

## Chapter 3: Foundational Results

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- Overview
- Safety Questions
- Turing Machine Mapping

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## Overview

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- Safety Questions
- HRU Model

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## What Is "Secure"?

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- Adding a **generic right**  $r$  where there was not one is "**leaking**"
- If a system  $S$ , beginning in initial state  $s_0$ , cannot leak right  $r$ , it is *safe with respect to the right  $r$* .
- What is a generic right?

**Generic rights correspond to a class of objects**

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## Definitions

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- **Definition 3-1.** When a generic right  $r$  is added to an element of the access control matrix not already containing  $r$ , that right is said to be leaked.
- **Definition 3-2.** If a system can never leak the right  $r$ , the system (including the initial state  $s_0$ ) is called *safe with respect to the right  $r$* . If the system can leak the right  $r$  (enter an unauthorized state), it is called *unsafe with respect to the right  $r$* .

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## Example

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- A computer system allows the network administrator to read all network traffic. It disallows all other users from reading this traffic. The system is designed in such a way that the network administrator cannot communicate with other users. Is this system safe?

**Yes, there is no way for the right  $r$  of the network administrator over the network device to leak.**

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## Safety Question

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- Does there exist an algorithm for determining whether a protection system  $S$  with initial state  $s_0$  is safe with respect to a generic right  $r$ ?
  - Here, “safe” = “secure” for an abstract model

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## Mono-Operational Commands

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- Answer: *yes*
- Sketch of proof:  
Consider minimal sequence of commands  $c_1, \dots, c_k$  to leak the right  $r$ .
  - Can merge all **creates** into oneWorst case: insert every right into every entry; with  $s$  subjects and  $o$  objects initially, and  $n$  rights, upper bound is  $k \leq n(s+1)(o+1)$

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## General Case

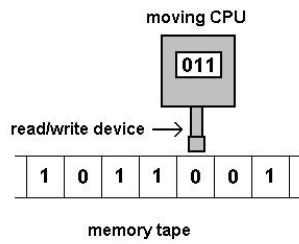
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- Answer: *no*
- Sketch of proof:  
**Reduce halting problem to safety problem**  
Turing Machine review:
  - Infinite tape in one direction
  - States  $K$ , symbols  $M$ ; distinguished blank  $b$
  - Transition function  $\delta(k, m) = (k', m', L)$   
means in state  $k$ , symbol  $m$  on tape location replaced by symbol  $m'$ , head moves to left one square, and enters state  $k'$
  - Halting state is  $q_f$ ; TM halts when it enters this state

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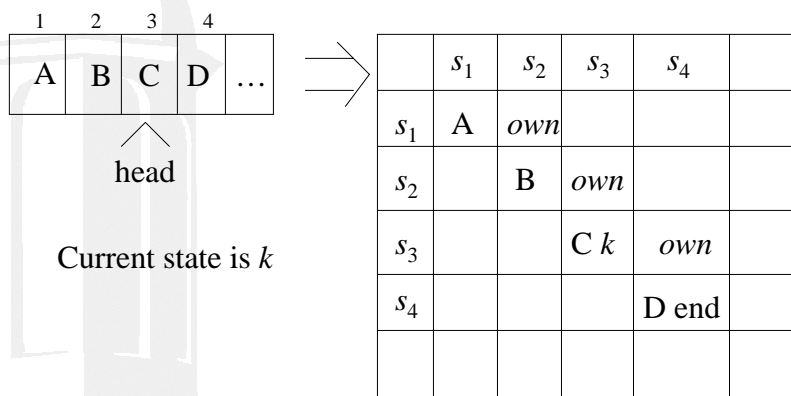
# Turing Machine



**It is undecidable whether a given state of a given protection system is safe for a given generic right.**

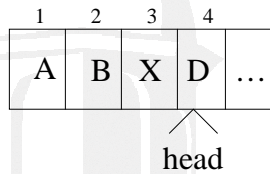
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# Mapping



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# Mapping



After  $\delta(k, C) = (k_1, X, R)$   
 where  $k$  is the current  
 state and  $k_1$  the next state

	$s_1$	$s_2$	$s_3$	$s_4$	
$s_1$	A	<i>own</i>			
$s_2$		B	<i>own</i>		
$s_3$			X	<i>own</i>	
$s_4$				D $k_1$ end	

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# Command Mapping

$\delta(k, C) = (k_1, X, R)$  at intermediate becomes

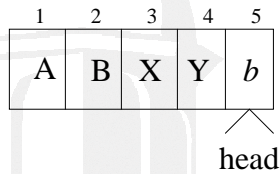
```

command  $c_{k,C}(s_3, s_4)$ 
if own in  $A[s_3, s_4]$  and  $k$  in  $A[s_3, s_3]$ 
    and  $C$  in  $A[s_3, s_3]$ 
then
    delete  $k$  from  $A[s_3, s_3]$ ;
    delete  $C$  from  $A[s_3, s_3]$ ;
    enter  $X$  into  $A[s_3, s_3]$ ;
    enter  $k_1$  into  $A[s_4, s_4]$ ;
end
    
```

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# Mapping



	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$
$s_1$	A	<i>own</i>			
$s_2$		B	<i>own</i>		
$s_3$			X	<i>own</i>	
$s_4$				Y	<i>own</i>
$s_5$					<i>b k<sub>2</sub> end</i>

After  $\delta(k_1, D) = (k_2, Y, R)$   
 where  $k_1$  is the current  
 state and  $k_2$  the next state

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# Command Mapping

$\delta(k_1, D) = (k_2, Y, R)$  at end becomes

```

command crightmostk,c( $s_4, s_5$ )
if end in  $A[s_4, s_4]$  and  $k_1$  in  $A[s_4, s_4]$ 
    and  $D$  in  $A[s_4, s_4]$ 
then
    delete end from  $A[s_4, s_4]$ ;
    create subject  $s_5$ ;
    enter own into  $A[s_4, s_5]$ ;
    enter end into  $A[s_5, s_5]$ ;
    delete  $k_1$  from  $A[s_4, s_4]$ ;
    delete  $D$  from  $A[s_4, s_4]$ ;
    enter  $Y$  into  $A[s_4, s_4]$ ;
    enter  $k_2$  into  $A[s_5, s_5]$ ;
end
    
```

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## Rest of Proof

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- Protection system exactly simulates a Turing Machine
  - Exactly 1 *end* right in ACM
  - 1 right in entries corresponds to state
  - Thus, at most 1 applicable command
- If TM enters state  $q_r$  then right has leaked
- If safety question decidable, then represent TM as above and determine if  $q_r$  leaks
  - Implies halting problem decidable
- Conclusion: safety question undecidable

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## Other Results

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- Set of unsafe systems is recursively enumerable
- Delete **create** primitive; then safety question is complete in **P-SPACE**
- Safety question for mono-conditional, monotonic protection systems is decidable
- Safety question for mono-conditional protection systems with **create**, **enter**, **delete** (and no **destroy**) is decidable.

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## Key Points

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- Safety problem undecidable
- Limiting scope of systems can make problem decidable

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