



Sketching Bode Plots by Hand

MECH 3140
Lecture #

Recall what a bode plot is

- It is the particular solution to a LTI differential equation for a sinusoidal input
- It shows how the output of a system will respond to different input frequencies (including constant inputs, i.e. $\omega=0$)
- It is plotted as gain and phase where gain is the $\frac{|output|}{|input|}$ and phase is the phase difference between the input sinusoid and output sinusoid

Why know how to sketch?

- Matlab can plot a bode plot and calculate the gain and phase from any transfer function
- So why do we need to know how to sketch (approximately draw) one?
 - Its not to be a pain
 - Its not because we don't like matlab
- By understanding how to sketch a bode plot we can do two things as systems engineer where we might not have the transfer function to begin with
 - Design
 - If I am designing a system I don't have a transfer function to put into matlab – I am building the transfer function
 - By understanding how to sketch a bode plot, I understand how to make a system with a given response
 - Analysis
 - I might just be given data to look at and need to understand what the system might be (i.e. be able to back out the transfer function from frequency response)
 - I might be analyzing a specific phenomena and by understanding how to sketch a bode plot I can understand what is happening or how to fix/change the phenomena.

Bode Sketching Rules

- You will notice that these sketching rules essentially come from the gain and phase calculations
 - The rules for 1st and 2nd order systems are “extended” by understanding how gains multiply and phases add.
 - From these relationships you will notice that the rule for a pole is inverted (or flipped) for the rule from a zero
 - Gain and phase of the numerator vs. denominator

$$G(\omega) = |H(j\omega)| = \frac{|num(j\omega)|}{|den(j\omega)|} = \frac{|a(j\omega)|}{|b(j\omega)|} \cdot \frac{|c(j\omega)|}{|d(j\omega)|}$$

$$\Phi(\omega) = \angle H(j\omega) = \angle num(j\omega) - \angle den(j\omega) = \angle a(j\omega) - \angle b(j\omega) + \angle c(j\omega) - \angle d(j\omega)$$

Bode Sketching Rules

- Draw your vertical and horizontal axis
 - Label horizontal axis ω (rad/s)
 - Label the top vertical axis Gain
 - Label the bottom vertical axis Phase (or ϕ)
- Mark the magnitude of all zeros and poles on the frequency axis
 - It is sometimes helpful to draw the poles and zeros on the s-plane (or at least calculate their magnitudes and damping ratios if they are under-damped)

Bode Sketching Rules (Gain)

- Start by sketching the asymptotes:
 - Start from the DC Gain
 - If the DC gain is 0 or ∞ then start with a line of correct slope (depending on the number of zeros or poles at the origin)
 - Move in frequency until you hit the frequency (in magnitude) of a pole or zero
 - This is the absolute value of the pole or zero if it is real
 - This is the ω_n of the pole or zero if it is complex
 - The slope then changes +1 for every zero and -1 for every pole
 - Note this means the slope would change +2 for a complex zero or -2 for a complex pole.

Bode Sketching Rules (Gain)

- Label the slope of the asymptotes
- Label the gain at DC and at the corners
 - Use your slopes and start from the DC gain
 - This requires you understand logarithmic plots (and their “slopes”)
 - $G_2/G_1 = \left(\frac{\omega_2}{\omega_1}\right)^{slope} \quad \therefore G_2 = G_1 \left(\frac{\omega_2}{\omega_1}\right)^{slope}$
 - If the DC Gain is 0 or ∞ , then you must select another portion of the graph to start your gain from
 - Generally this means calculating the gain from the transfer function for a particular frequency to use as your starting point

Bode Sketching Rules (Gain)

- Calculate (and label) the gain at the corners
 - Real Pole: $0.707G_c$
 - Real Zero: $1.414G_c$
 - Complex Poles: $\frac{G_c}{2\zeta}$
 - Complex Zeros: $2\zeta G_c$
- Draw the Gain line

Bode Sketching Rules (Phase)

- Start with the phase at $\omega=0$
 - This will be zero unless there is a pole or zero at the origin
 - In which case the phase will be
 - $90 \cdot z$ (where z is the number of zeros at the origin)
 - $-90 \cdot p$ (where p is the number of poles at the origin)
 - The phase changes $+90/\text{zero}$ and $-90/\text{pole}$ at the magnitude of the zero or pole
 - Half of the phase change ($+45/\text{zero}$, $-45/\text{pole}$) occurs at the magnitude of the eigenvalue

Comments

- Note that these sketching rules produce very rough ideas for the bode plot (or the frequency response)
 - But they provide insight into design and analysis.

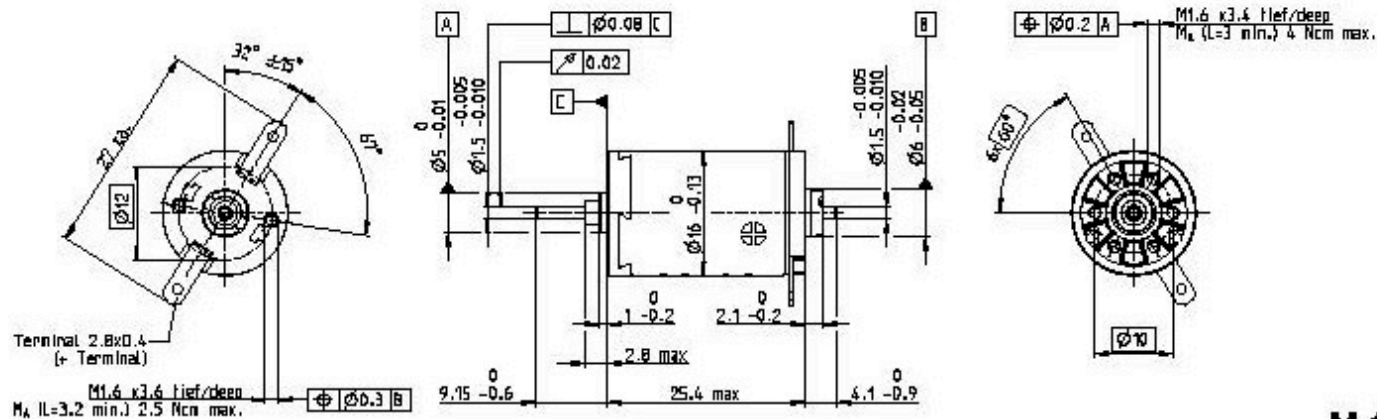
Maxon DC16 Spec Sheet



maxon DC-max

DC-max 16 S Precious Metal Brushes DC motor $\varnothing 16$ mm

Key Data: 2/4.3 W, 4.1 mNm, 11 000 rpm



M 1:1

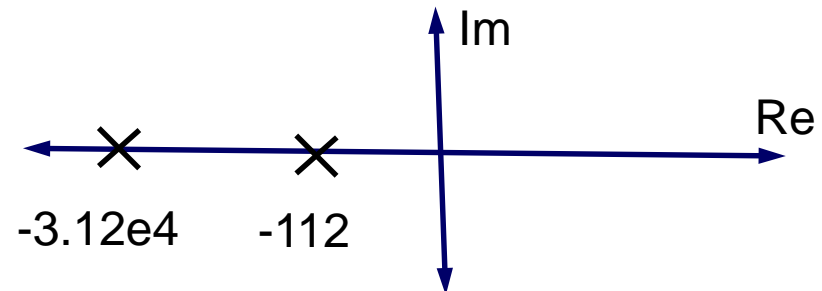
Motor Data

1_ Nominal voltage	V	6	12	24
2_ No load speed	rpm	7890	7560	7470
3_ No load current	mA	14.7	6.90	3.40
4_ Nominal speed	rpm	4830	4390	4210
5_ Nominal torque (max. continuous torque)	mNm	4.06	3.92	3.80
6_ Nominal current (max. continuous current)	A	0.577	0.267	0.128
7_ Stall torque	mNm	10.5	9.44	8.75
8_ Stall current	A	1.46	0.629	0.289
9_ Max. efficiency	%	81	80	80
10_ Terminal resistance	Ω	4.10	19.1	83.2
11_ Terminal inductance	mH	0.140	0.610	2.49
12_ Torque constant	mNm/A	7.19	15.0	30.3
13_ Speed constant	rpm/V	1330	637	315
14_ Speed/torque gradient	rpm/mNm	758	809	864
15_ Mechanical time constant	ms	8.87	8.92	9.00
16_ Rotor inertia	gcm ²	1.12	1.05	0.994

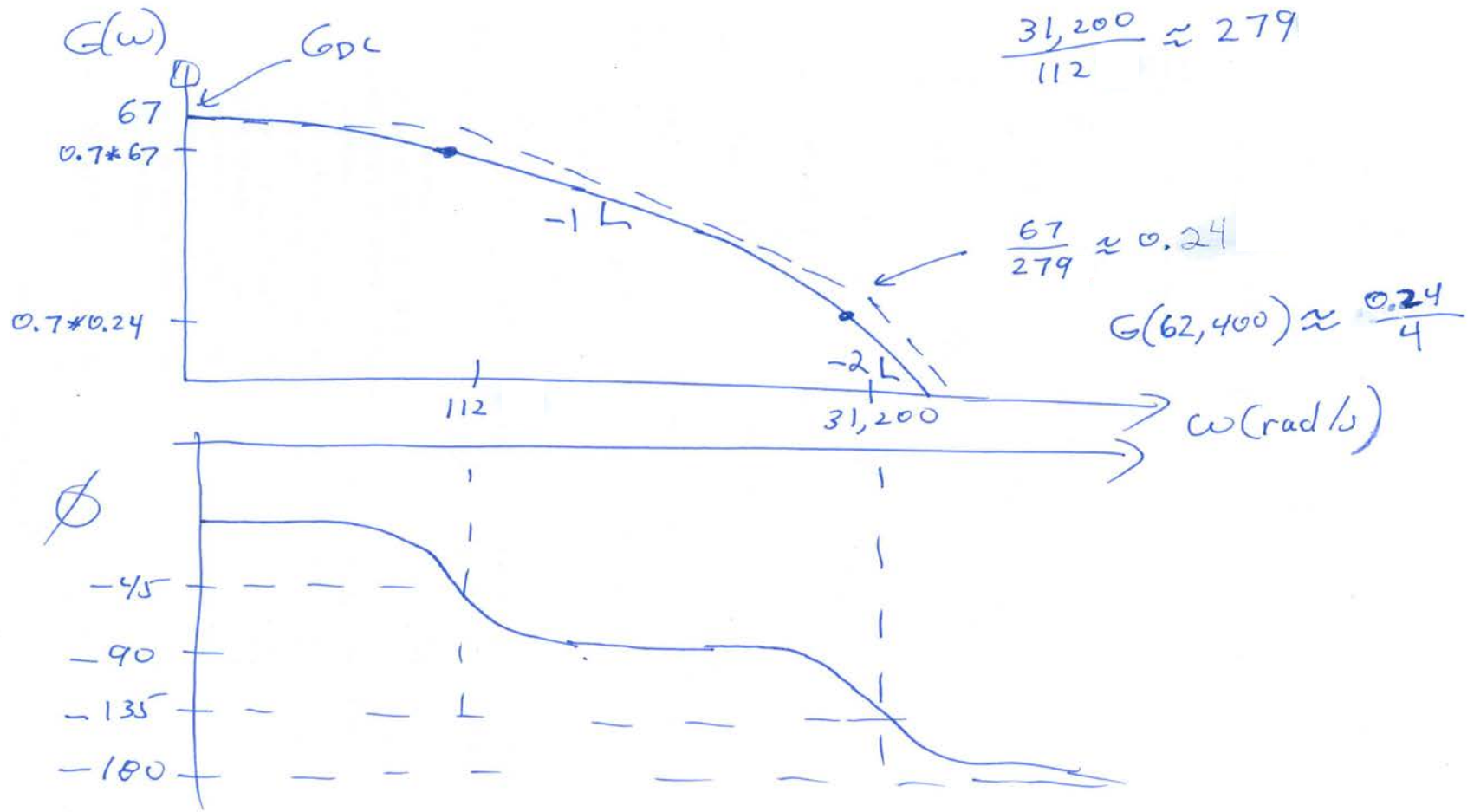
Example #1 (12 V Maxon DC 16 Motor)

$$\frac{\omega(s)}{V_s(s)} = \frac{K_I}{LJs^2 + (JR + Lb)s + (K_I K_b + Rb)} = \frac{0.015}{6.405e^{-11}s^2 + 2.006e^{-6}s + 2.249e^{-4}}$$

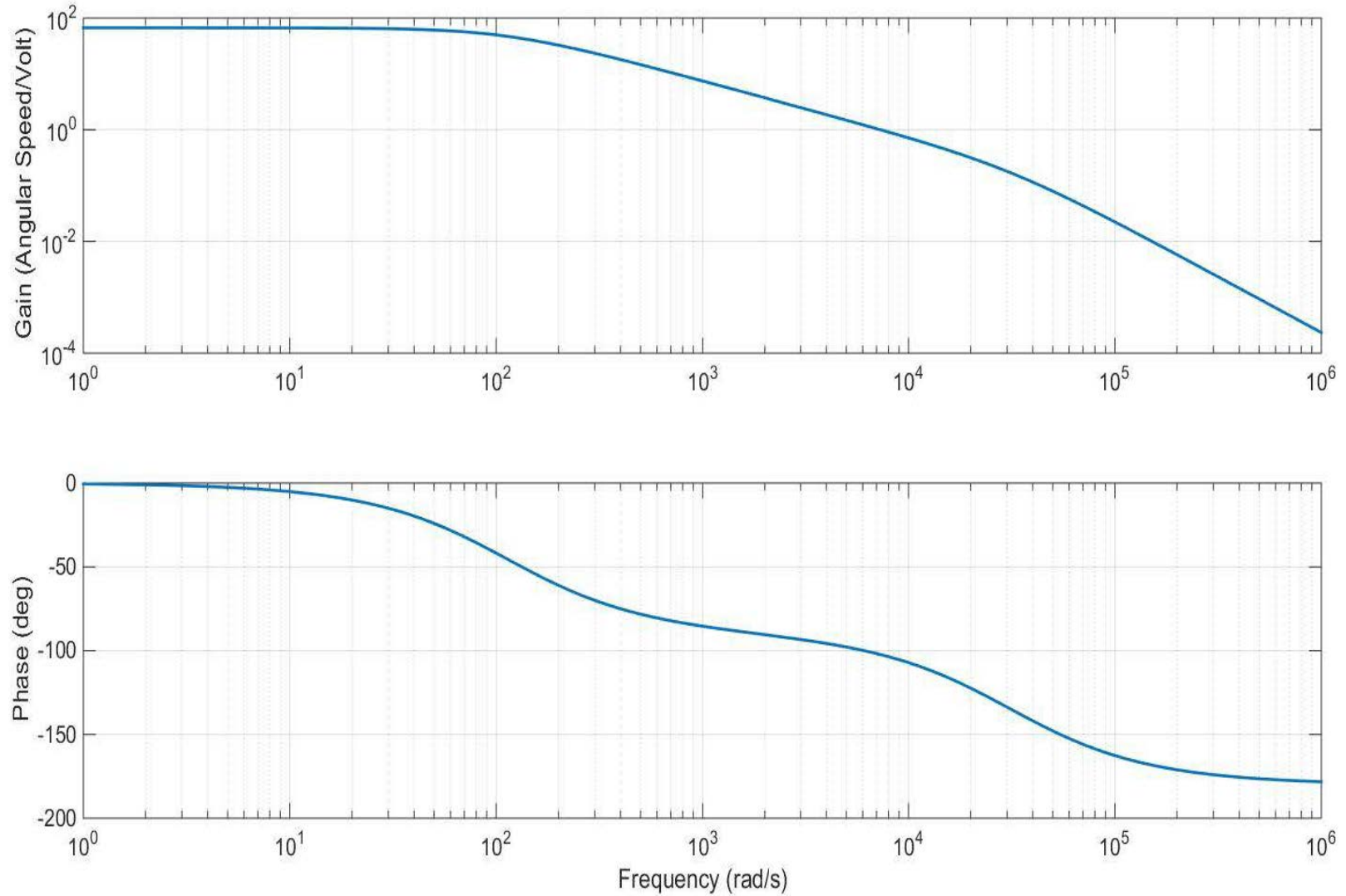
- Two eigenvalues: $S = -3.12e4, -112$
- $\tau_1 = 0.032$ ms, $\tau_2 = 8.9$ ms
- Note the dominant time constant is the “mechanical” time constant on the data sheet.
- $G_{DC} = 66.7$ rad/sec/Volt (this matches the no load speed on the data sheet).



Hand Sketch



>> bode(Ki,[J*L J*R+L*b Ki*Kb+R*b])

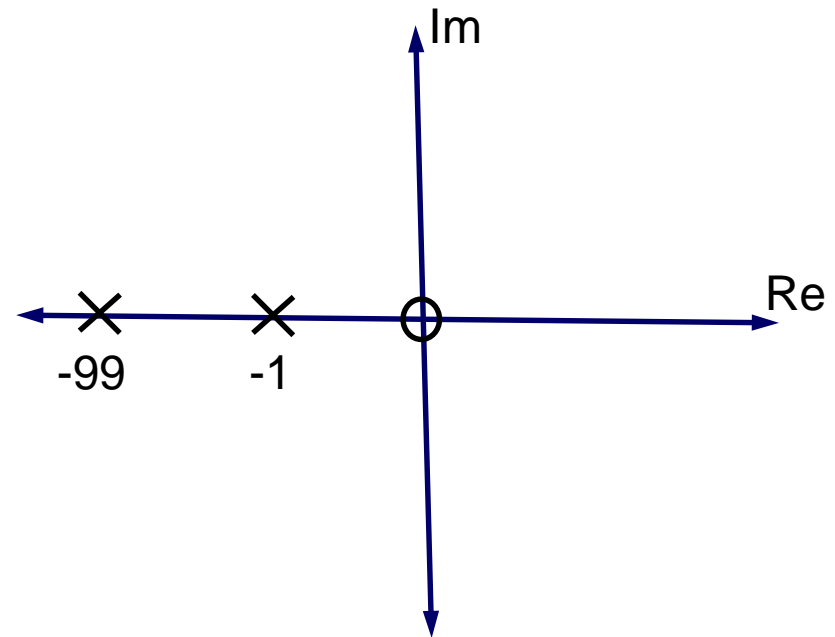


Example #2 (LRC Circuit)

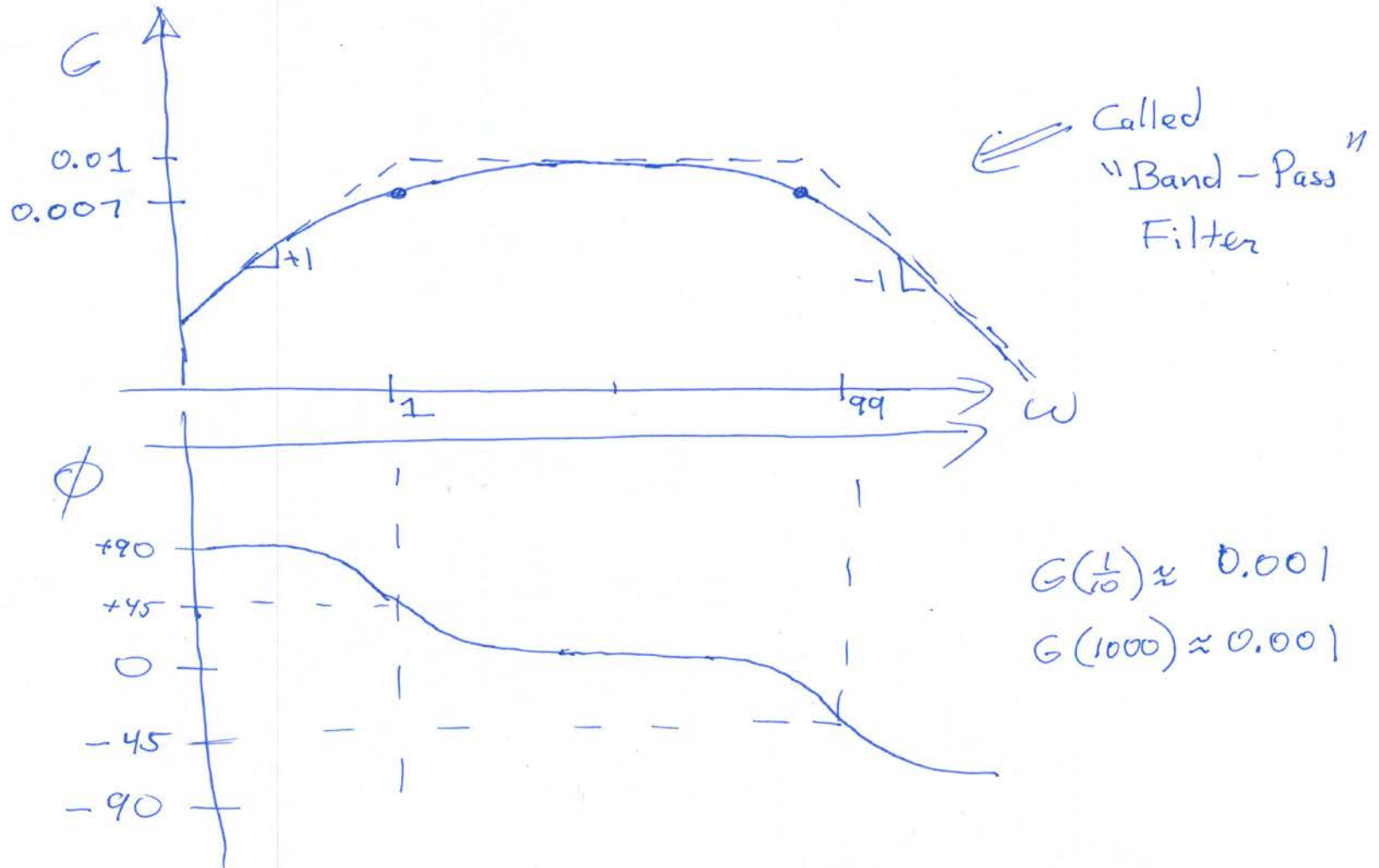
$$\frac{I(s)}{V(s)} = \frac{Cs}{LCs^2 + RCs + 1} = \frac{0.01s}{0.01s^2 + s + 1}$$

L=1 H
R=100 Ω
C=10 mF

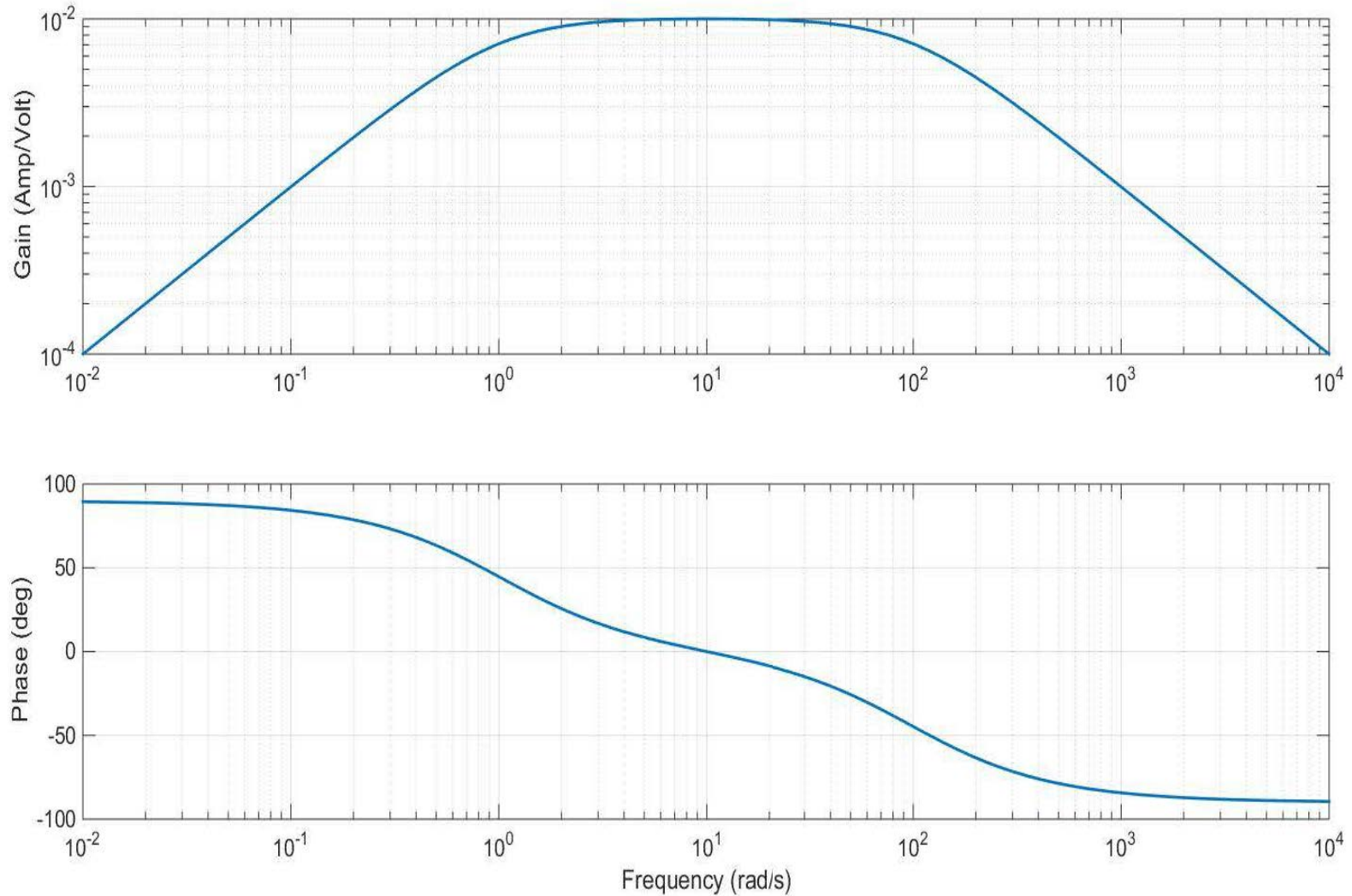
$$G(10) = \frac{|0.1j|}{|10j + (1 - 1)|} = 0.01$$



Hand Sketch



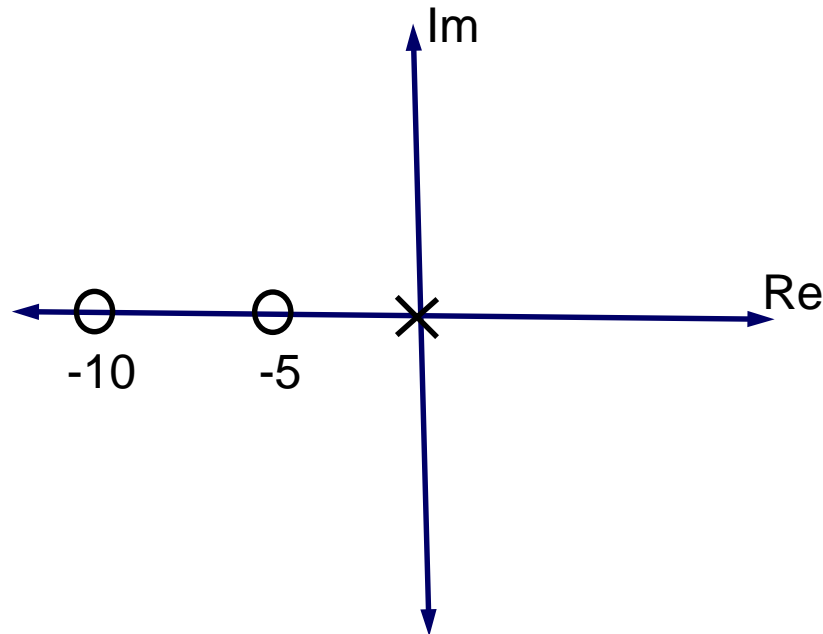
>> bode([C 0],[L*C R*C 1])



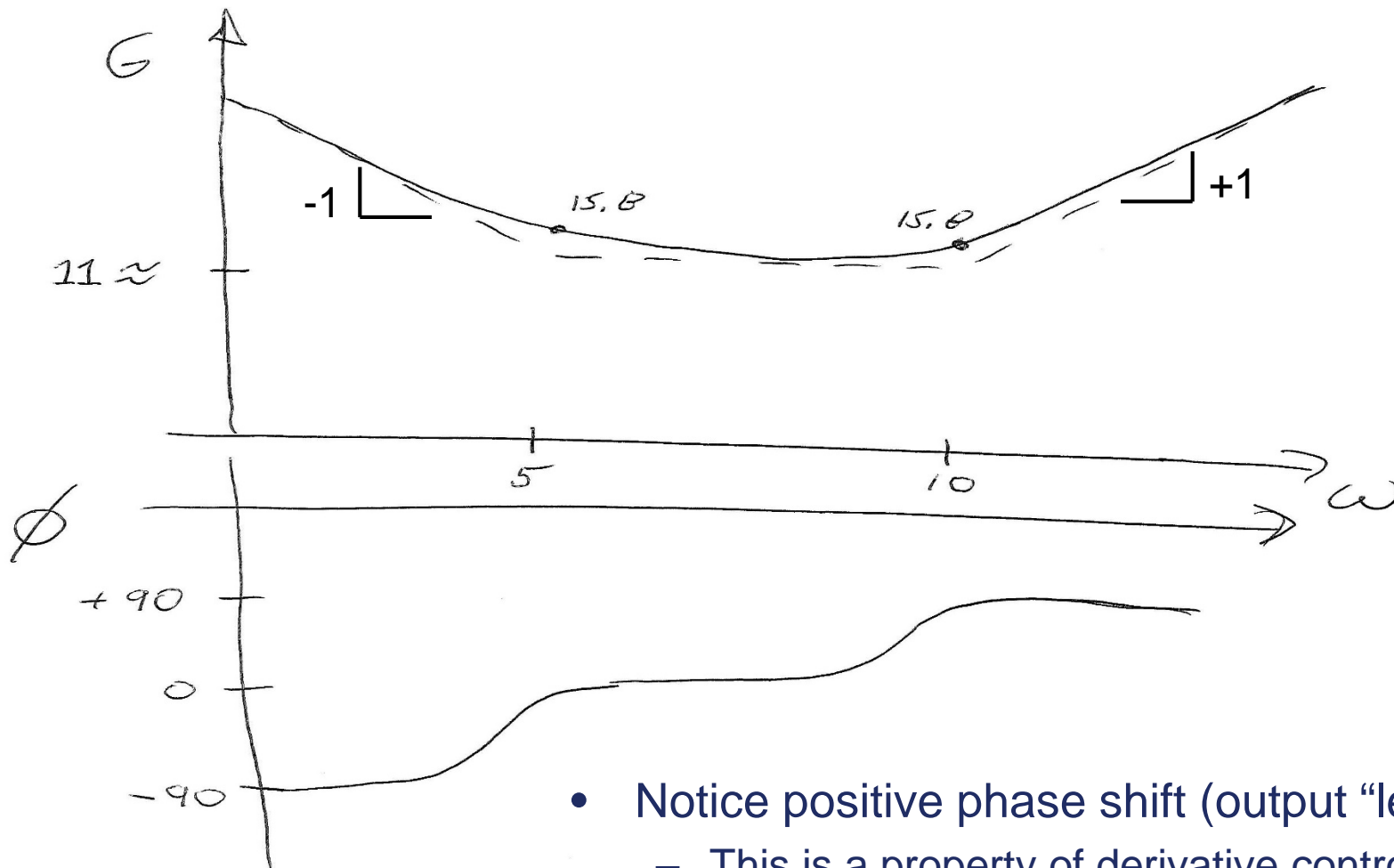
Example #3 (PID Controller)

$$F(t) = K_p e + K_d \dot{e} + K_I \int e dt$$

$$\frac{F(s)}{e(s)} = K_p + K_d s + \frac{K_I}{s} e = \frac{K_d s^2 + K_p s + K_I}{s} = \frac{s^2 + 15s + 50}{s}$$

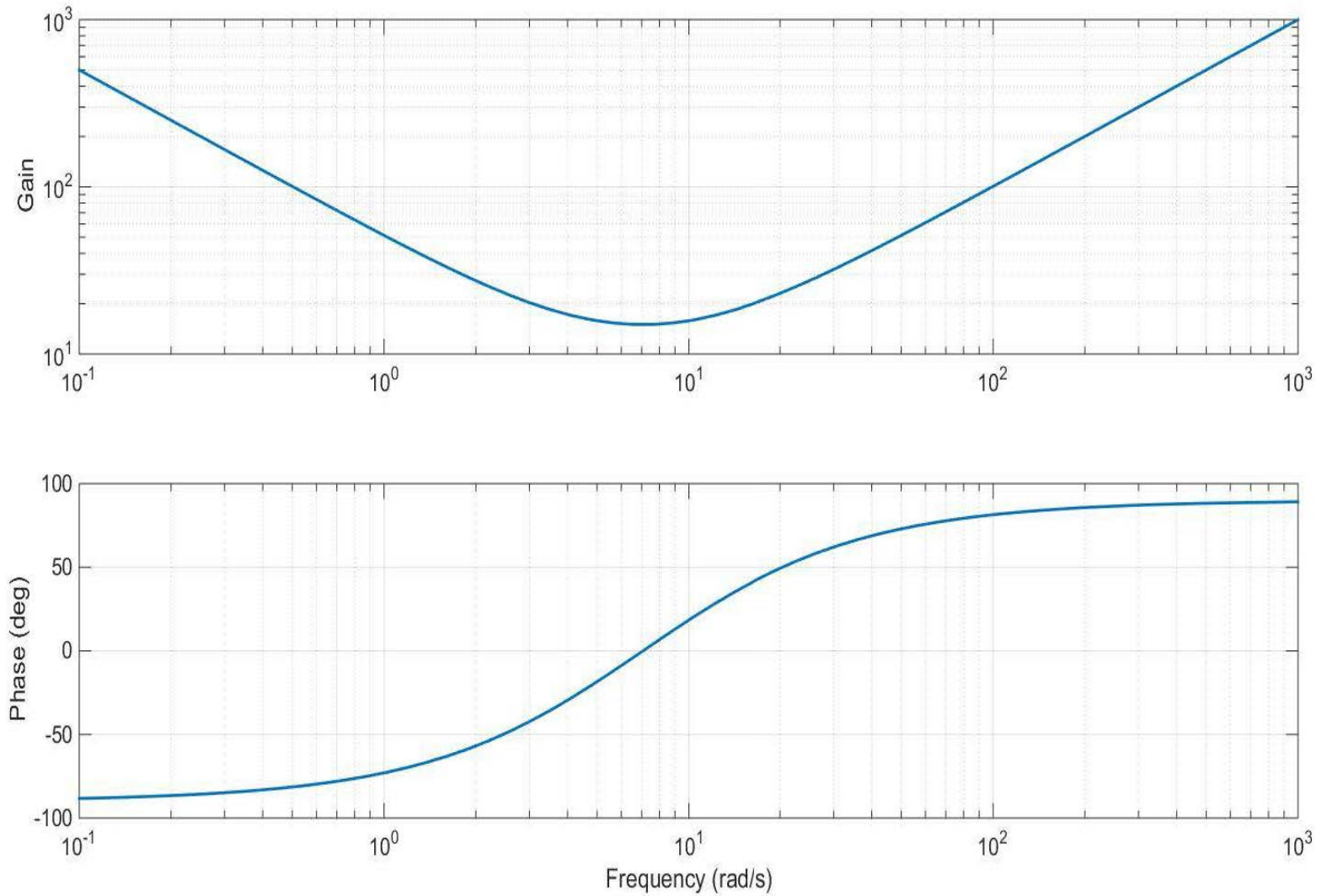


Hand Sketch



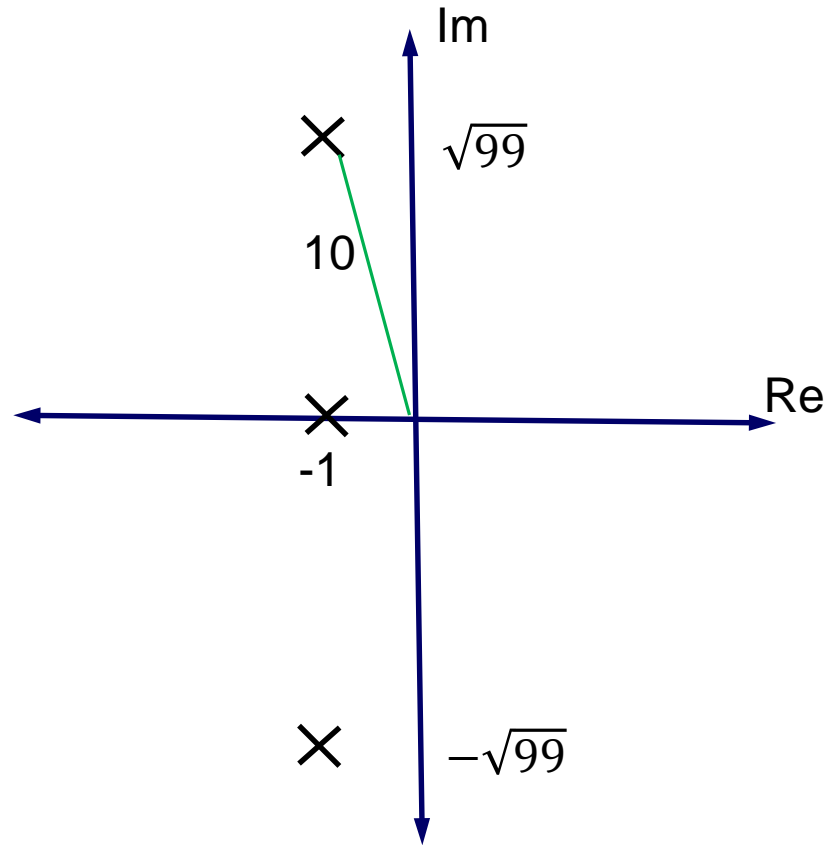
- Notice positive phase shift (output “leads” input)
 - This is a property of derivative control
 - It “anticipates” the output by using the derivative
 - However, notice the increasing gain
 - Amplifies noise!

```
>> bode([1 15 50],[1 0])
```

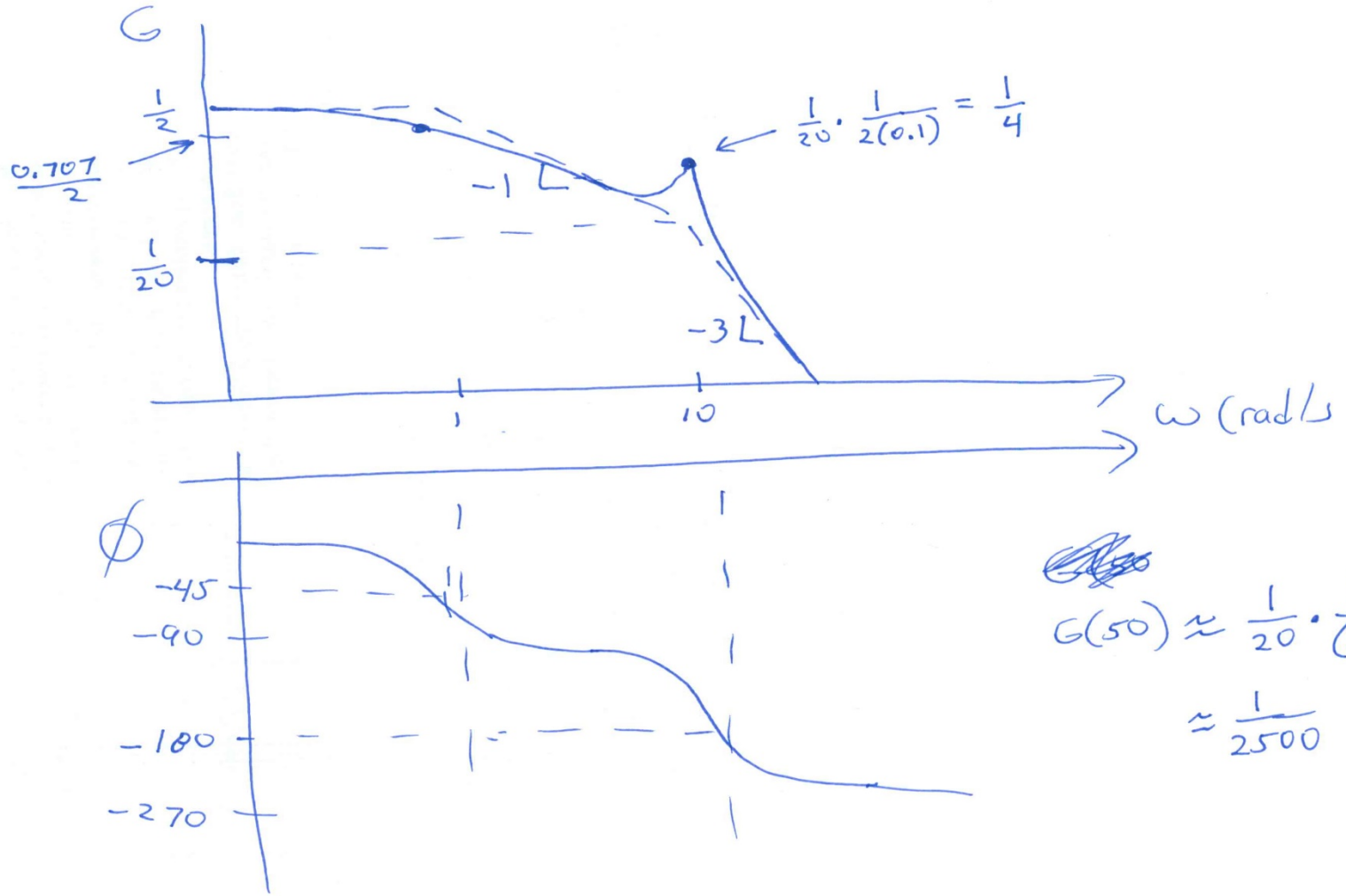


Example #4 (Random System)

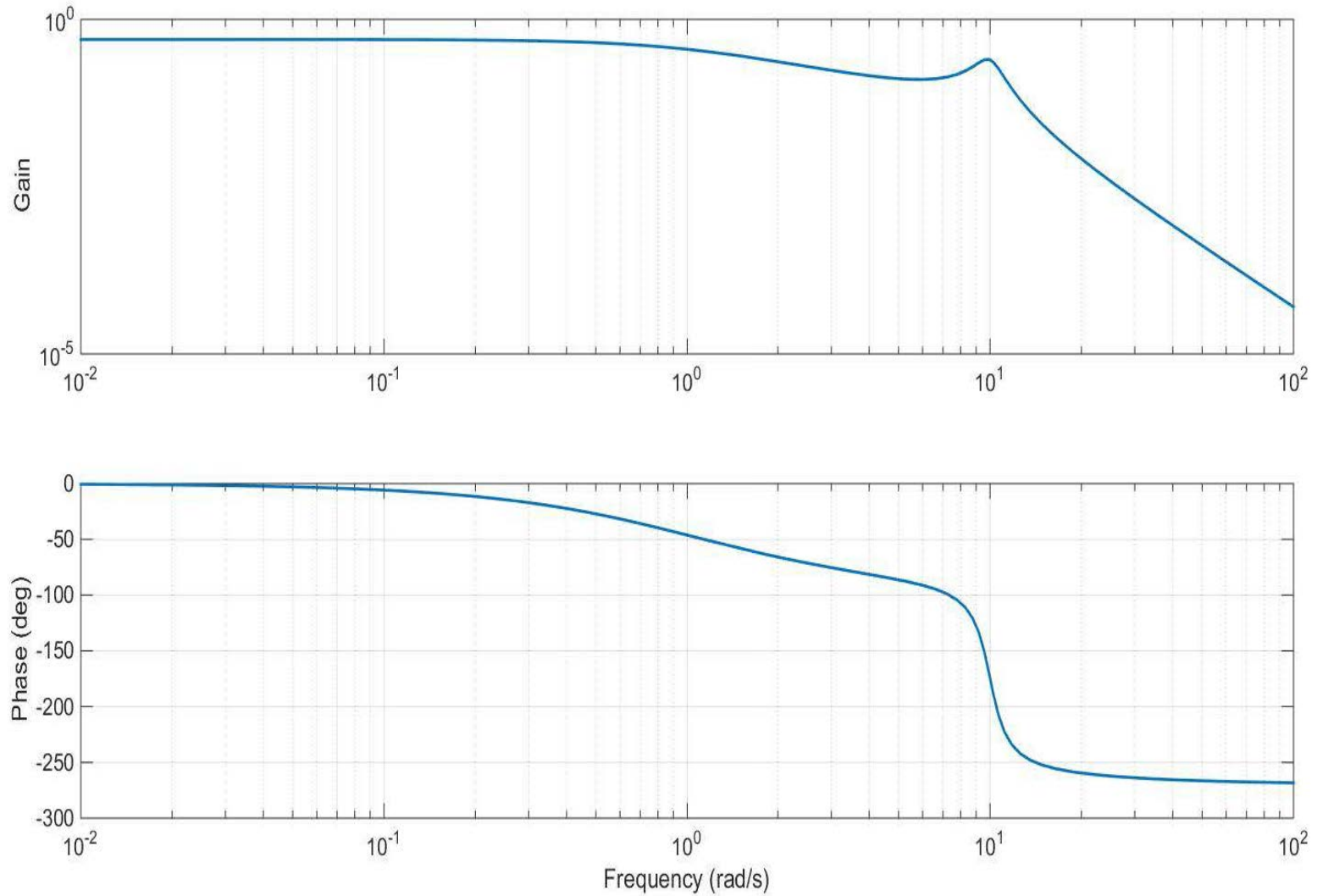
$$H(s) = \frac{50}{(s^2 + 2s + 100)(s + 1)} = \frac{50}{s^3 + 3s^2 + 102s + 100}$$



Hand Sketch

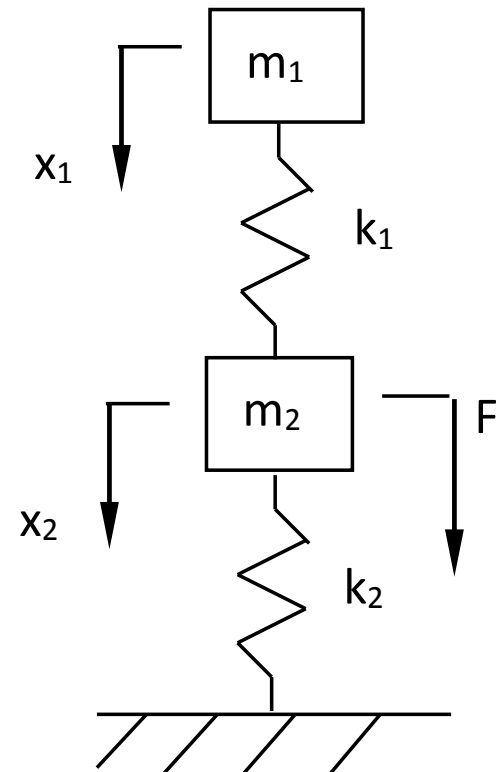


```
>>bode(50,[1 3 102 100])
```



Example #5 (1/4 Car Model)

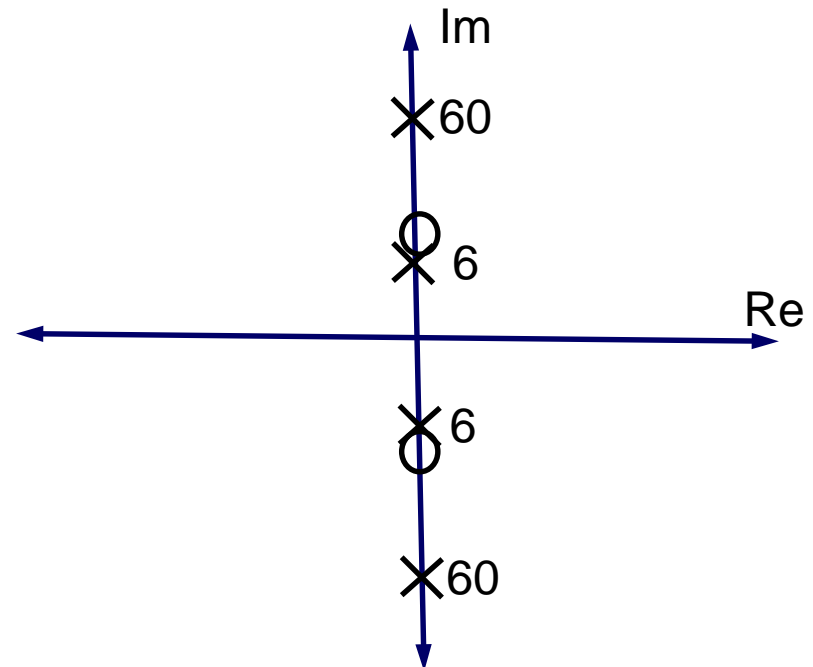
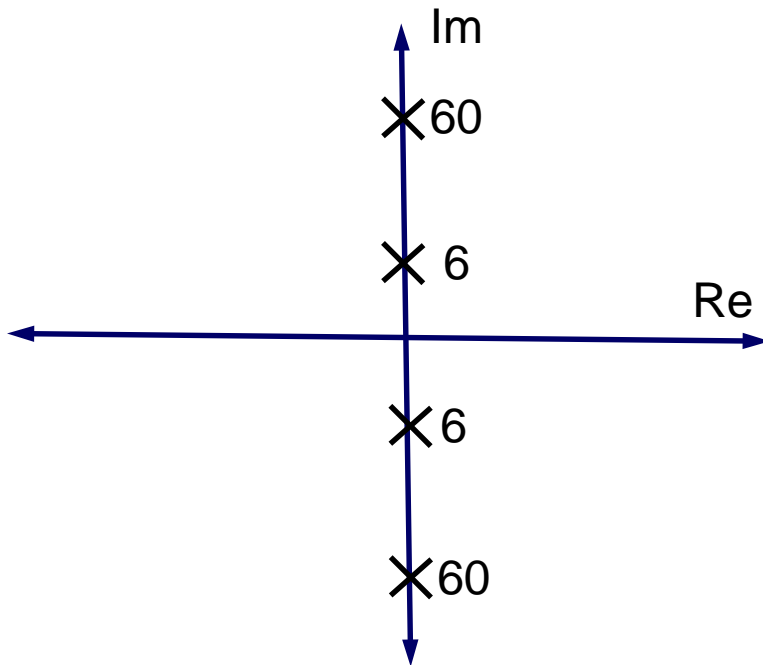
- Used to model vehicle ride (vertical motion only)
- For this example, we will ignore the damping



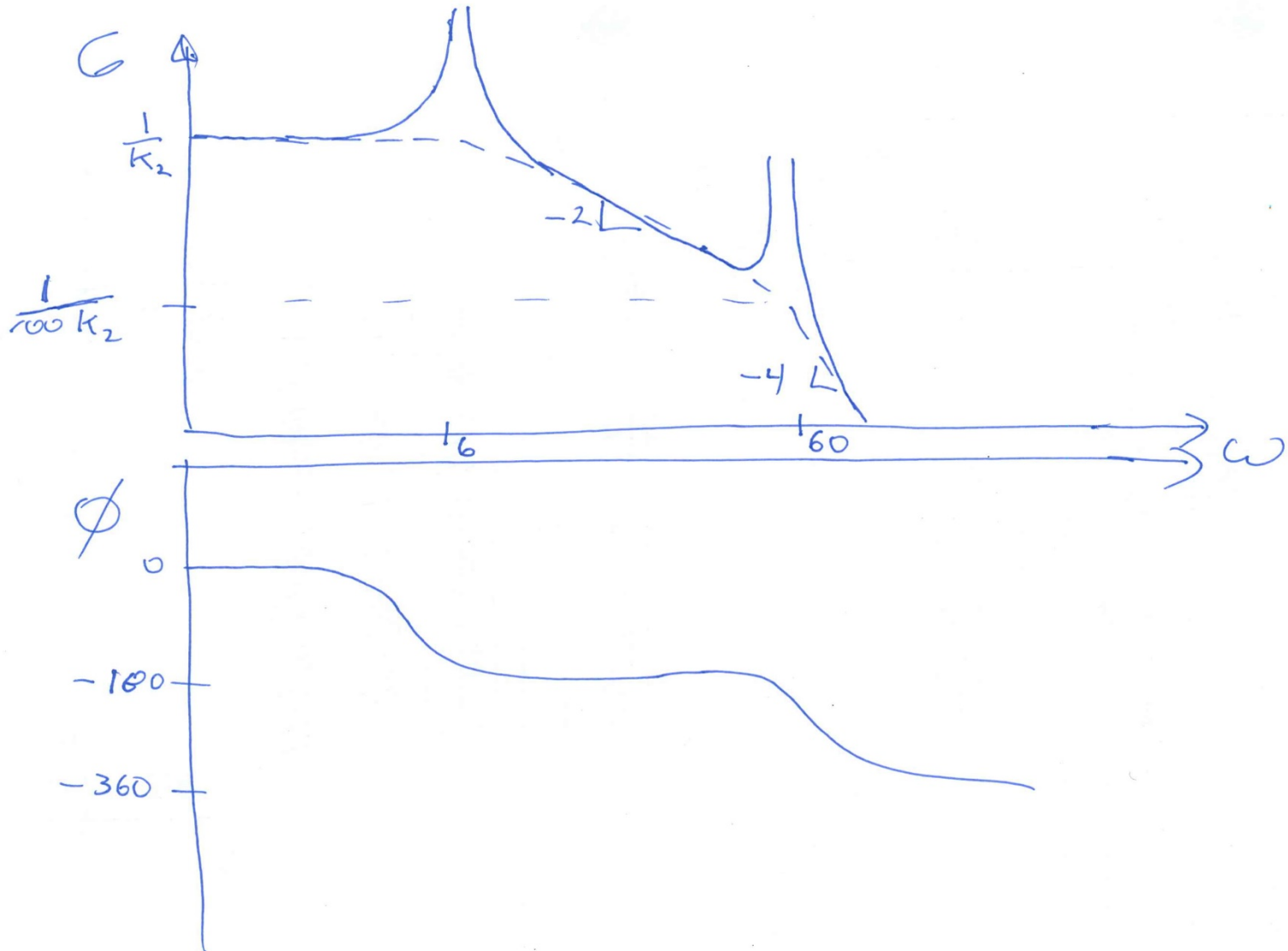
¼ Car Transfer Functions

$$\frac{x_1(s)}{F(s)} = \frac{k_1}{m_1 m_2 s^4 + (m_1 k_1 + m_1 k_2 + m_2 k_2) s^2 + k_1 k_2}$$

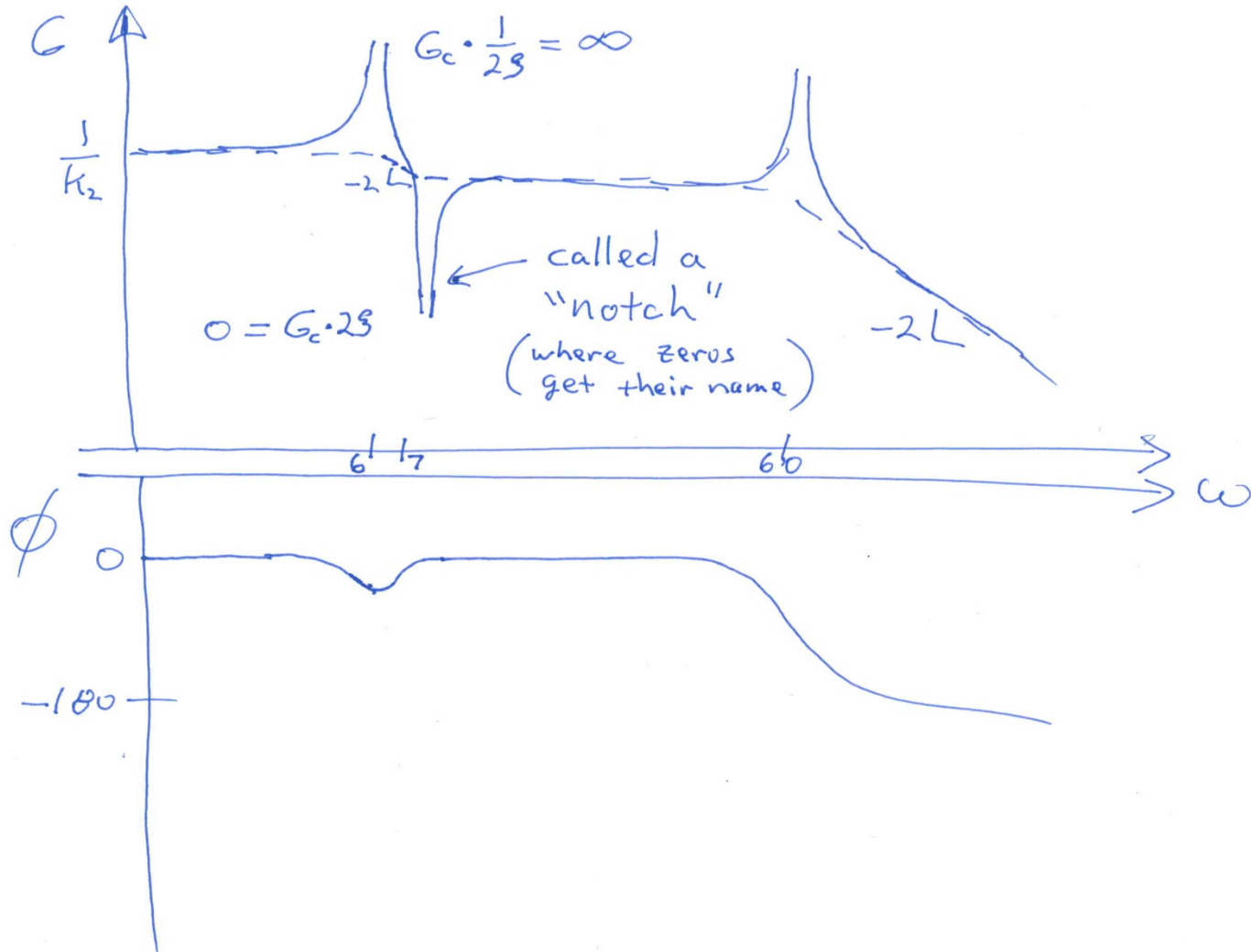
$$\frac{x_2(s)}{F(s)} = \frac{m_1 s^2 + k_1}{m_1 m_2 s^4 + (m_1 k_1 + m_1 k_2 + m_2 k_2) s^2 + k_1 k_2}$$



X_1 Hand Bode Sketch (Car)

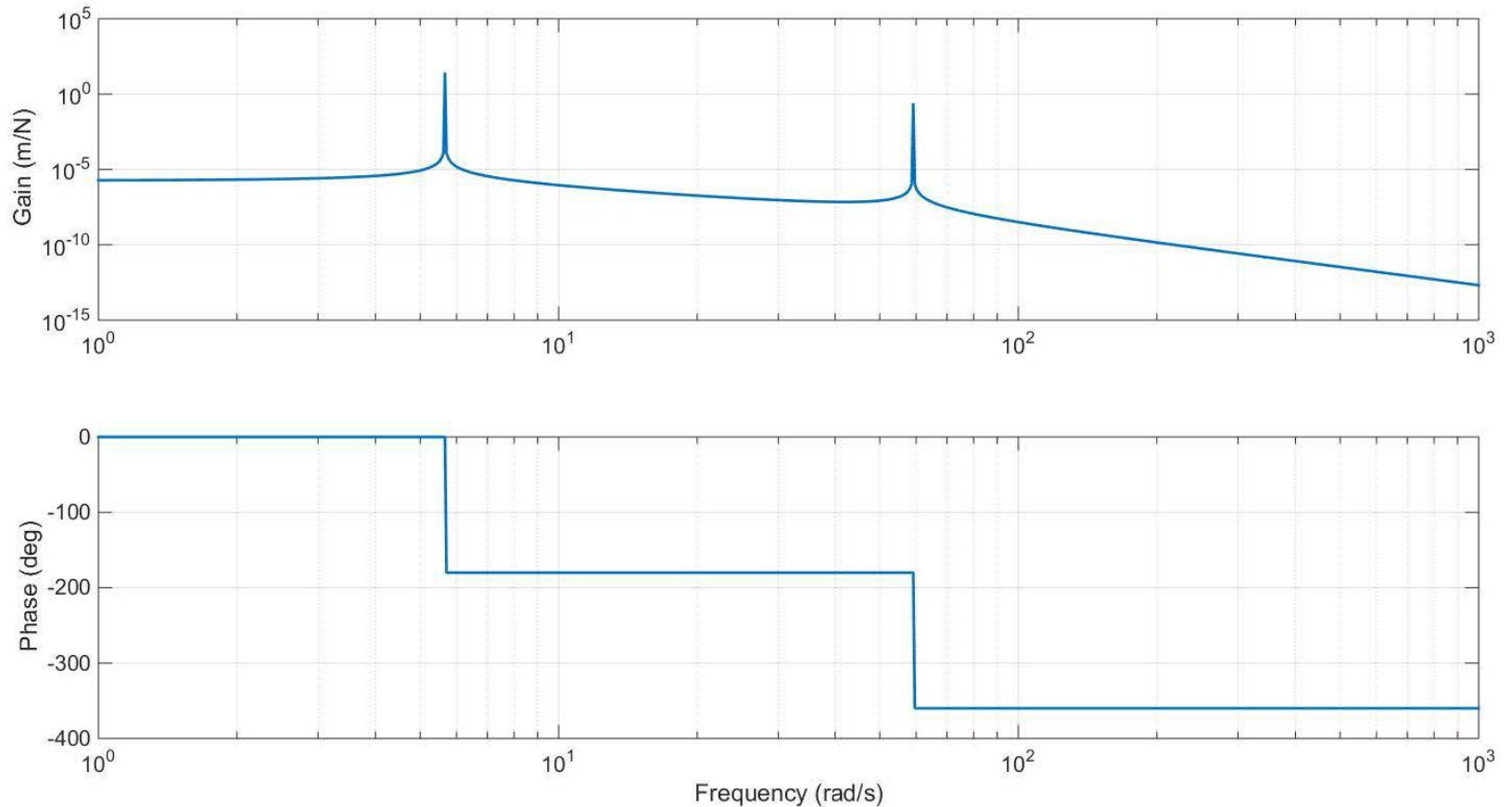


X_2 Hand Bode Sketch (Wheel)



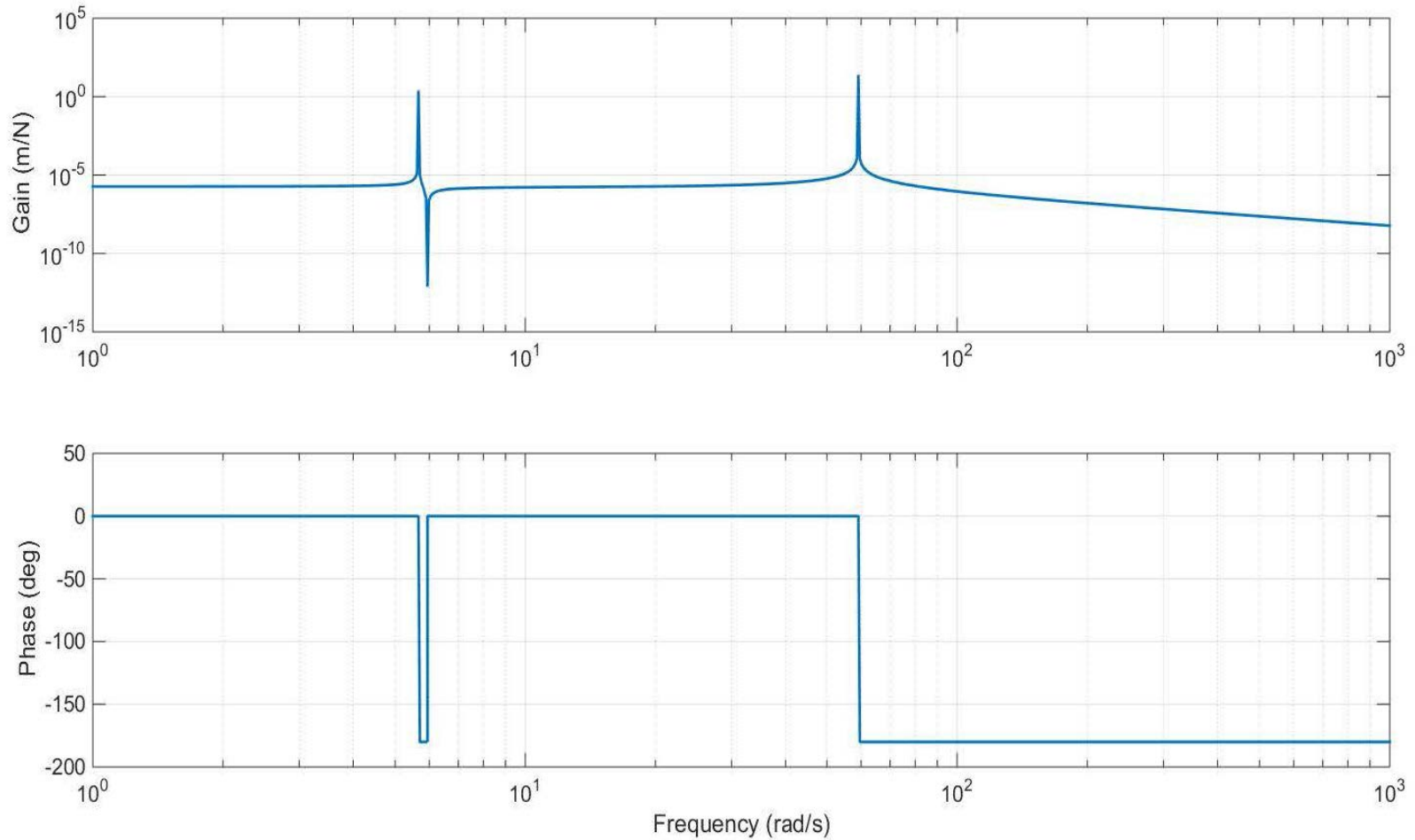
X_1 Bode Plot (Car)

```
>>bode(k1,[m1*m2 0 m1*(k1+k2)+m2*k2 0 k1*k2])
```



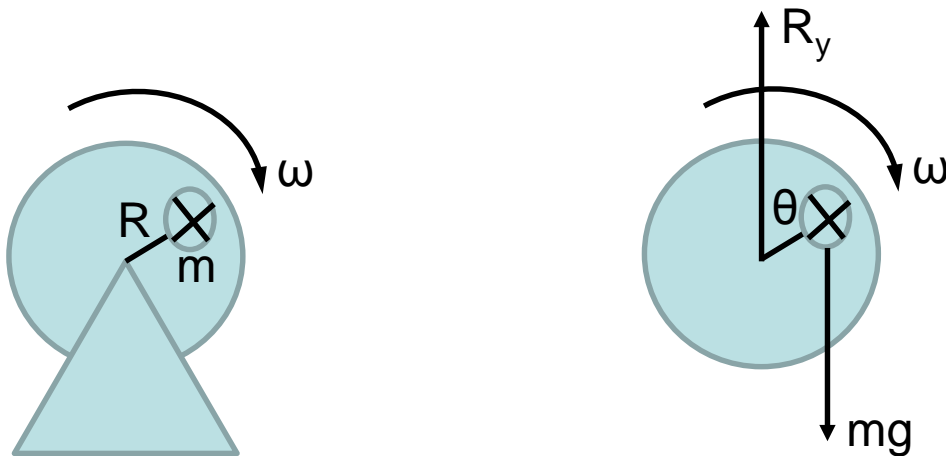
X_2 Bode Plot (Wheel)

```
>>bode([m1 0 k1],[m1*m2 0 m1*(k1+k2)+m2*k2 0 k1*k2])
```



Effect of Wheel Imbalance

- What is the force acting on the wheel due to a wheel imbalance
 - Same as the reaction forces on the pendulum you did in a prior Matlab homework



$$\sum F_y = m\ddot{y} = R_y - mg$$

$$\ddot{y} = R\ddot{\theta} \sin(\theta) - R\dot{\theta}^2 \cos(\theta)$$

If $\omega = \text{const}$

$$\ddot{y} = -R\dot{\theta}^2 \cos(\theta)$$

$$\theta = \omega t + \theta_0$$

$$R_y = mg - mR\omega^2 \sin(\omega t + \theta_0)$$

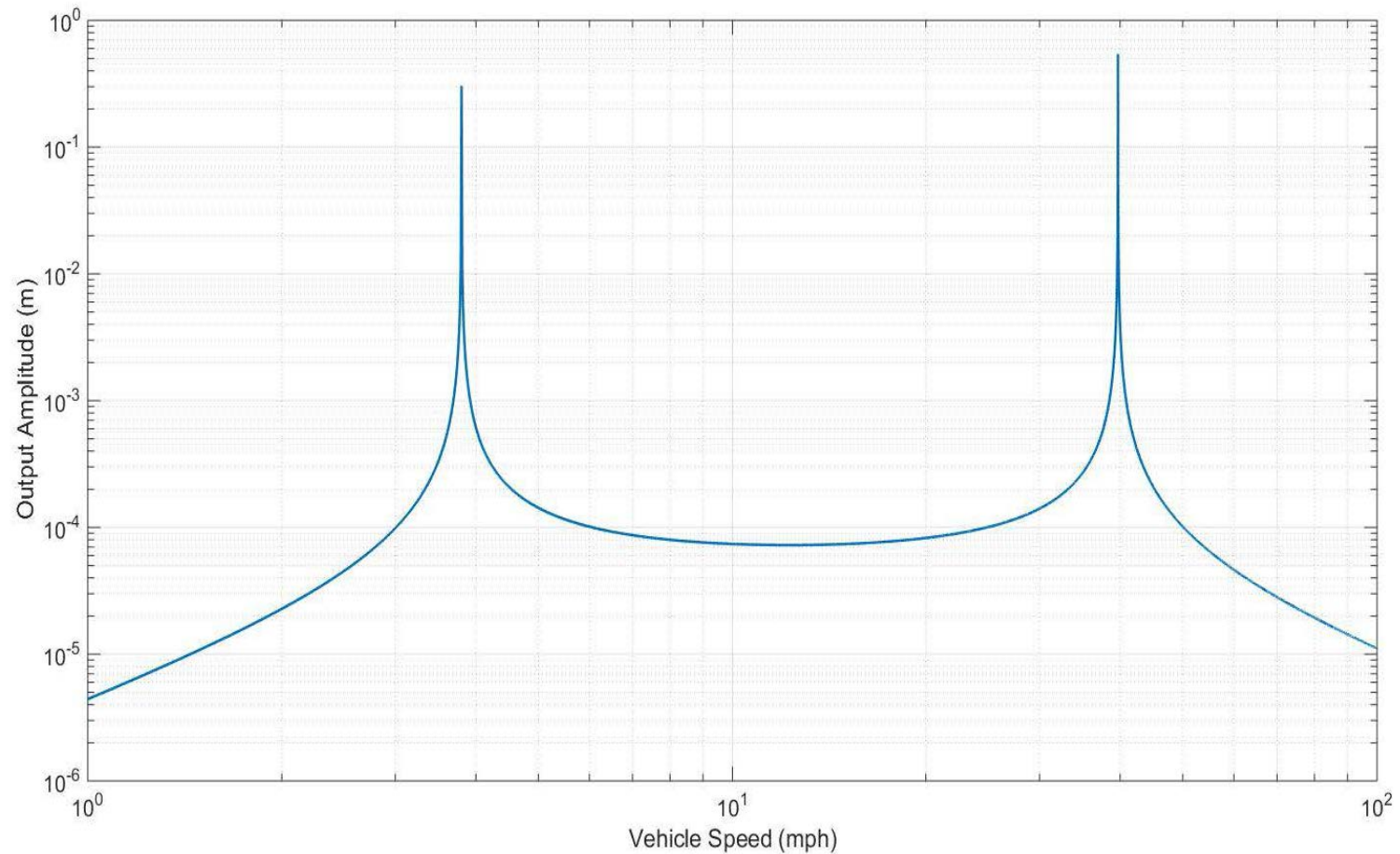
Effect of Wheel Imbalance

- It creates a sinusoidal force input!
 - Frequency of the sinusoidal input is the speed of the wheel
 - Notice the magnitude of the input force increases by frequency (i.e. speed) squared!
 - 2nd order low pass filter would not remove

$$R_y = mg - mR\omega^2 \sin(\omega t + \theta_0)$$

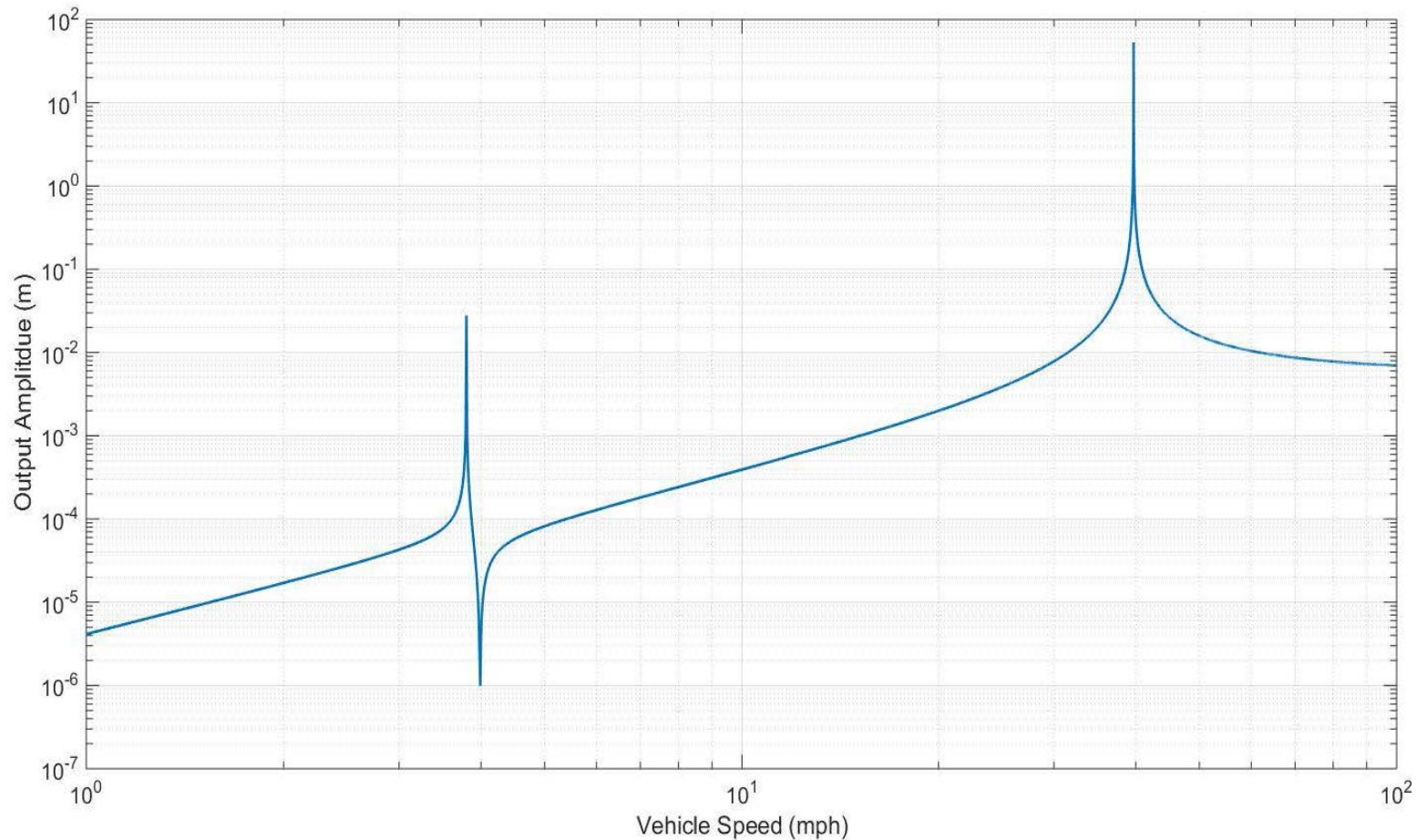
$|x_1|$ (Car position)

- Remember the Bode Plot is $\frac{|output|}{|input|}$



$|x_2|$ (Wheel position)

- Remember the Bode Plot is $\frac{|output|}{|input|}$



Calculating Gain & Phase Graphically

- Also, since the gain and phase are calculated by evaluating the magnitude and phase of the transfer function at $j\omega$, you can also do it graphically.
- Recall that $j\omega$ is essentially the imaginary axis on the s-plane
 - Can find the gain and phase by calculating lengths from the imaginary axis to the poles and zeros and the angles from the poles and zeros to the frequency on the imaginary axis:

$$G(\omega) = \frac{\text{product of the lengths from } \omega \text{ on the imaginary axis to all the zeros}}{\text{produce of the lengths from } \omega \text{ on the imaginary axis to all the zeros}}$$

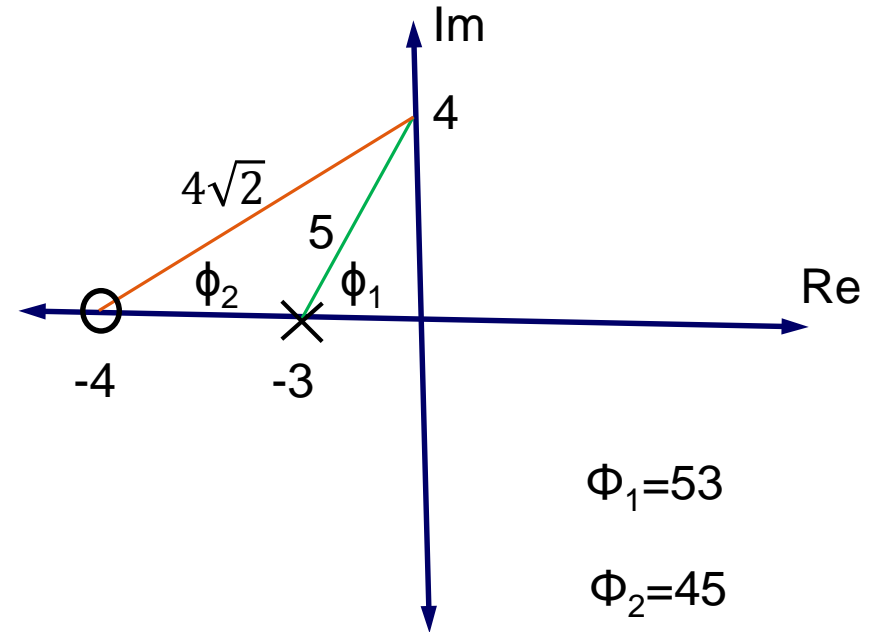
$$\phi(\omega) = \sum \phi_{zeros} - \sum \phi_{poles}$$

Graphical Example #1

$$H(s) = \frac{s + 4}{s + 3}$$

$$G(4) = \frac{4\sqrt{2}}{5}$$

$$\phi(4) = -7 \text{ deg}$$



Graphical Example #2

$$H(s) = \frac{s + 3}{s^2 + 12s + 52}$$

$$G(4) = \frac{5}{6 * 10} = 0.0833$$

$$\phi(4) = 0 \text{ deg}$$

