Antennas
Part 3

- Small Loop Antenna
  - – Vector Magnetic Potential
  - – Current in a Loop
  - – Electric and Magnetic Fields
  - – Antenna Characteristics

Vector Magnetic Potential

Line Source

$$\underset{\left(\frac{H\cdot A}{m}\right)}{A} = \frac{\mu\,(H/m)}{4\pi}\int_{c} \underset{(A)}{I}\,\frac{e^{-jkR}\,(-)}{\underset{(\cancel{m})}{R}}\,d\ell'\ (\cancel{m})$$

↑ contour (or) Line integral

Surface Source

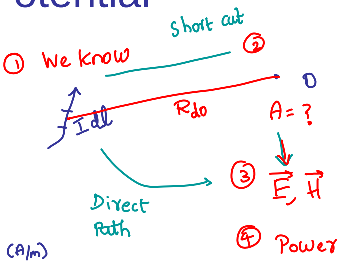
$$A = \frac{\mu}{4\pi}\iint_{s} J_{s}\,\frac{e^{-jkR}}{R}\,ds'$$

↑ surface

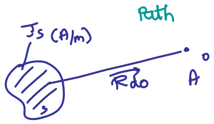
Volume Source

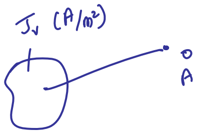
$$A = \frac{\mu}{4\pi}\iiint_{V} J_{v}\,\frac{e^{-jkR}}{R}\,dv'$$

↑ volume



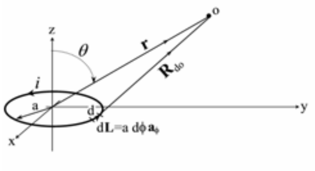
④ Power
- Pattern
- Beam solid angle
- Directivity
- Beam width
- Radiated Power
- Radiation Res.





Ref: C. A. Balanis, *Antenna Theory: Analysis and Design*, John Wiley, 2005.

Small Loop Antenna



A small loop of current located in the x-y plane centered at the origin is shown. Such a small loop is known as a small loop antenna or sometimes a magnetic dipole.

Step 1: Current Density

$$\vec{I}d\ell = I_s a d\phi \mathbf{a}_\phi$$

Step 2: Vector Magnetic Potential

$$\mathbf{A}_{os} = \frac{\mu_o I_s a}{4\pi}\oint \frac{e^{-j\beta R_{do}}}{R_{do}} d\phi \mathbf{a}_\phi$$

We assume the loop is electrically small, or a << λ, and A_{os} is found in the far-field.

Solving the integral remains quite complicated, involving a pair of series expansions and some dexterous coordinate transformations

$$\mathbf{A}_{os} = \frac{\mu_o I_s S}{4\pi r^2}\left(1 + j\beta r\right) e^{-j\beta r} \sin\theta \mathbf{a}_\phi \qquad \text{where} \qquad S = 4\pi r^2$$

Small Loop Antenna

Step 3: Electric and Magnetic Fields

The magnetic field is given by

$$\mathbf{B}_{os} = \nabla \times \mathbf{A}_{os},$$

$H_{os} = \frac{1}{\mu}(\nabla \times A)$

(A/m) $\frac{1}{(\cancel{H}/m)}$ $(\frac{1}{m})$ $(\frac{\cancel{H}A}{\cancel{m}})$

$$\mathbf{H}_{os} = \frac{-\omega\mu_o I_s S\beta}{4\pi\eta_o r}\sin\theta e^{-j\beta r}\mathbf{a}_\theta$$

The electric field is given by

$$\mathbf{E}_{os} = -\eta_o \mathbf{a_r} \times \mathbf{H}_{os}.$$

$$\mathbf{E}_{os} = \frac{\omega\mu_o I_s S\beta}{4\pi r}\sin\theta e^{-j\beta r}\mathbf{a}_\phi$$

Step 4: Antenna Parameters

Power Density:

The power density vector is then

$$\mathbf{P}(r,\theta) = \left(\frac{\omega^2\mu_o^2 I_o^2 S^2 \beta^2}{32\eta_o \pi^2 r^2}\right)\sin^2\theta \mathbf{a_r},$$

S = area of the Loop

where

$$P_{\max} = \frac{\omega^2\mu_o^2 I_o^2 S^2 \beta^2}{32\eta_o \pi^2 r^2},$$

Antenna Pattern Solid Angle:

The normalized power function is the same as for the Hertzian dipole

$$\Omega_p = \frac{8\pi}{3}$$

Directivity:

$$D_{\max} = \frac{4\pi}{\Omega_p} = 1.5$$

Small Loop Antenna

Total Radiated Power and Radiation Resistance :

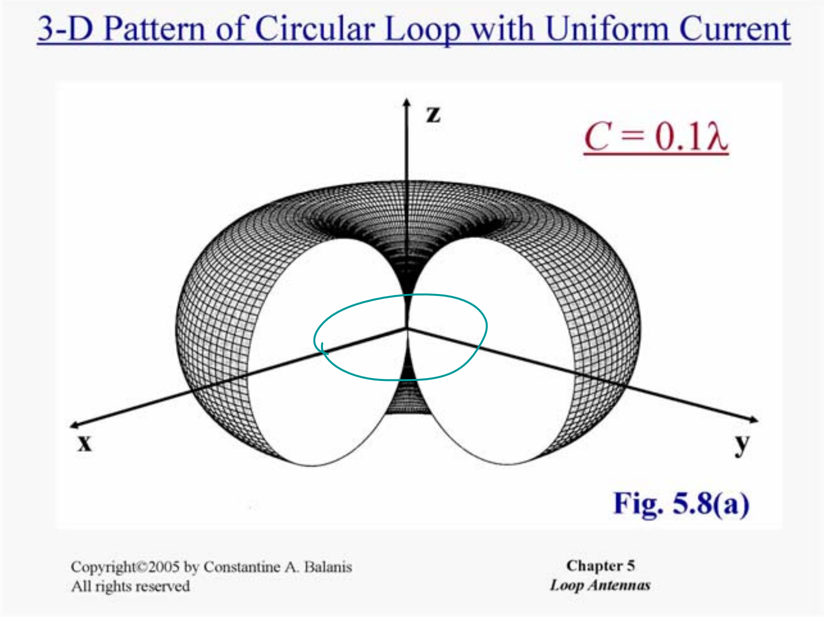
using $\lambda = \frac{c}{f}$

$$P_{rad} = \frac{4\eta_o \pi^3 I_o^2}{3}\left(\frac{S}{\lambda^2}\right)^2 \qquad R_{rad} = 320\pi^4\left(\frac{S}{\lambda^2}\right)^2 \Omega = 320\pi^4\left(\frac{Sf^2}{c^2}\right)^2$$

The fields for the small loop antenna are very similar to that of a Hertzian dipole. Since it is the dual for the Hertzian (electric) dipole, the small loop antenna is often called a magnetic dipole.

The magnetic dipole equations are also valid for a multiturn loop, so long as the loops remain small compared to wavelength. For an N-loop coil, $S = N\pi a^2$ in the above equations. The loops are not required to be circular. To use the equations for a square coil of N loops, each of side length b, $S = Nb^2$.

Increasing the diameter of the loop antenna results in an increase in the radiation resistance, and hence the efficiency. Wrapping the loops around a ferrite core (i.e. a ferrite-loop antenna) is a way of concentrating magnetic flux in the loops and making them appear larger. This is a common approach for constructing compact receiving antennas for AM radio.



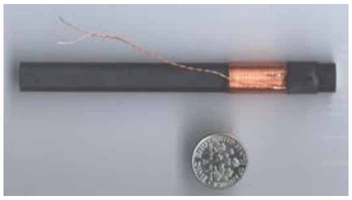
3-D Pattern of Circular Loop with Uniform Current

Fig. 5.8(a)

Chapter 5
Loop Antennas

Practical Example

Small Loop Antenna



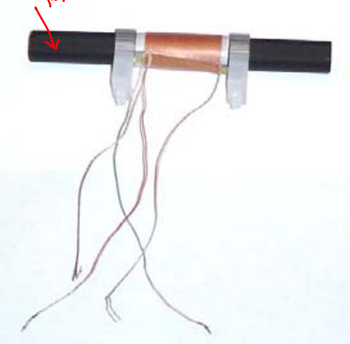
Part No	Length	Thickness	Inductance	Q	Number of Turns	Price
VLC139	3.89 in	0.35 inch	1.5 mH	110	139	$ 6.95

A precision VLF antenna coil. Operates from 1 KHz to 60 KHz. External capacitors allow tuning. If you add an 0.1 uF capacitor it will tune 13 KHz. If a 0.3 uF capacitor is used then the coil will tune 8 KHz. Will work down to 1000 Hz as a general audio pickup coil. Can hear tweeks. If your receiver can pull in tweeks loudly with this unit, then you've got a very good receiver! Will go lower if you add more turns. A nice 3.89 inch ferrite rod wound with 139 wire turns and a small capacitor prewired to the coil, upper frequency limit is 60 KHz unless you clip the onboard capacitor.

http://www.stormwise.com/page27.htm

Practical Example

Small Loop Antenna



$\mu_r = 125$

This AM / LW loopstick antenna is for tuning between 100 KHz - 1710 KHz. When used with our AM variable capacitor, above, this AM core will the full AM band and down to about 310 KHz. This tuning range is with the coil centered as in the photo.

Specifications:

Size: 3.5 inches long by 0.25 inches diameter.

Coil inductance value (centered on rod) is: 788 uH.

Permeability value: Est.125.

Description:

The unit contains two coils. One coil (the green and red) are the smaller coil. The black and the white are the longer coil. The coils are isolated from each other, but can be connected make one continuous coil having a tap. The above tuning results were tested only with the white and black wires used (the larger coil side).

The red and green wires are isolated from larger section and can be used to couple an outdoor long-wire antenna.

A very useful part!

AM-LW BAR-1 is $ 4.95 each + s/h.

ships US MAIL ONLY. ADD $ 4.00 s/h.

http://www.stormwise.com/page27.htm

This precision brand-new Air Variable Capacitor has a range of 15 pF to 384 pF.

This capacitor is useful for receiver and antenna tuning in the longwave, AM, and shortwave bands from
50 KHz to 10 MHz.

10 MHz 50 kHz

Most capacitors of this type go from 20 pF to 365 pF. Our unit will go up to 384 pF. This gives you a little extra room to center the band or tune just slightly lower.

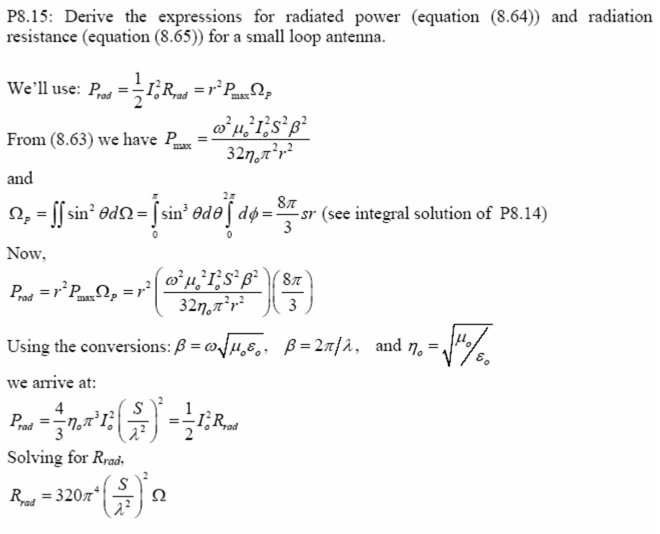
Use this capacitor to tune your ferrite rod antenna. Peak up the signal to your longwave radio by many S-units with this low loss capacitor! Improve your reception with the sensitivity that only an Air Variable Capacitor can provide!

Air Variable Capacitors offer sharp, narrow-band tuning. Get an Air Variable Capacitor to get the highest "Q" factor in your antenna!

Size (inches): 1.38 inches wide by 1.31 inches deep by 1.19 high. Built-in frame ground lug for easy connection. 50V rms max.

Stormwise Part # AMVC384 is $ 18.00 each + $ 8.00 s/h.

Example



P8.15: Derive the expressions for radiated power (equation (8.64)) and radiation resistance (equation (8.65)) for a small loop antenna.

We'll use: $P_{rad} = \frac{1}{2} I_o^2 R_{rad} = r^2 P_{\max} \Omega_P$

From (8.63) we have $P_{\max} = \dfrac{\omega^2 \mu_o{}^2 I_o^2 S^2 \beta^2}{32 \eta_o \pi^2 r^2}$

and

$$\Omega_P = \iint \sin^2\theta \, d\Omega = \int_0^{\pi} \sin^3\theta \, d\theta \int_0^{2\pi} d\phi = \frac{8\pi}{3} sr \text{ (see integral solution of P8.14)}$$

Now,

$$P_{rad} = r^2 P_{\max} \Omega_P = r^2 \left(\frac{\omega^2 \mu_o{}^2 I_o^2 S^2 \beta^2}{32 \eta_o \pi^2 r^2} \right) \left(\frac{8\pi}{3} \right)$$

Using the conversions: $\beta = \omega \sqrt{\mu_o \varepsilon_o}$, $\beta = 2\pi / \lambda$, and $\eta_o = \sqrt{\mu_o / \varepsilon_o}$

we arrive at:

$$P_{rad} = \frac{4}{3} \eta_o \pi^3 I_o^2 \left(\frac{S}{\lambda^2} \right)^2 = \frac{1}{2} I_o^2 R_{rad}$$

Solving for R_{rad},

$$R_{rad} = 320 \pi^4 \left(\frac{S}{\lambda^2} \right)^2 \Omega$$