Name: Key

Exam # 2 **ELEC 5760/6760** **Mon 11/28/22**

Constants: $\pi = 3.14159$, $\varepsilon_o = 8.854$ pF/m, 1 atm = 101.325 kPa, 1 G = 9.8 m/s^2,

Equations: $PE = mgh$, $KE = \frac{1}{2}mv^2$, $P_s = \rho gh$, $F_{PPA} = \frac{\varepsilon_o \varepsilon_r A V^2}{2d^2}$

$V_{PI} = \sqrt{\frac{8kd_o^3}{27A\varepsilon_o\varepsilon_r}}$, $A_{circle} = (\pi)r^2$, $d = a(\frac{m}{k}) = aS$, $y(t) = \frac{2m\Omega A_x}{c^2\omega_n}\cos(\omega_n t)$

$P_t = P_s + \frac{\rho v^2}{2}$, $f_d = \frac{f_s}{1 + \frac{v_{object}}{v_{wave}}}$, $V_{rms} = \frac{V_{amplitude}}{\sqrt{2}}$, $C_{PPA} = \frac{\varepsilon_o \varepsilon_r A}{d}$

Laplace Transforms: $\mathcal{L}[cu(t)] = \frac{c}{s}$, $\mathcal{L}[ce^{-at}] = \frac{c}{s+a}$, $\mathcal{L}[cte^{-at}] = \frac{c}{(s+a)^2}$

Problems:

1) An object fell onto a hard surface where its velocity then went to 0 m/s in 100 ms. The on-board shock sensor indicated an average acceleration of -500 m/s^2 during the impact. If G = 9.8 m/s^2, what height did the object fall from? **(10 points)**

$a \approx \frac{\Delta V}{\Delta t} \rightarrow \Delta V = |a|\Delta t = (500)(100\times10^{-3}) = 50$ m/s

$KE = PE$

$\frac{1}{2}mV^2 = mgh \rightarrow \frac{1}{2}V^2 = gh$

$h = \frac{V^2}{2g} = \frac{(50)^2}{2(9.8)} = 127.55$m

Match the question with an answer by writing the letter of the answer in the blank next to the question. No answer is used more than once. (30 points)

Questions

1) This instrument uses optical wavelength absorption to determine gas content: D
2) Transmissibility can be measured using this optical instrument: E
3) When a reflected wave from a moving object has a different frequency: I
4) In a PPA, this pulls the electrodes toward each other: N
5) A type of MEMS fluidic actuator: F
6) A bimorph is made of two materials with different values of: M
7) Measures 3 axis acceleration and 3 axis rotation: B
8) Measures pressure with respect to a full vacuum: O
9) The principle of buoyancy is also known as: P
10) Closed loop accelerometers and pressure sensors make use of: C
11) This is a 1 axis inertial sensor: H
12) The acceleration of a typical mechanical shock event has a characteristic: K
13) MEMS gyroscopes make use of the: G
14) All energy dissipative systems experience this: L
15) Two high Q MEMS gyros in close proximity can experience this problem: A

Answers to choose from

A. Intermode Coupling
B. IMU
C. Force Feedback
D. Spectrometer
E. Interferometer
F. Flow FET
G. Coriolis Force
H. Gyroscope
I. Doppler Shift
J. Diaphragm
K. Half Sine Pulse
L. Thermal Noise
M. CTE
N. Electrostatic Force
O. Absolute Pressure Sensor
P. Archimedes' Principle

2) An object is moving through a fluid with a density of 2 g/cm^3 at 10 m/s. Its pressure sensor measures a total pressure of 105 kPa using a gauge pressure sensor. What is the static pressure of the object in the fluid? **(10 points)**

$$\rho: \left(\frac{2g}{cm^3}\right)\left(\frac{1kg}{1000g}\right)\left(\frac{100cm}{m}\right)^3 = 2000\ kg/m^3$$

$$P_t = P_s + \frac{\rho V^2}{2}$$

$$\text{or: } P_s = P_t - \frac{\rho V^2}{2}$$

$$= 105{,}000 - \frac{(2000)(10)^2}{2}$$

$$= 5000\ Pa$$

$$= 5\ kPa$$

3) A parallel plate actuator (PPA) consists of two square electrodes 100 μm across, separated by 10 μm, in a vacuum. The spring constant is 50 N/m. What is the pull-in voltage? **(10 points)**

$$V_{PI} = \sqrt{\frac{8k d_0^3}{27A\varepsilon_0\varepsilon_r}} = \sqrt{\frac{8(50)(10\times10^{-6})^3}{27(100\times10^{-6})^2(8.854\times10^{-12})(1)}}$$

$$= 409.05\ V$$

4) What is the amplitude of motion along the sense axis for a certain MEMS gyroscope that has a mass of 1 μg, $c = 0.1 \times 10^{-6}$ Kg/s, $f_n = 10$ kHz, and $A_x = 1$ μN when it experiences and angular rate of 180°/s? **(10 points)**

$m = 1 \times 10^{-9}$ kg

$$y(t) = \frac{2 m \Omega A_x}{c^2 \omega_n}$$

$$= \frac{2(1\times10^{-9})(180)(\frac{\pi}{180})(1\times10^{-6})}{(0.1\times10^{-6})^2(2\pi)(10{,}000)} = \frac{(1\times10^{-9})(1\times10^{-6})}{(0.1\times10^{-6})^2(10{,}000)}$$

$$= 10\,\mu m$$

5) A parallel plate actuator (PPA) consists of two round electrodes, 500 μm in diameter, separated by 10 μm, in a vacuum. If one electrode is connected to ground and the other electrode is connected to a high frequency AC voltage (100 V amplitude), what is the average force produced by the PPA? **(10 points)**

$$F = \frac{\varepsilon_0 \varepsilon_r A V^2}{2 d^2} = \frac{(8.854\times10^{-12})(1)(\pi)(250\times10^{-6})^2(100)^2}{(2)(2)(10\times10^{-6})^2} = 43.46\,\mu N$$

note: $V = \frac{100}{\sqrt{2}} = V_{rms}$

$$V^2 = \frac{100^2}{2}$$

6) What does GCA stand for? **(5 points)**

Gap Closing Actuator

7) For a nitinol SMA actuator, what does SMA stand for? **(5 points)**

Shape Memory Alloy

8) What is the measurand for a vibrating proof mass MEMS gyroscope? **(5 points)**

Angular Rate

9) The proof mass for a certain open-loop MEMS accelerometer experiences a displacement of 10 μm due to a 10 m/s^2 acceleration. What is the natural frequency of the accelerometer in Hz? **(5 points)**

$$d = a\frac{m}{K} = aS = \frac{a}{\omega_n^2}$$

$$S = \frac{d}{a} = \frac{10\times10^{-6}}{10} = 1\times10^{-6}\ s^2$$

$$\omega_n = \frac{1}{\sqrt{S}} = \frac{1}{\sqrt{1\times10^{-6}}} = 1000\ rad/s$$

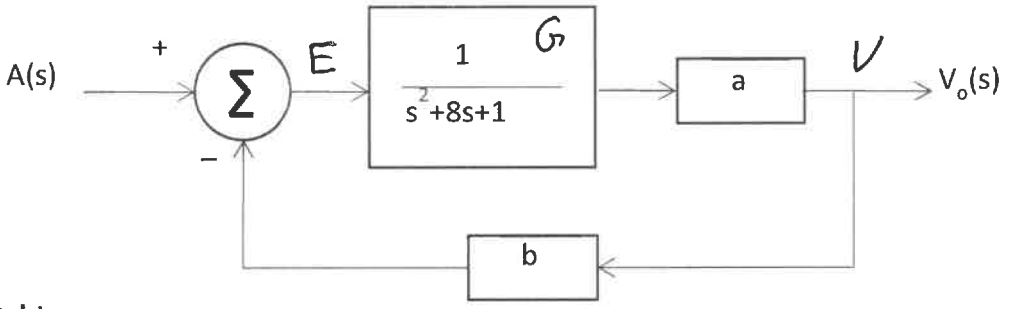
$$f_n = \frac{\omega_n}{2\pi} = \frac{1000}{2\pi} = 159.16\ Hz$$

Bonus Question (10 points)

A certain open loop MEMS accelerometer has the following transfer function, where the damping ratio is not equal to one:

$$G(s) = \frac{1}{s^2 + 8s + 1}.$$ want: $\zeta = 1$, $G_{CL}|_{DC} = 1$

It is placed in a closed-loop controller (shown below) to adjust the damping ratio to one and the DC gain to one [$G_{cl}(s) = 1$ at DC]. Select values for the two gains, a and b, to achieve this goal for the closed loop system. Show all steps.



$E = A(s) - Vb$

$V = EGa = Ga(A(s) - Vb)$

$V(1 + Gab) = Ga\,A(s)$

$$\frac{V(s)}{A(s)} = \frac{Ga}{1+Gab} = \frac{a}{(s^2+8s+1)\left(1+\frac{ab}{s^2+8s+1}\right)} = \frac{a}{s^2+8s+1+ab}$$

$2\zeta\omega_n = 8 \;\Big|_{\zeta=1} \rightarrow 2\omega_n = 8 \rightarrow \omega_n = 4$

$1 + ab = \omega_n^2 = 4^2 = 16$

$\therefore ab = 15$

For $G_{CL}|_{DC} = 1$: $a = 1 + ab = 16$

$\therefore b = \frac{15}{a} = \frac{15}{16}$

$a = 16$

$b = \frac{15}{16}$

Blank sheet for Calculations