MECH 5970/6970
Homework #3
Due: 10/23/2017

1. This problem is to look at noise models.
   a) Perform 1000 run monte-carlo simulation to demonstrate that the error from the sum OR difference of two random measurements is expressed by
      \[ \sigma_{a\pm b} = \sqrt{\sigma_a^2 + \sigma_b^2} \]
   b) Perform a 1000 run monte-carlo simulation (of 10 minutes) to look at the error growth of a random walk (integrated white noise). Use a white noise with 1-sigma value of 0.1 and 0.01 and compare the results. Plot the mean and standard deviation of the monte-carlo simulation along with one run of the simulation (show that the random walk is zero mean with a standard deviation is \( \sigma_w = \sigma_w \Delta t \sqrt{t} \))
   c) Perform a 1000 monte-carlo simulation to look at the error growth of a 1st order markov process (integrated filtered noise) of the form \( \dot{x} = -\frac{1}{\tau} x + w \). Use the same noise characteristics as above and compare the results with a 1 second and 100 second time constant (this results in 4 combinations). Comment on how changing the time constant and changing the standard deviation of the noise effects the error. Show that the 1st order markov process is zero mean with a standard deviation of is \( \sigma_x = \sigma_w \Delta t \sqrt{\left(\frac{A^2 - 1}{A^2 - 1}\right)} \) where \( A = \left(1 - \frac{\Delta t}{\tau}\right) \). Note that for a positive time constant (i.e. stable system) the standard deviation has a steady state value.

2. Show that the differential GPS problem is linear. In other words derive the following expression:
   \[ \Delta \rho = [uv_x \ uv_y \ uv_z \ 1] \begin{bmatrix} r_x \\ r_x \\ r_x \\ c\delta t_{ab} \end{bmatrix} \]
3. Set up your own 2D planar trilateration problem. Place the SVs at four corners of (0,0) (1000,0), (1000,1000), and (0 1000). Generate a range measurement for a base station at (500,500) and a user at (501,500).
   a. Solve for the position of the user using 2 SVs and then 4 SVs assuming no clock errors. How does the PDOP change for the two cases?
   b. Solve for the position of the user assuming you need to solve for the user clock bias. What is the PDOP with all 4 satellites.
   c. Calculate a differential solution between the base and user using a single difference model and assuming you must solve for a clock bias between the base station and user. What is the PDOP with all 4 satellites?
   d. Calculate a differential solution between the base and user using a double difference model to remove the clock bias between the base station and user. What is the PDOP with all 4 satellites?
   e. Assuming the range error is zero mean with unit variance, what is the order of accuracy in the above 4 solution methods?