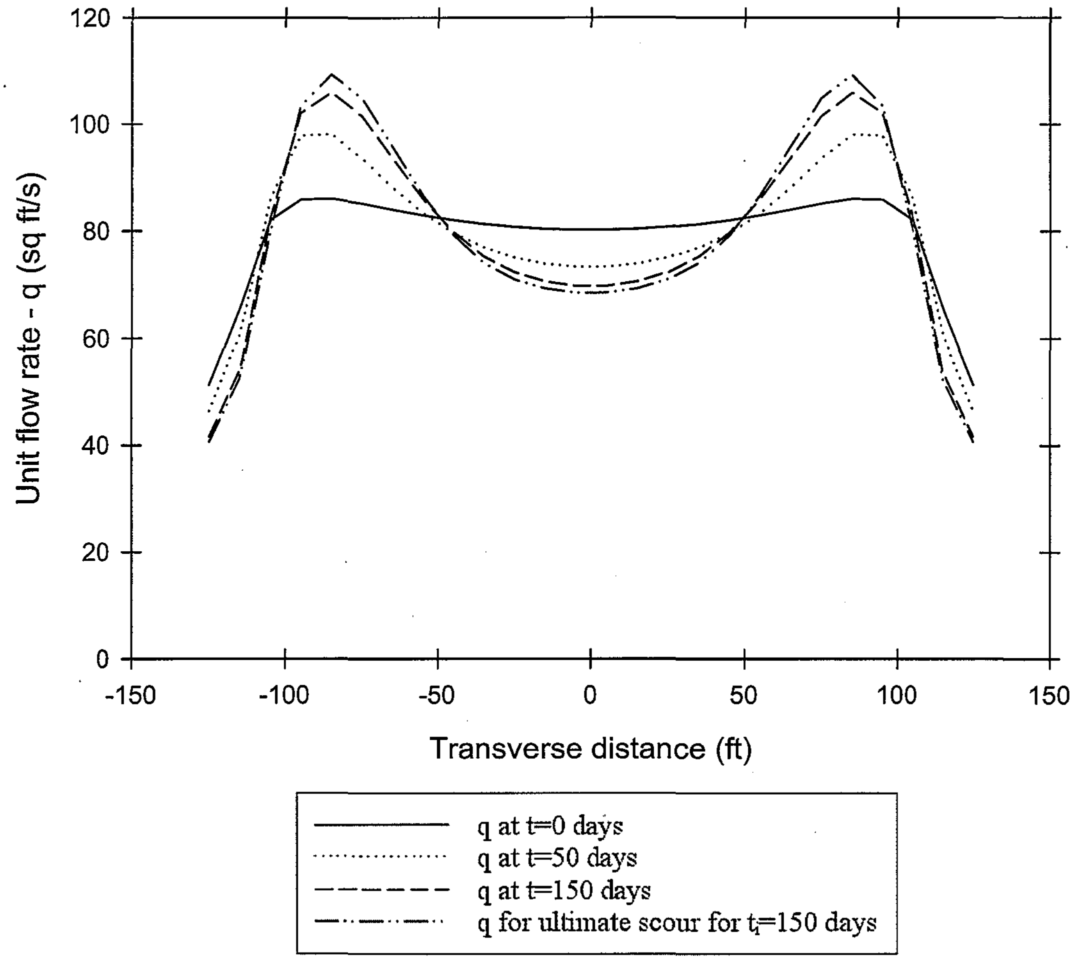

Figure 14. Transverse unit flow rate (q) profile at midpoint of contraction (x = 0 ft) at times 0, 50, and 150 days and for ultimate scour based on conditions for t_i = 150 days

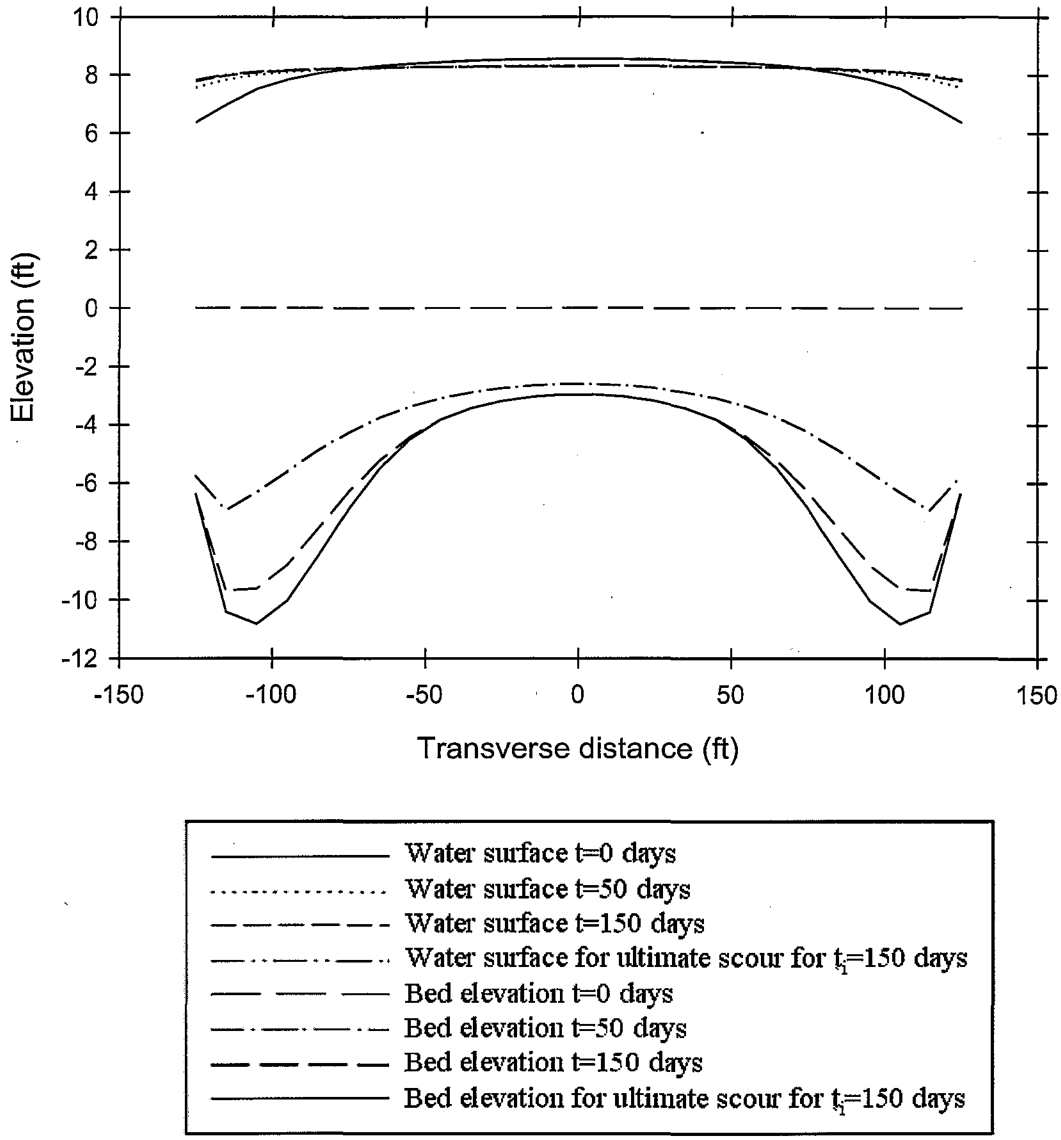

Figure 15. Transverse bed and water surface profiles at entrance to contraction (x = -30 ft) at times 0, 50, and 150 days and for ultimate scour based on conditions for t_i = 150 days

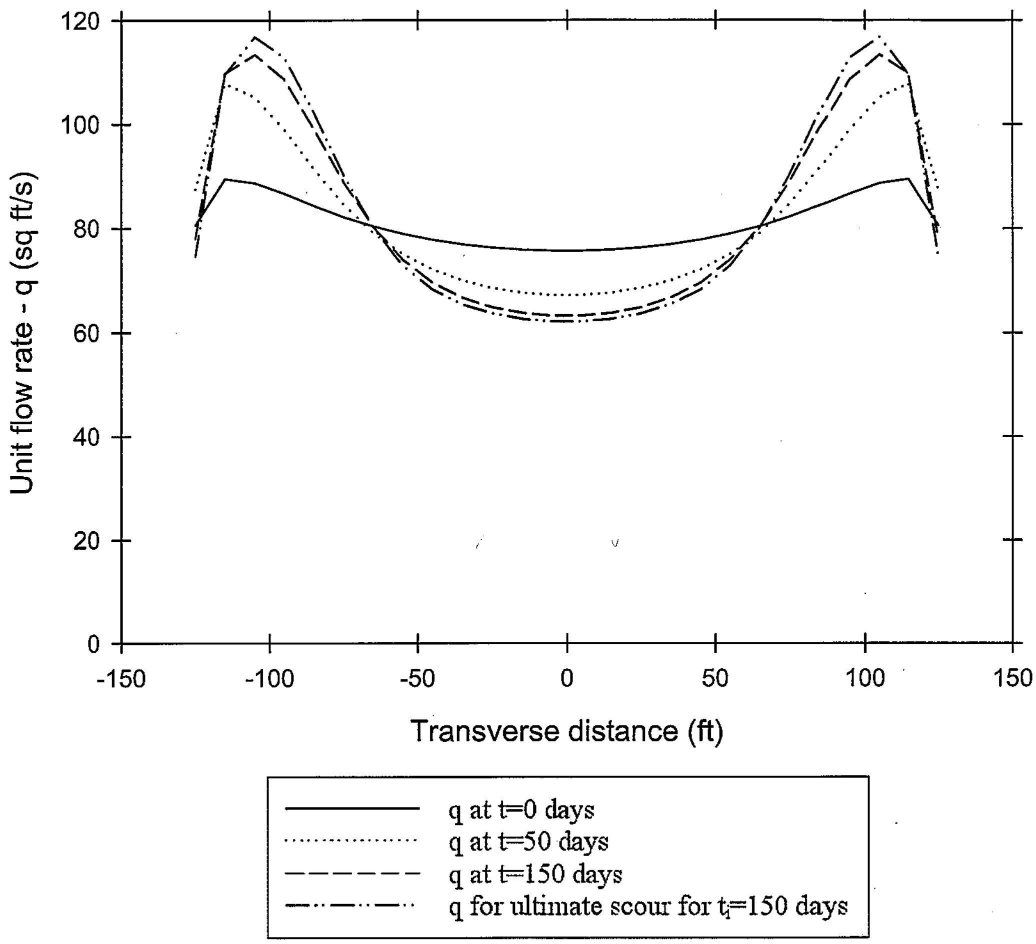

Figure 16. Transverse unit flow rate (q) profile at entrance to contraction (x = -30 ft) at times 0, 50, and 150 days and for ultimate scour based on conditions for t_i = 150 days

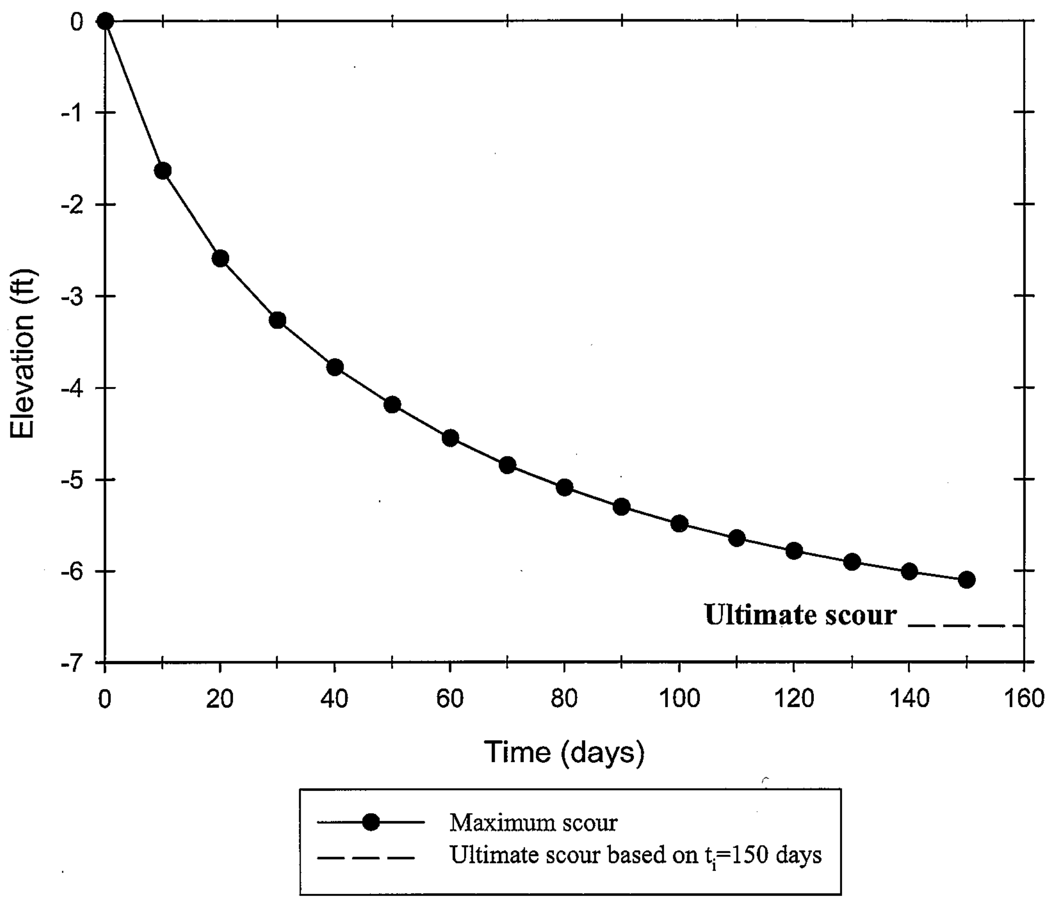

Figure 17. Maximum scour depth along centerline over time and ultimate scour based on conditions for $t_i = 150$ days

3.2 Ultimate Scour Results

The ultimate scour was calculated based on the q_i, Re, and H_i at times 0, 50, and 150 days. One-day time steps were chosen to reach 50 days and 10-day time steps were used between days 50 and 150. Basing the ultimate scour on the data for the 150-day case gives a more accurate representation of the ultimate scour depths that may occur rather than basing it on the data for an earlier time. As time goes on, the scouring of the bed results in changes in the flow pattern which dictate where subsequent scour will be most concentrated. Plan views of the ultimately scoured beds, based on the times mentioned previously in this section, are depicted in Figure 18. Longitudinal profiles of the ultimately scoured beds and corresponding water surface elevations based on times t_i = 0, 50, and 150 days are shown in Figure 19. The transverse profiles of the same features are depicted in Figure 20 for the entrance of the contraction (x = -30 ft).

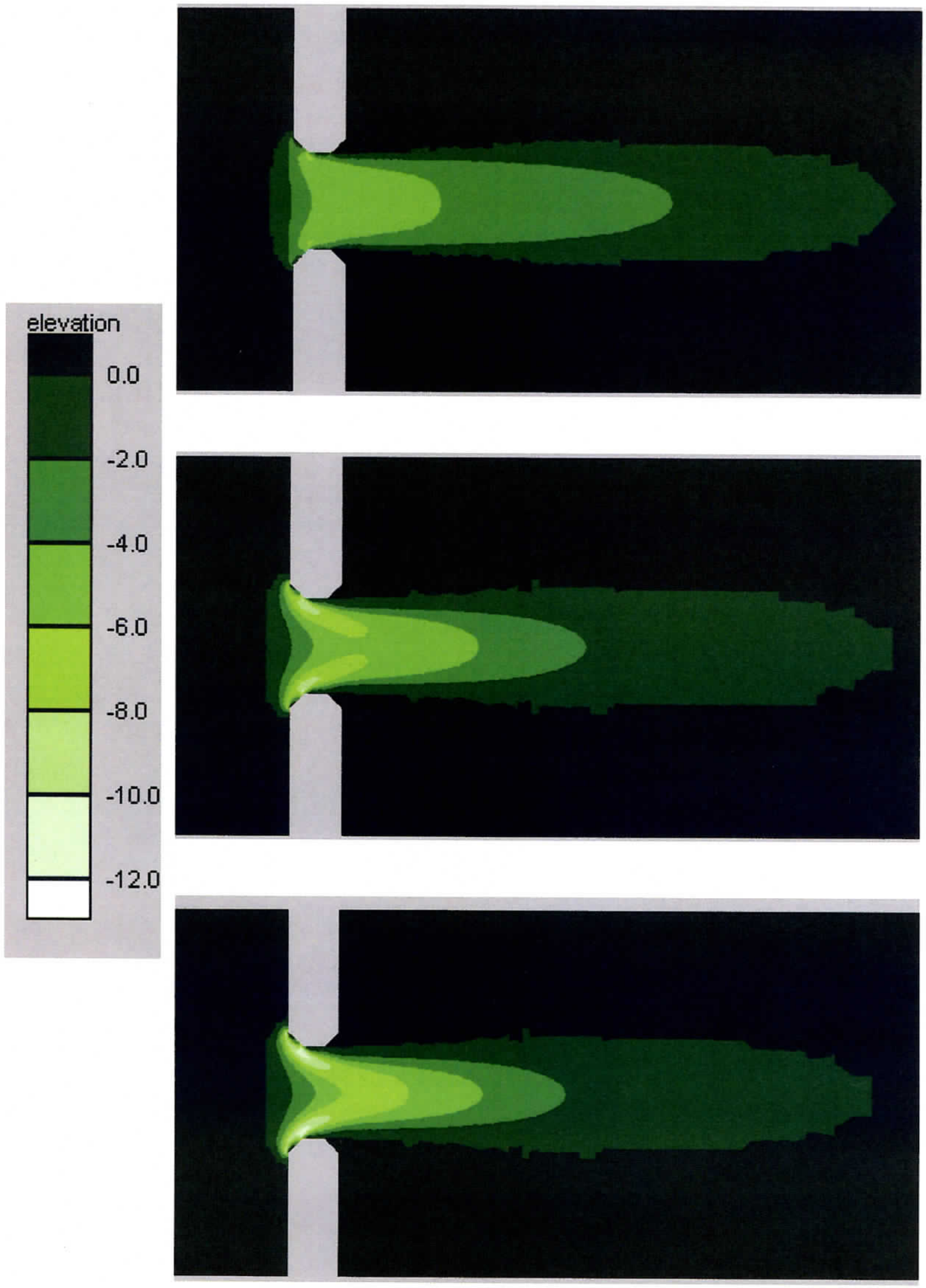

Figure 18. Ultimate scour (ft) based on q_i, Re, and H_i at times $t_i = 0$, 50, and 150 days (top to bottom)

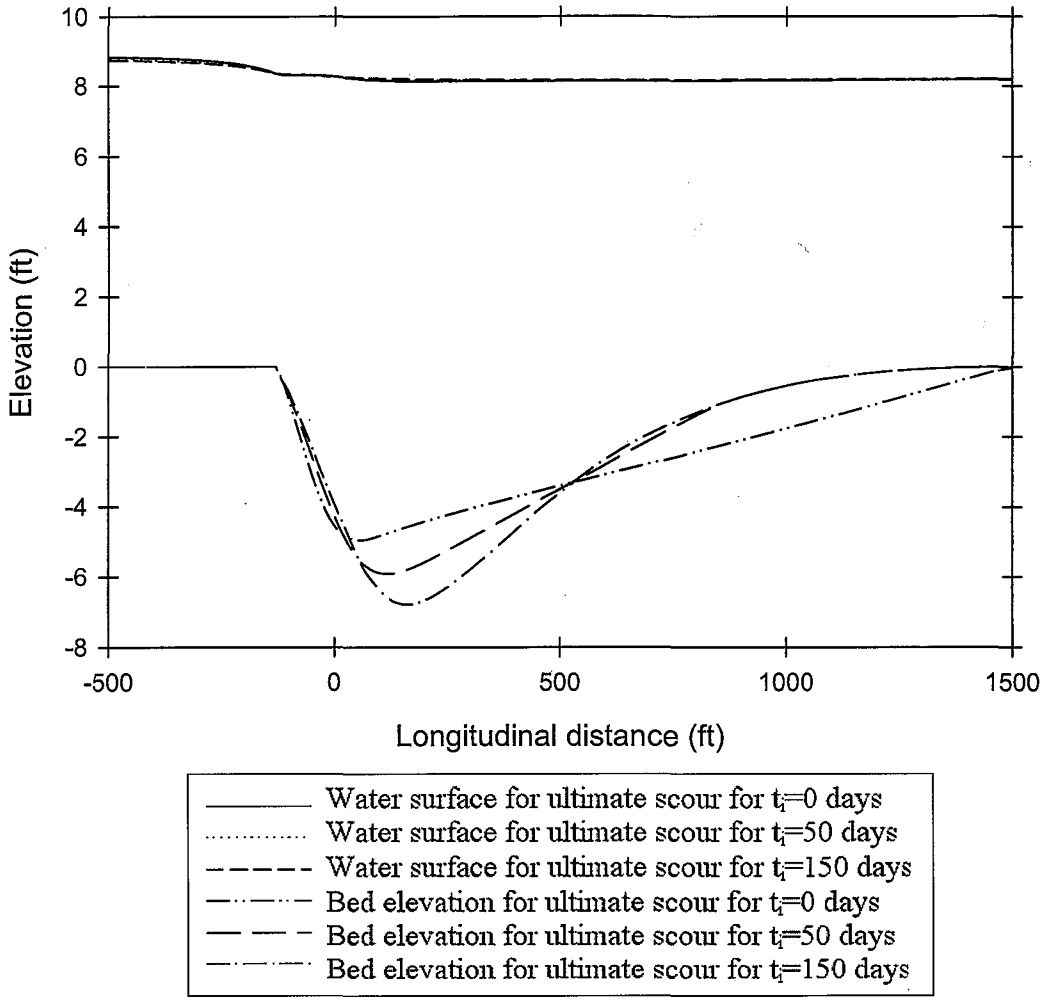

Figure 19. Longitudinal plot of ultimate water surface and ultimately scoured bed elevation along centerline from x = -500 to 1500 ft based on based on q_i, Re, and H_i for times t_i = 0, 50, and 150 days

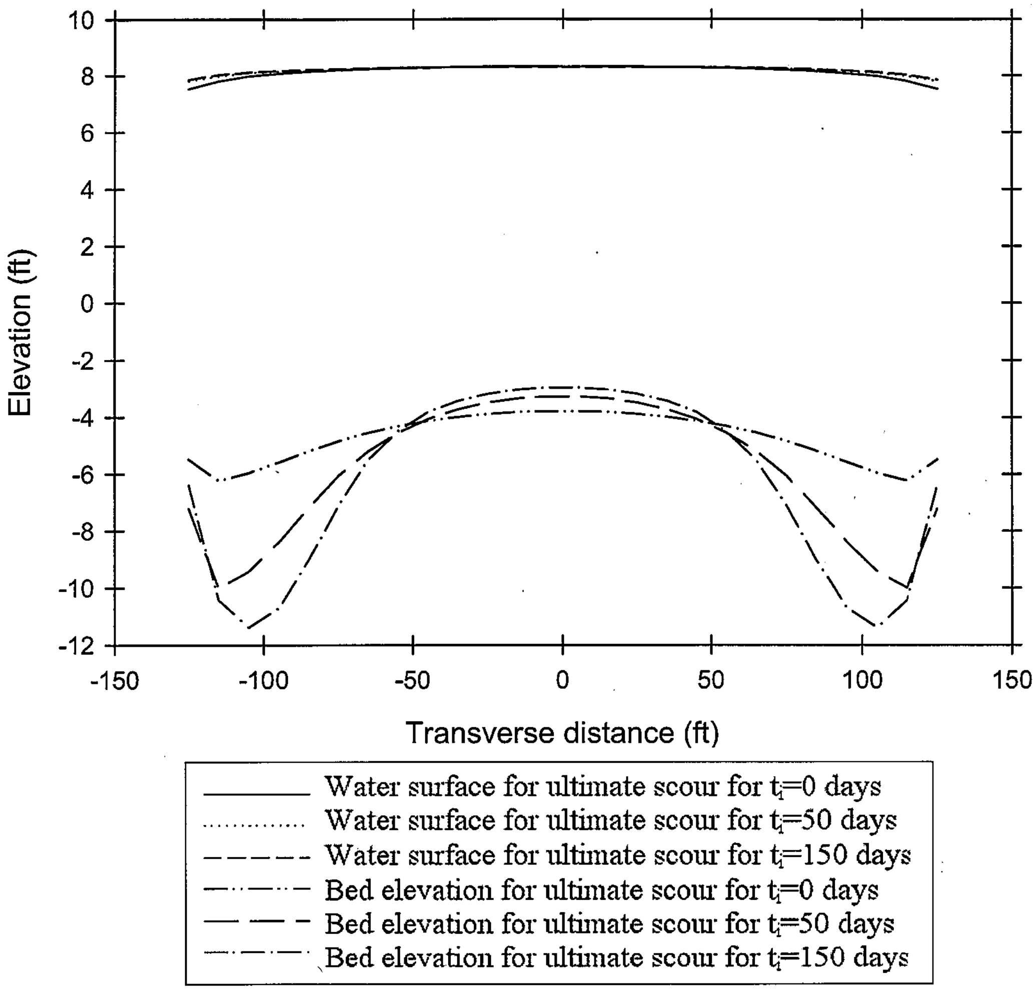

Figure 20. Transverse plot of ultimate water surface and ultimately scoured bed elevation at entrance to contraction (x = -30 ft) based on based on q_i, Re, and H_i for times t_i = 0, 50, and 150 days

4. FURTHER INVESTIGATIONS INTO METHODOLOGY

The next analyses look at varying the time step size, using a second order correction method in the modeling procedure, comparing the smooth case ($k_s = 0$ ft) to a rough case ($k_s = 0.000974$ ft), and comparing the Henderson implicit formula for open channel flow to the Swamee-Jain explicit formula for pipe flow for calculating f.

4.1 Effect of Time Step Size

The calculation of a scour rate allows the modeler to calculate the scour that is expected to occur over a specified time interval. The size of that time interval may be dictated by flow information available; daily flow data are available through the United States Geological Survey (USGS) for many streams and rivers. In this analysis, time intervals of 1, 5, 10 days were used to approach the scour that would be realized after 50 days of flow. The maximum scour along the centerline was plotted in Figure 21 for each case, with these data points being retrieved at 10 days intervals as before. These plots are labeled uncorrected in reference to the next section which sets forth a method for adjusting the model with a second order Euler correction. Naturally, smaller time steps will provide a more accurate approach to determining the time dependent scour. Even so, after 50 days, the difference in the bed elevation is less than 0.5 ft when comparing the 1 and 10-day time steps.

4.2 Effect of Second Order Correction

A second order correction (corrected method) was performed to compare with thus far presented first order calculations (uncorrected). In the previous calculations, the initial model with a given bed elevation is run. Then, the data calculator is used to calculate the friction factor, shear stress, rate of scour (rate), and the scour occurring over

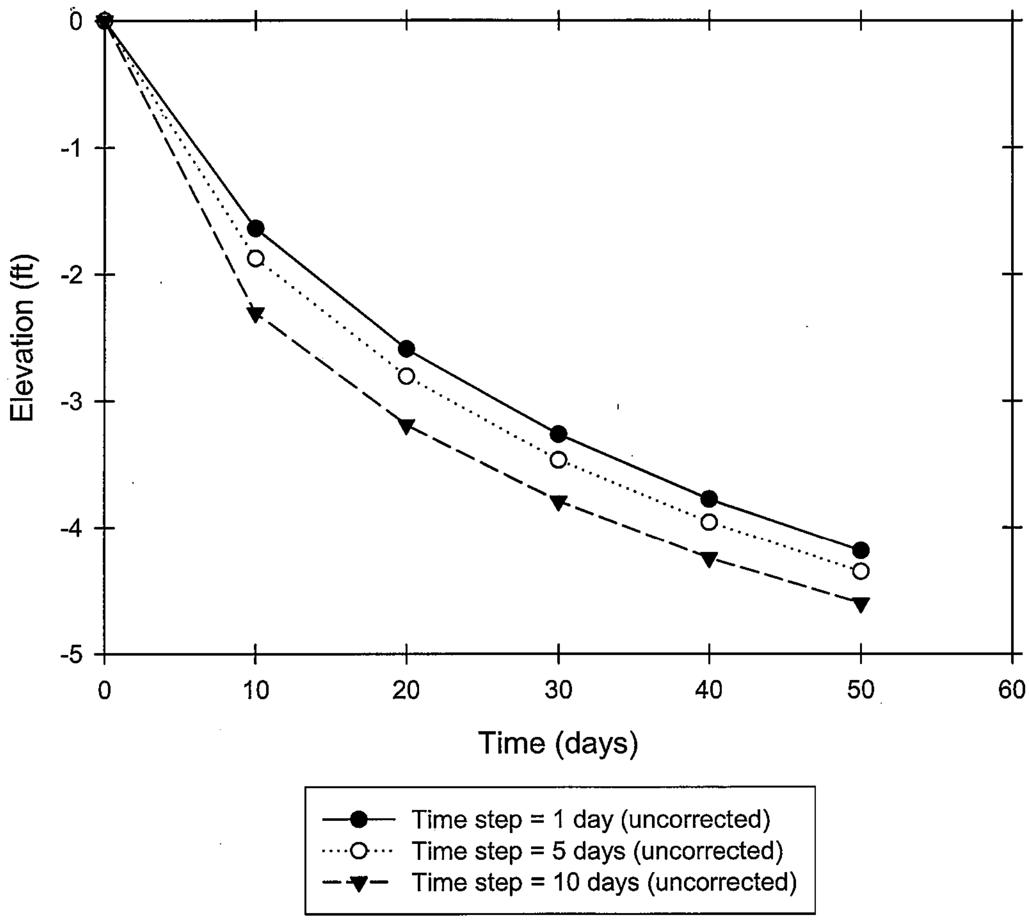

Figure 21. Maximum scour along centerline over time for varied time step size

a specified time interval for that rate using the first order Euler approximation:

$$\text{New Bed Elevation} = \text{Initial Bed Elevation} - (\text{Initial Rate of Scour} * \text{Time Step}) \quad (13)$$

The correction method is more intensive requiring that the model be run again with this new bed elevation, and that the same calculations be performed for the new output. However, rather than calculate the new bed using the method described above, a second order Euler approximation is now used which employs the rate and elevation from the initial model to obtain a more accurate approximation of the new bed elevation. The following equation describes this second order approximation:

$$\text{New Bed Elevation} = \text{Initial Bed Elevation} - \left(\left(\frac{\text{Initial Rate} + \text{New Rate}}{2}\right) * \text{Time Step}\right) \quad (14)$$

Figure 22 depicts varied time step sizes for the corrected case calculated using this second order approximation. Time step size is less important for when the correction is used. Although the uncorrected 1-day time step plot is not included in this plot for considerations of clarity, it lies virtually on top of the corrected 1-day time step plot. As expected, the smaller the time step, the closer the corrected and uncorrected plots. Since the uncorrected method requires about half as much time as the corrected, and since 1-day flow data is commonly available, the uncorrected 1-day case was used for analyses in this study.

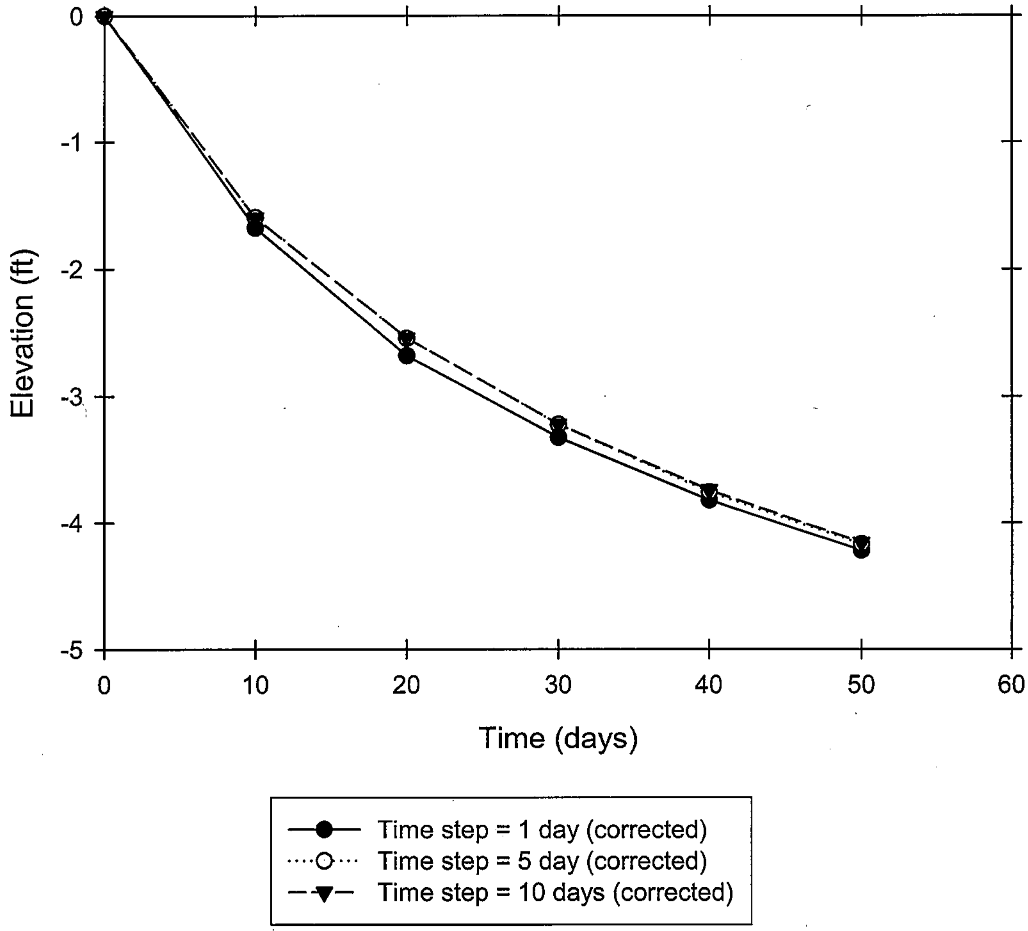

Figure 22. Maximum scour along centerline over time for varied time step size using the second order Euler correction method

4.3 Effect of Roughness

Initially, all the models were performed with the assumption of a smooth boundary (roughness height (k_s) equal to zero). A more realistic model would take into account the effects of roughness on hydraulic behavior. For the "rough" case, k_s was calculated based on Soil No. 1, which has a median particle size (D_{50}) of 0.06 mm. The assumption that $k_s = 0.5*D_{50}$, as used in Güven et al. (2002), resulted in a k_s of 0.000974 ft. Figure 23 depicts a longitudinal profile of the friction factor for the rough and smooth cases at time zero prior to any scour. All other conditions were kept identical and the time dependent scour was calculated up to 150 days using 1-day time steps up to 50 days and 10-day time steps from then on. A plot of the maximum scour depth along the longitudinal centerline is shown in Figure 24. Again, data points were taken at 10-day intervals.

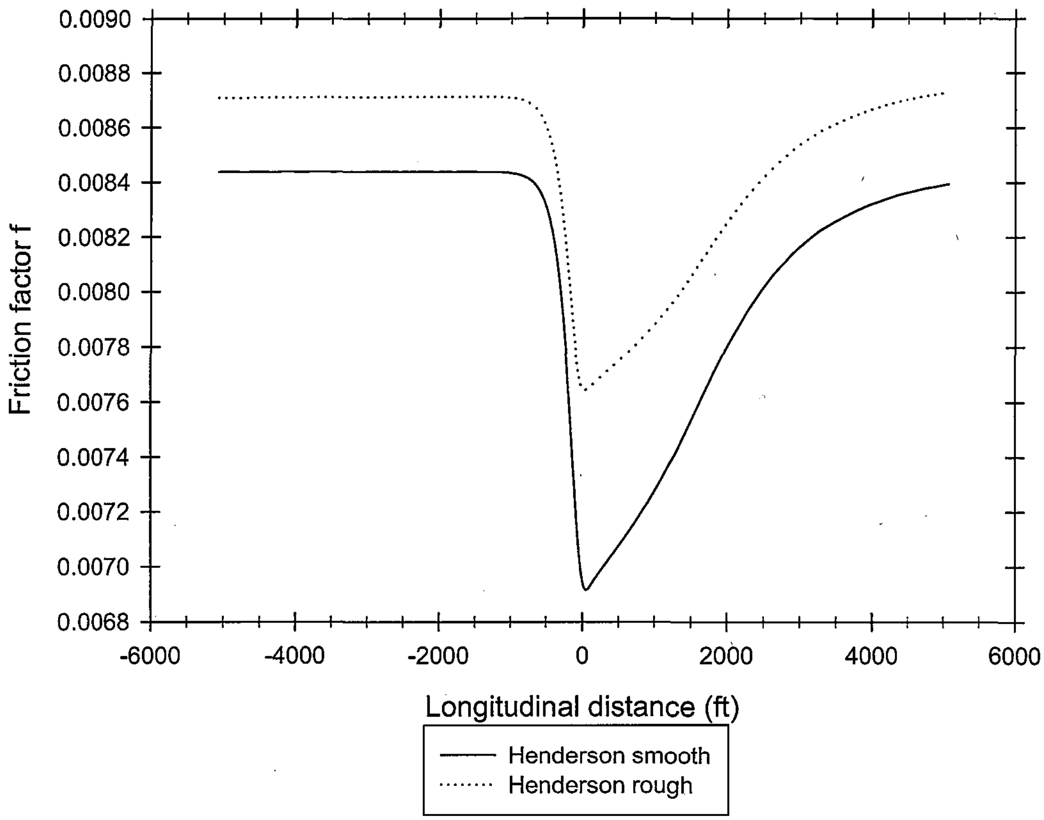

Figure 23. Longitudinal plot of friction factor at t = 0 along centerline calculated based on the Henderson formula; smooth versus rough cases

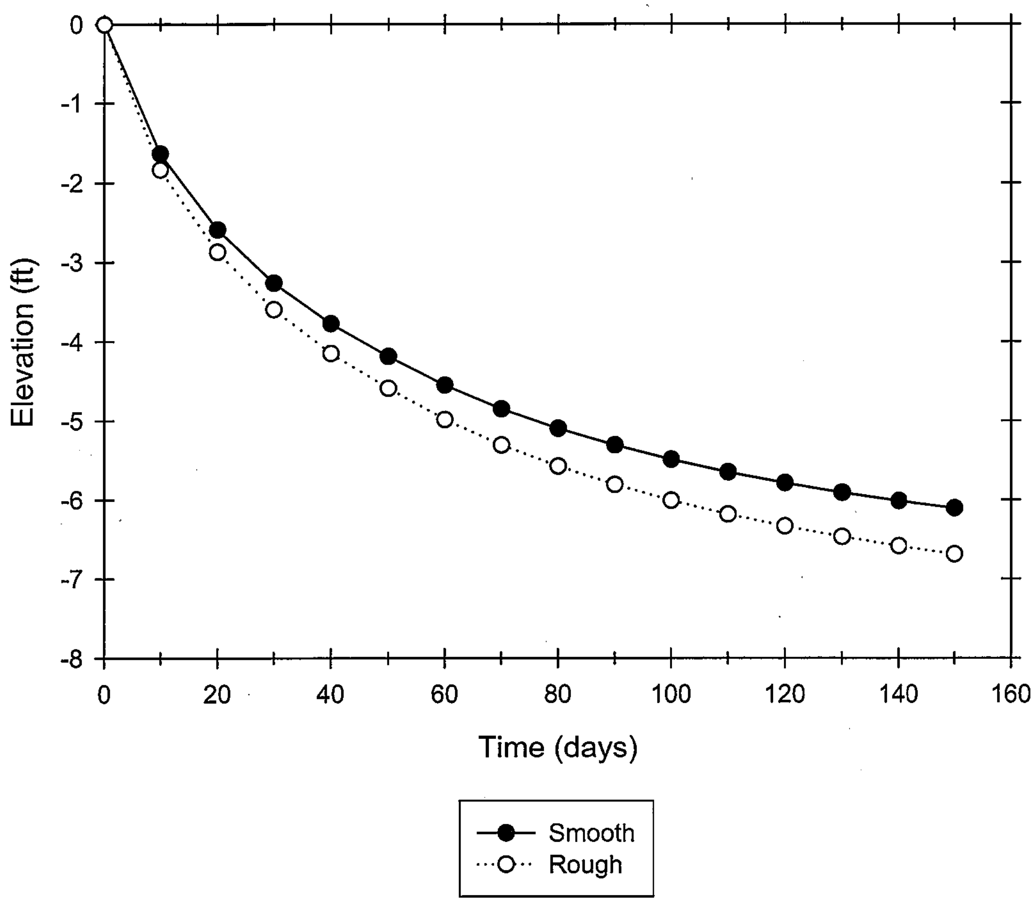

Figure 24. Maximum scour along longitudinal centerline over time for smooth and rough cases using Henderson formula for calculating f

4.3 Effect of Swamee-Jain Explicit Formula Versus Henderson Implicit Formula to Calculate f

In the section that discusses the calculation of time-dependent scour, a method is proposed to calculate the friction factor f employing both the Swamee-Jain explicit formula and the Henderson implicit formula. The proposed method uses Swamee-Jain as an initial estimation of f to be used in subsequent calculations using the Henderson formula, which calculates f as a function of f. The Henderson formula is then used to calculate f in an iterative fashion; this method is referred to as Henderson. To conserve steps and time, the use of the Swamee-Jain explicit formula alone as a fairly accurate estimate of f was investigated. Figure 25 is a plot of f along the longitudinal centerline for the base case prior to scouring calculated using the two different methods. Then, an analysis was done using the same time step and data point retrieval scheme as the previous sections in this chapter. The two methods to compute f are compared in Figure 26.

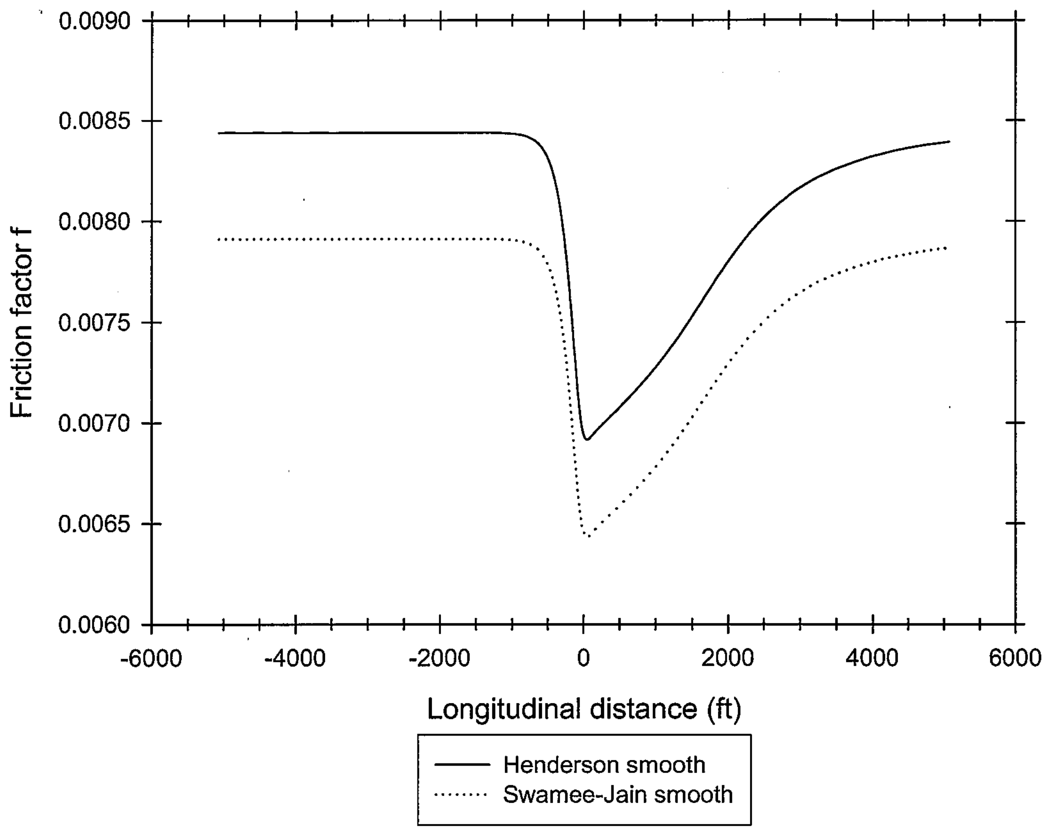

Figure 25. Longitudinal plots of friction factor at t = 0 along centerline calculated based on Henderson versus Swamee-Jain formulas for calculating f

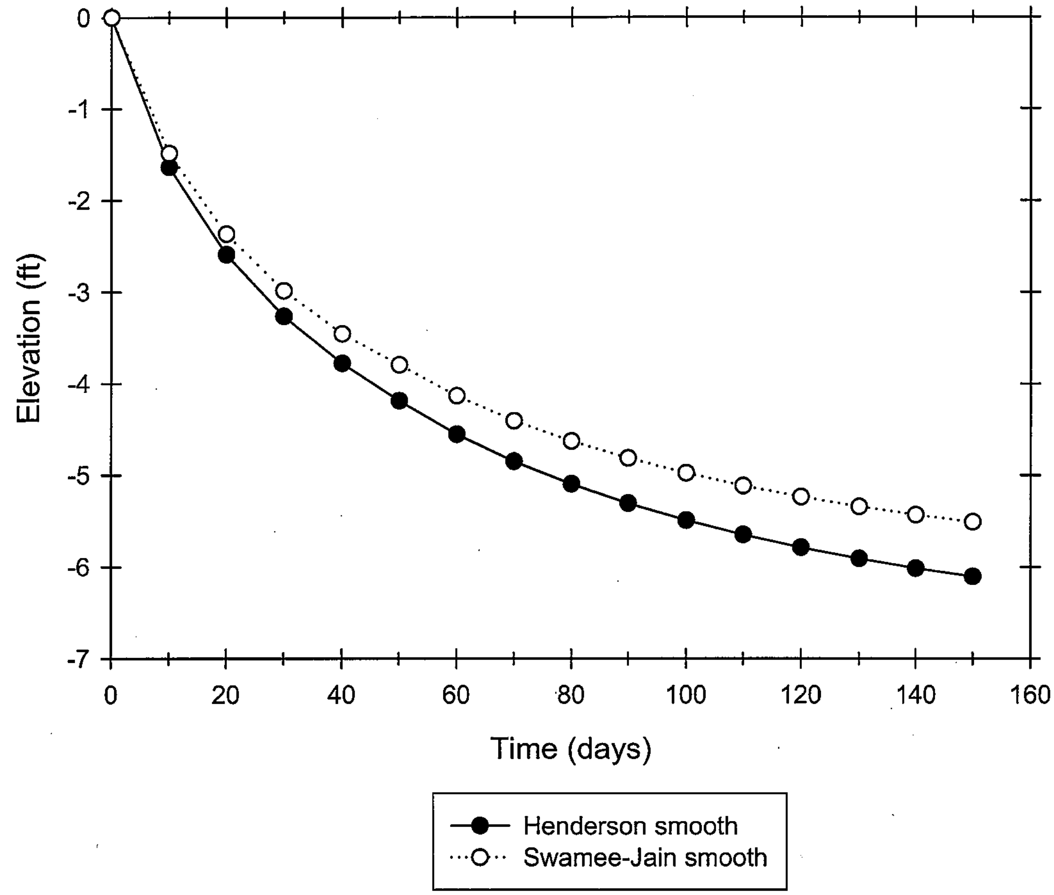

Figure 26. Comparison of maximum scour along centerline over time using Swamee-Jain explicit and Henderson implicit formulas to compute the friction factor

5. CONCLUDING REMARKS

The methods described in this report allow for the calculation of time-dependent scour with a user-defined time step. Flow data, variable or constant, can be used to calculate scour and update the bed of the model. In addition, an ultimate scour can be calculated based on the model results obtained for a particular time. The authors suggest that the corrected method be used and that smaller time steps be used, at least initially (since more scour occurs during this period). It is also suggested that the friction factor be calculated with the Henderson formula, without ignoring roughness, since this approach appears to be more realistic.

Physical modeling, comparison with field measurements, and comparison with three-dimensional modeling are still needed to investigate whether two-dimensional modeling is an accurate representation of complex three-dimensional hydraulics at bridge sites.

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