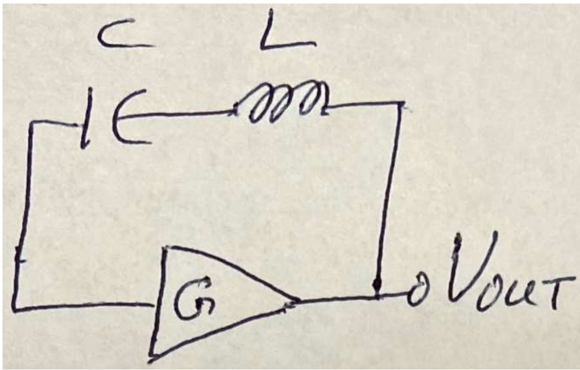
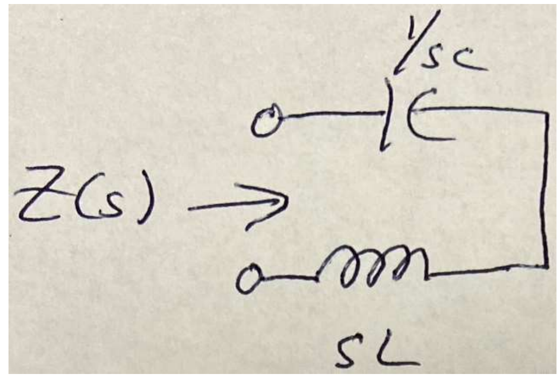
Lecture 9/24/24

Capacitive Sensing

1. Capacitor Interface Circuitry: LC Oscillators



and



Assume that $G = 1\underline{|0^o}$.

G is the amplifier gain.

$Z(s) = \frac{1}{sC} + sL \;\rightarrow\; Z(j\omega) = \frac{-j}{\omega C} + j\omega L$

At $\omega = \frac{1}{\sqrt{LC}} \rightarrow Z(j\omega) = 0\underline{|0^o}$, i.e. a short, allowing the circuit to oscillate at this frequency.

So, $\omega_n = \frac{1}{\sqrt{LC}}$: assume L is fixed and C is our sensor.

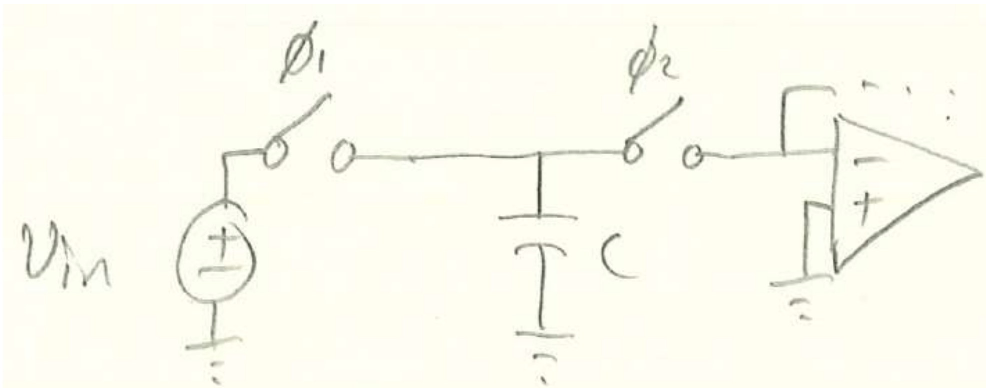
The output, V_{OUT}, is a sinusoidal signal where $f \propto \frac{1}{\sqrt{C}}$: the frequency is a nonlinear function of C(t) or C(x).

This is the common operation of LC oscillators.

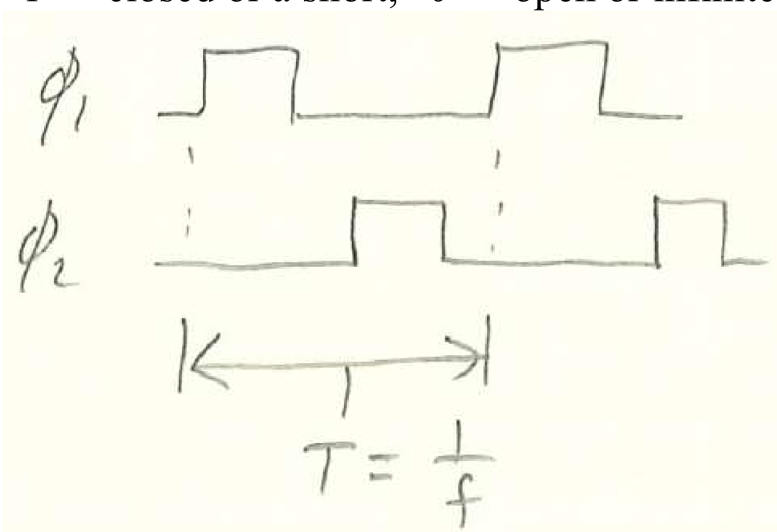
2. Capacitor Interface Circuity: Switched-Capacitor Circuits

Switching a capacitor on and off can make it behave like a resistor, under certain conditions.

Consider:



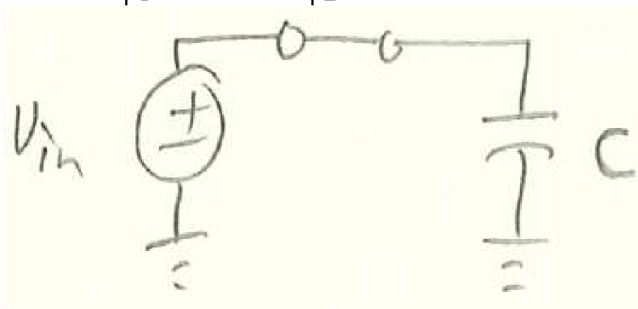
ϕ_1 and ϕ_2 are clocked waveforms that turn on and off the two switches: "1" = closed or a short, "0" = open or infinite resistance:



Both switches can be off at the same time, but never both on at the same time. They both operate at the same frequency, f, with period T = 1/f.

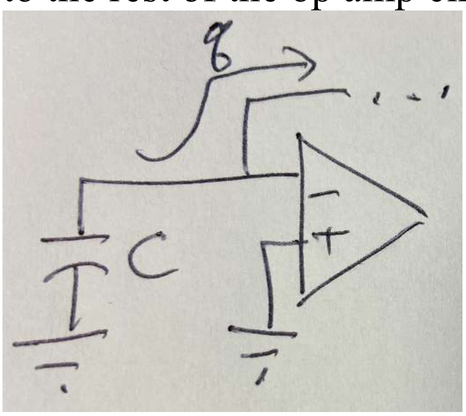
The rest of the op amp circuit (feedback network, etc.) is not shown. The ϕ_2 switch is connected to a virtual ground through the op amp circuit.

When $\phi_1 = 1$ and $\phi_2 = 0$:



Charge, q, is stored on C: $q = Cv_{in}$.

Next, when $\phi_1 = 0$ and $\phi_2 = 1$, q is transferred through the virtual ground to the rest of the op amp circuit, fully discharging C:



For a given T, q charge moves through the circuit. Therefore, we can model this as:

$$i = \frac{dq}{dt} \approx \frac{\Delta q}{\Delta t} = \frac{q}{T} = \frac{Cv_{in}}{T} = Cfv_{in}$$

The current, i, is proportional to v_{in}. Therefore, we can define an equivalent resistance, R_{eq}:

$$R_{eq} = \frac{v_{in}}{i} = \frac{1}{Cf}$$

This is called a switched-capacitor resistor.

It is applicable to low-pass systems where f >> system bandwidth.

A LPF can be added at the circuit output to attenuate switching noise.

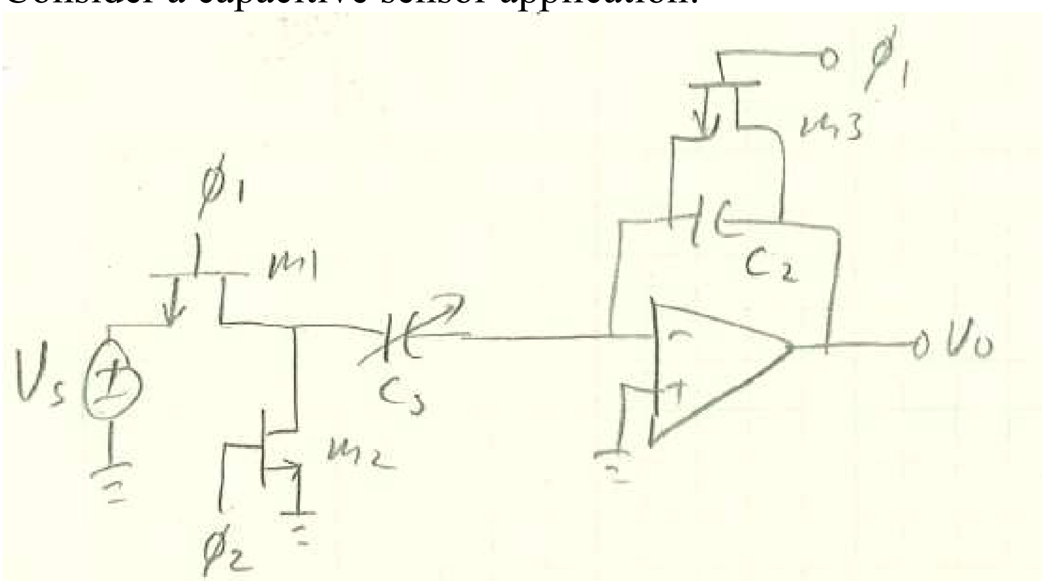
It can be used as a frequency tunable resistor.

In CMOS analog ICs, it's often used for resistors because on-chip capacitors have much tighter fabrication tolerances than on-chip resistors, particularly for audio signal processing applications:

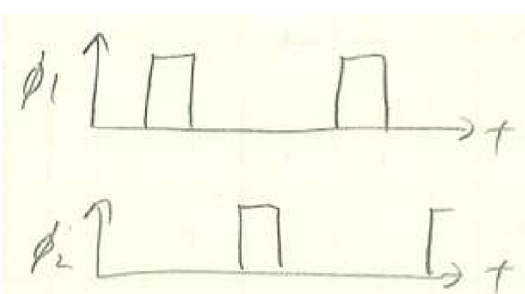
For a 20 kHz BW → use f ~ 200 kHz

MOSFETS (single of paired) are typically used as the switches.

Consider a capacitive sensor application:

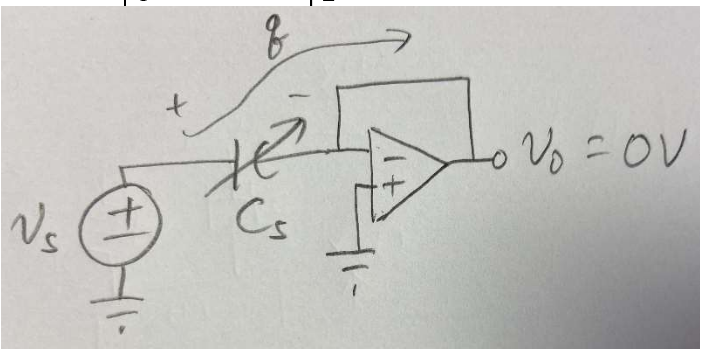


M1, M2, and M3 are MOSFET analog switches. C_s is the sensor.



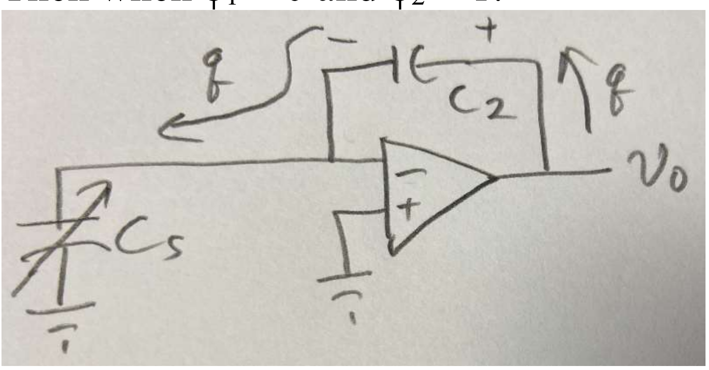
non-overlapping clock waveforms

When $\phi_1 = 1$ and $\phi_2 = 0$:



Charge $q = C_s v_s$ is stored on C_s.

Then when $\phi_1 = 0$ and $\phi_2 = 1$:



Charge q flows out of C_s to ground, requiring q to flow through C_2:

$v_o = \frac{q}{C_2} = \frac{C_s v_s}{C_2}$ → $\phi_2 = 1$ and $\phi_1 = 0$. However, $v_o = 0$ V otherwise.

Assuming that ϕ_2 has a 50% duty cycle, averaged over time: $v_o \approx \frac{C_s v_s}{2C_2}$.

But, if you only measure v_o when $\phi_2 = 1$, then use $v_o = \frac{C_s v_s}{C_2}$.

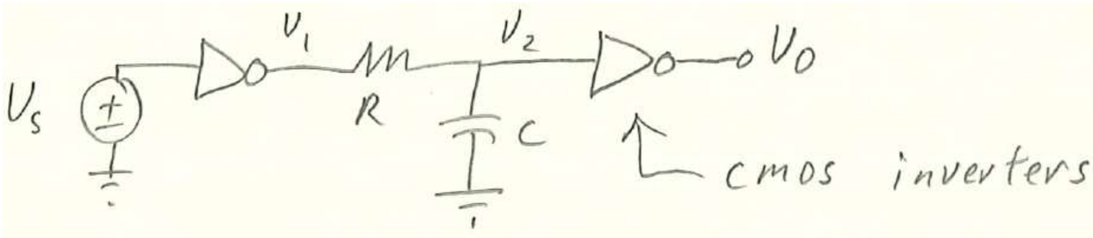
With fixed v_s and C_2, C_s is converted to a voltage, v_o, proportional to it.

Therefore, this circuit is called a charge amplifier, and has a gain of v_s/C_2.

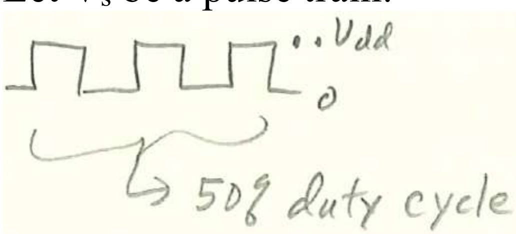
Lecture 9/24/24

3. Capacitor Interface Circuity: Phase or State Delay Circuits

Consider this circuit:



Let V_s be a pulse train:



$V_{tr} = \frac{V_{dd}}{2} \rightarrow$ inverter trip voltage

$V_2 = V_1(1 - e^{-t/RC})$

Solve for t = f(RC) for $V_1 = V_{dd}$ and $V_2 = V_{tr}$

Therefore, $\frac{V_{dd}}{2} = V_{dd}(1 - e^{-t/RC})$

Or: $\frac{1}{2} = 1 - e^{-t/RC}$

Or: $e^{-t/RC} = \frac{1}{2}$

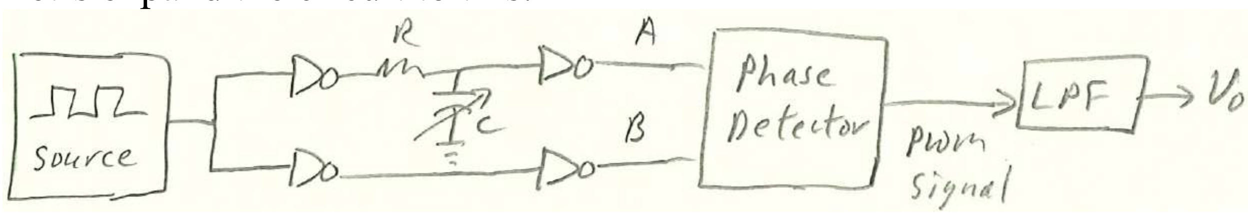
Leading to: $t = -RC\ln(0.5) = 0.693RC$

The last inverter squares up the signal out of the RC subcircuit.

Therefore, the pulse train is delayed 0.693RC s through the RC subcircuit.

Note: this assumes that C fully charges to V_{dd} or discharges to 0 V before the next state change → otherwise the delay is not a linear function of C.

Let's expand the circuit to this:



Now there are two parallel inverter paths, one with RC_s and one without.

RC_s delays signal A compared to signal B.

The Phase Detector compares the logical signals, A and B, and produces a pulse width modulated (PWM) output signal.

Consider the Exclusive OR (EXOR) gate:

A B	Q
0 0	0
0 1	1
1 0	1
1 1	0

} only high when $A \neq B$

Therefore, pulse width (and duty cycle) is proportional to how out of phase A and B are (0° to 180°).

In phase: Q = 0

180° out of phase: Q = 1

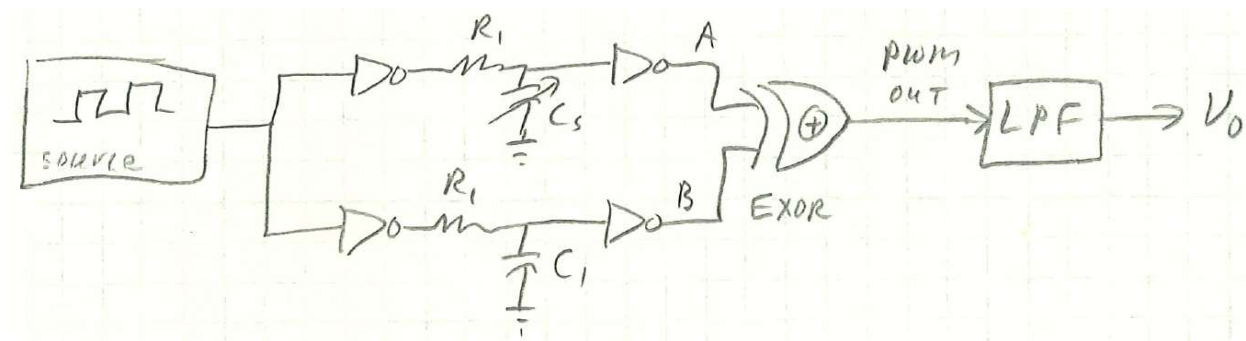
The average or DC value of the PWM signal is proportional to the PWM duty cycle, which is proportional to C_s. The low pass filter (LPF) produces V_o, which primarily consists of this DC term, which is proportional to C_s.

To optimize this capacitive sensor interface circuit:

(1) For the EXOR phase detector: we do not want a phase difference near 0° or 180° to avoid phase jitter issues.

(2) To avoid excessive nonlinear distortion, limit the phase delay range from the sensor to 45°, and place that about 90° (67.5° to 112.5°).

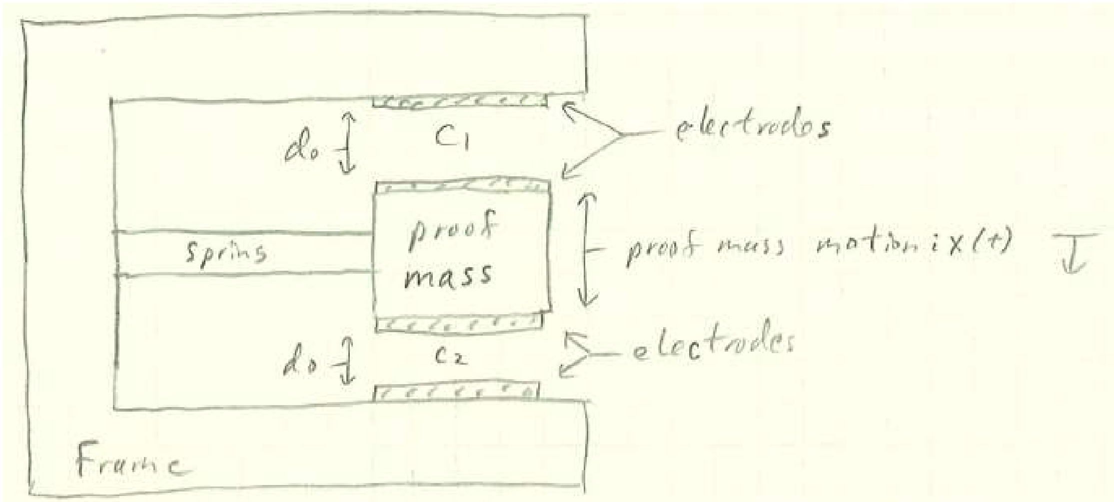
(3) Add a fixed RC delay stage in the lower inverter chain to achieve this:



Signal A's minimum delay differs from signal B by 67.5°, and signal A's maximum delay differs from signal B by 112.5°. One of the two resistors can be potentiometer so that the interface circuit can be tuned.

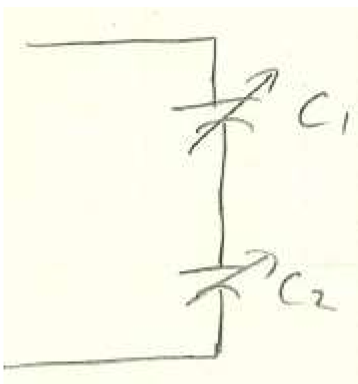
4. Capacitor Interface Circuity: AC Voltage Division

Sometimes the MEMS device is designed to produce a differential capacitance:

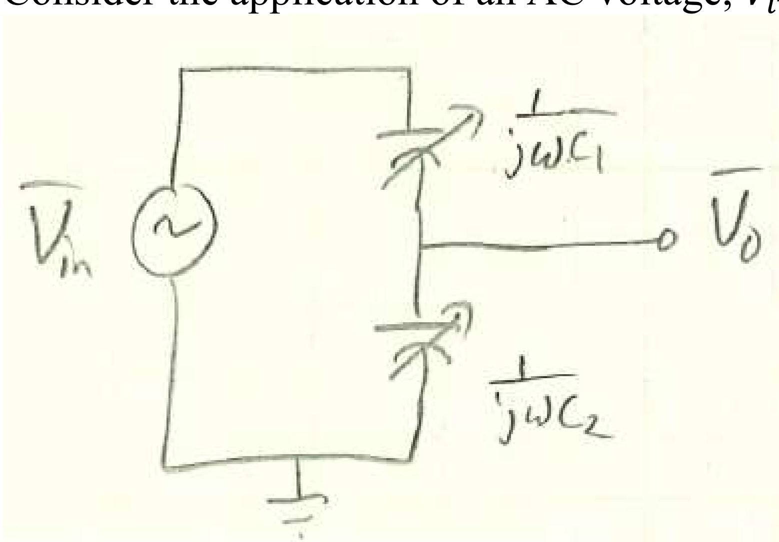


$C_1 = \frac{\varepsilon_o \varepsilon_r A}{d_1} = \frac{\varepsilon_o \varepsilon_r A}{d_o + x(t)}$ and $C_2 = \frac{\varepsilon_o \varepsilon_r A}{d_2} = \frac{\varepsilon_o \varepsilon_r A}{d_o - x(t)}$

The circuit model is:



Consider the application of an AC voltage, $\bar{V}_{in}$:



$$\frac{\bar{V}_o}{\bar{V}_{in}} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1}+\frac{1}{j\omega C_2}} = \frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{C_1}{C_2+C_1}$$ → An AC voltage divider

Let $\bar{V}_{in} = V_1 \sin(\omega t)$

Therefore: $\bar{V}_o = V_1 \sin(\omega t)\left[\frac{C_1}{C_2+C_1}\right]$ → select ω >> ω_{MEMS}

$$\frac{C_1}{C_2+C_1} = \frac{\frac{\varepsilon_o \varepsilon_r A}{d_o+x(t)}}{\frac{\varepsilon_o \varepsilon_r A}{d_o-x(t)}+\frac{\varepsilon_o \varepsilon_r A}{d_o+x(t)}}$$

$$= \frac{\frac{1}{d_o+x(t)}}{\frac{1}{d_o-x(t)}+\frac{1}{d_o+x(t)}}$$

$$= \frac{d_o-x(t)}{d_o+x(t)+d_o-x(t)}$$

$$= \frac{d_o-x(t)}{2d_o}$$

$$= 0.5\left(1-\frac{x(t)}{d_o}\right)$$

$$\therefore\ \bar{V}_o(t) = V_1 \sin(\omega t)\left[0.5\left(1-\frac{x(t)}{d_o}\right)\right]$$

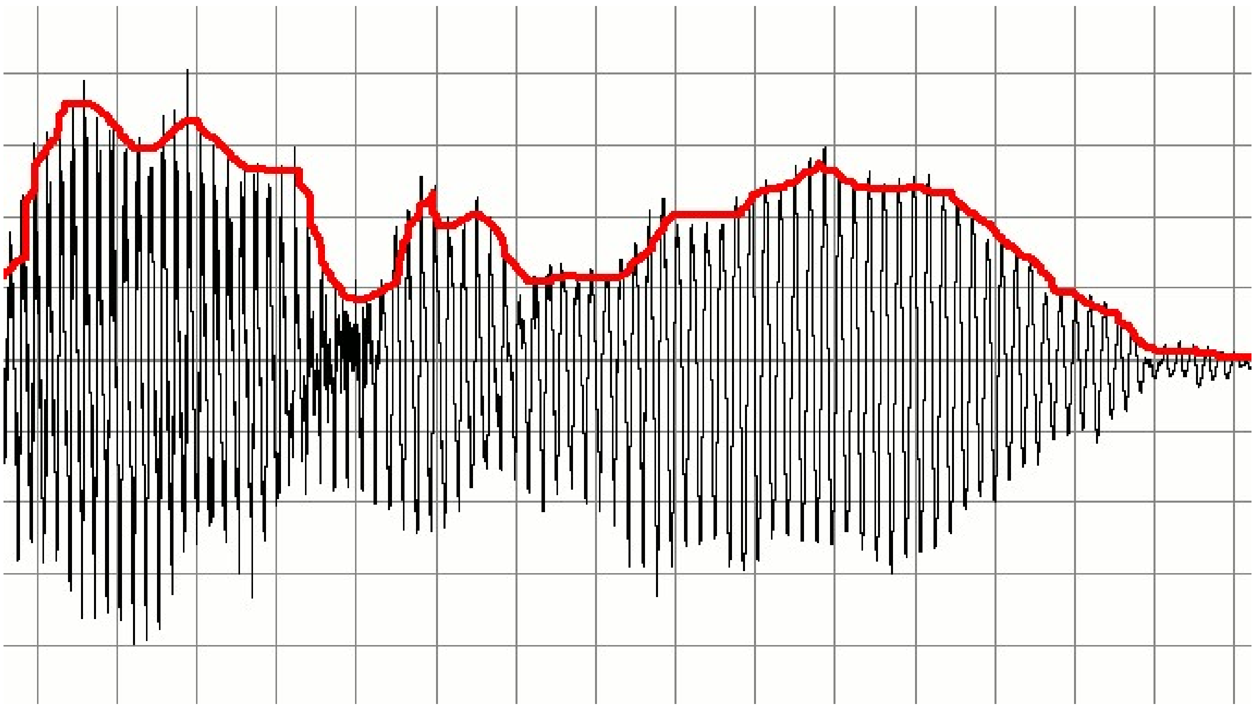
The amplitude of $\bar{V}_o(t)$ is a linear function of x(t).

So, how do we recover the amplitude of $\bar{V}_o(t)$, $V_1\left[0.5\left(1-\frac{x(t)}{d_o}\right)\right]$?

It is desirable to produce a DC voltage proportional to $V_1\left[0.5\left(1-\frac{x(t)}{d_o}\right)\right]$.

a. Envelope Detection

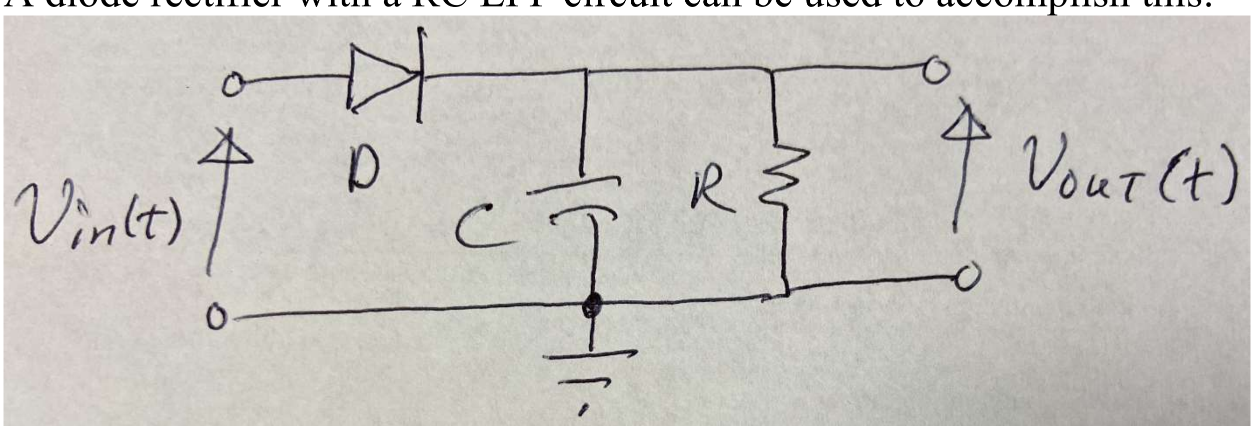
An envelope detector is a diode circuit that recovers the demodulated envelope of an AM modulated signal.



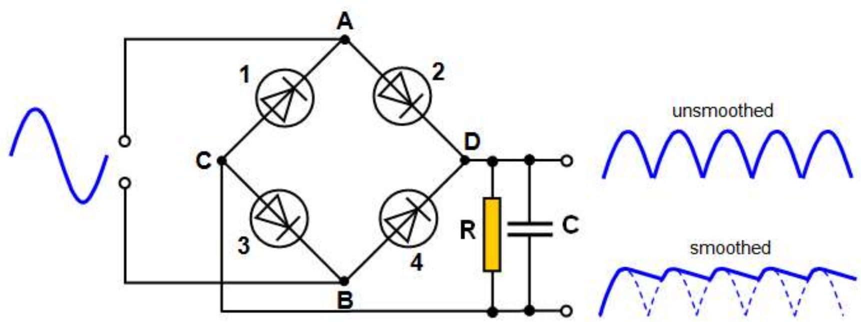
Curtesy: https://en.wikipedia.org/wiki/Envelope_detector#/media/File:C_Envelope_follower.png

The black waveform is the AM signal. The red waveform is the envelope or message we wish to recover.

A diode rectifier with a RC LPF circuit can be used to accomplish this:



More advanced rectifier circuits, such as using a four diode full wave bridge rectifier, or using an op amp based “super diode” could also be used for envelope detectors:



Curtesy: https://www.schoolphysics.co.uk/age16-19/Electronics/Semiconductors/text/Rectification_/index.html

b. Synchronous Demodulator (Lock-in Amplifier)

Consider this:

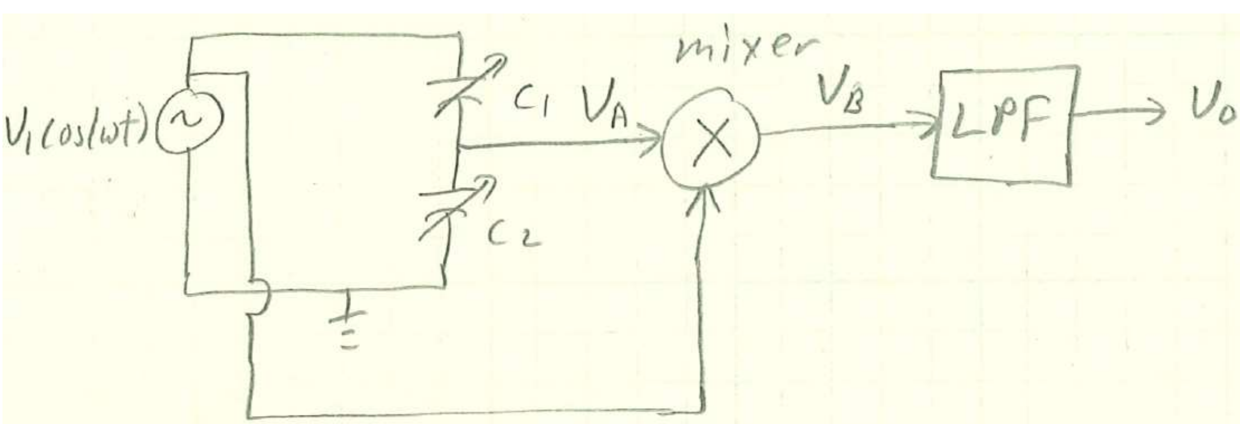
$$Acos(\omega t) \times Bcos(\omega t) = 0.5AB[\cos(\omega t - \omega t) + \cos(\omega t + \omega t)]$$

$$= 0.5AB[\cos(0) + \cos(2\omega t)]$$

$$= 0.5AB + 0.5ABcos(2\omega t)$$

↑ DC term ↑ AC term at 2ωt

So let’s connect our differential capacitive sensor into a new circuit:



$V_A = V_1\cos(\omega t)\left[0.5\left(1-\frac{x(t)}{d_o}\right)\right]$ → same form as with the AC voltage divider

$V_B = V_A V_1 \cos(\omega t)$

$$= 0.5\,V_1^2\left[0.5\left(1-\frac{x(t)}{d_o}\right)\right] + 0.5\,V_1^2\cos(2\omega t)\left[0.5\left(1-\frac{x(t)}{d_o}\right)\right]$$

↑ DC term ↑ AC term at 2ωt

The LPF attenuates the AC term so that:

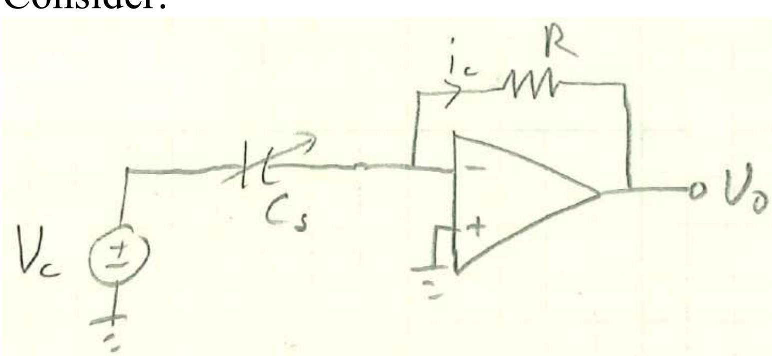
$$V_o \approx 0.5\,V_1^2\left[0.5\left(1-\frac{x(t)}{d_o}\right)\right] = V_c\left(1-\frac{x(t)}{d_o}\right)$$

where V_c is a constant: $V_c = 0.5\,V_1^2(0.5)$

Now, V_o is a DC voltage that is a linear function of x(t).

5. Capacitor Interface Circuity: <u>Transimpedance amplifier</u> (TIA)

Consider:



<u>Note</u>: some op amps are <u>not</u> stable with the input tied to a capacitor, and the output will break into high frequency oscillation.

But assuming the op amp configuration is stable: $v_o = -i_c R$

Note: for this inverting amplifier circuit, since the input is a current and the output is a voltage, the gain is a resistance with units of Ω.

For a fixed capacitor: $i_c = C\frac{dv_c}{dt}$. Note: as a differentiator of v_c, the TIA is noisy: it amplifies high frequency noise.

However, the more general case is: $i_c = C\frac{dv_c}{dt} + v_c\frac{dC}{dt} = C\frac{dv_c}{dt} + v_c\frac{\partial C}{\partial x}\frac{dx}{dt}$

a. If v_c is a constant, V_c, then $i_c = V_c\frac{\partial C}{\partial x}\frac{dx}{dt}$

If the capacitor's electrodes are in relative motion, then $\frac{dx}{dt}$ is a velocity term. In steady state, the time varying $C_s(t)$ pumps i_c into the circuit.

b. If $V_c = V_A \sin(\omega t)$ and $\omega >> \omega_{MEMS}$,

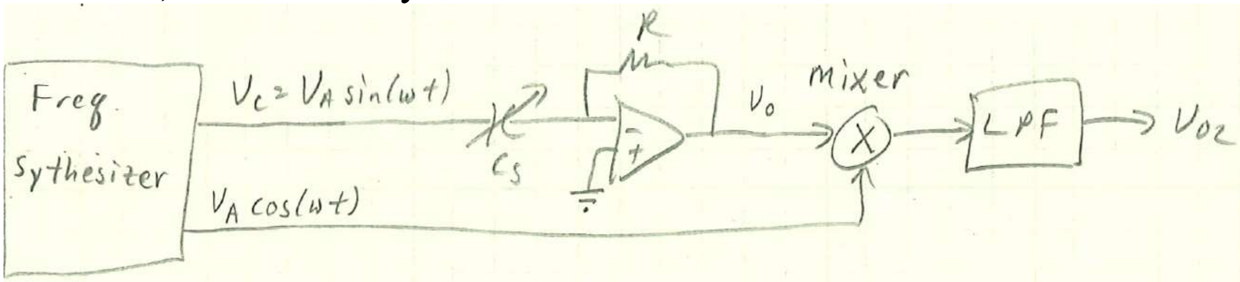
then for short time periods of several V_c cycles, C_s is nearly constant and

$v_c\frac{dC_s}{dt} \approx 0$ (i.e. a very small change during the measurement time)

So: $i_c \approx C_s\frac{dV_c}{dt} = C_s V_A \omega \cos(\omega t)$

And finally: $v_o \approx -C_s R V_A \omega \cos(\omega t)$ for quick measurements of C_s.

However, we can add synchronous demodulation here too:



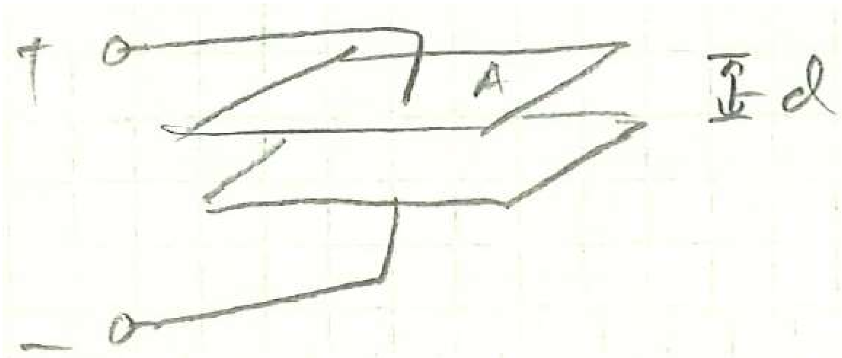
$\therefore V_{o2} = LPF[V_o \times V_A \cos(\omega t)] = -0.5 C_s R V_A^2 \omega = k C_s$

where k is a constant: $k = -0.5 R V_A^2 \omega$

So once again, V_{o2} is a DC voltage proportional to C_s, our sensor's capacitance.

Capacitive Fringing Field Sensors

Consider:

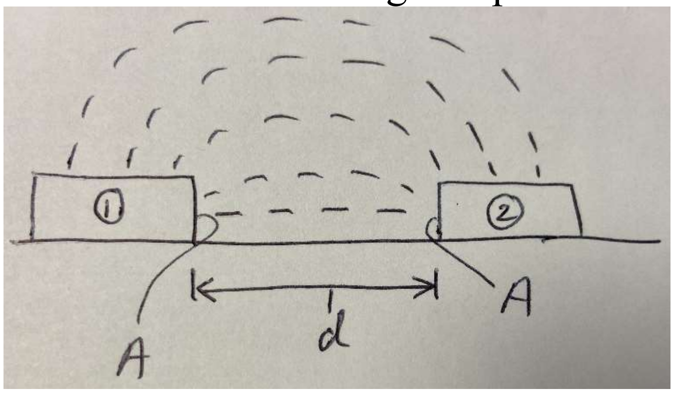


For $A \gg d^2$: $C \approx \frac{\varepsilon_o \varepsilon_r A}{d}$, and fringing effects are small,

But $A \approx d^2$ or if $d^2 > A$, $C \neq \frac{\varepsilon_o \varepsilon_r A}{d}$.

Actually now, $C > \frac{\varepsilon_o \varepsilon_r A}{d}$ due to fringing effects.

Consider the case using two planar electrodes:

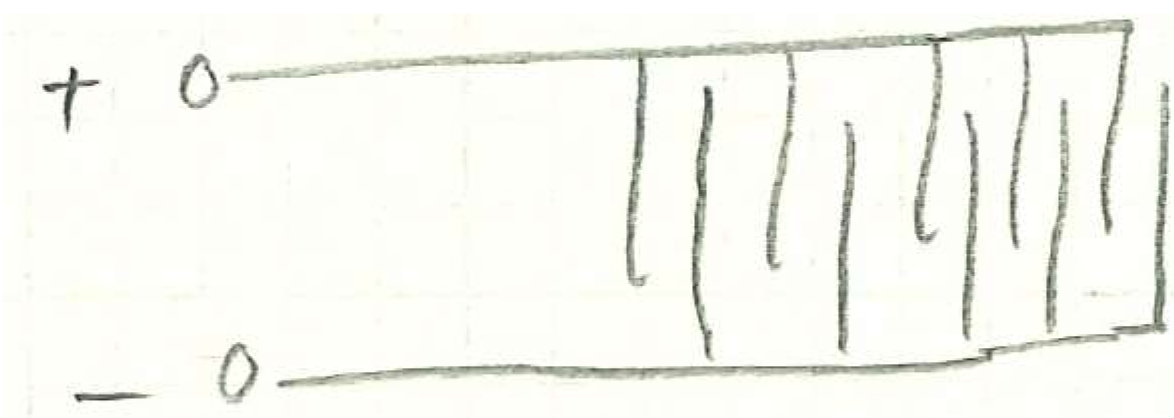


1 and 2 are the electrodes, d is the distance between them, and A is the area of each electrode facing each other. The dashed lines represent electric flux lines.

The capacitance between 1 and 2 can be modeled by:

$C \approx \frac{\varepsilon_o \varepsilon_r A\gamma}{d}$, where γ is a fringing scale factor and $\gamma > 1$.

Often, the two electrodes are arranged in an interdigitated electrode (IDE) layout on a planar surface, realizing a capacitive fringing field sensor:



Let n = number of interdigitated fingers.

$$C \approx \frac{(n-1)\varepsilon_o \varepsilon_r A\gamma}{d}$$

Sometimes, the IDE is coated with a thin insulating layer, such as polyimide (PCB technology) or silicon dioxide (MEMS technology).

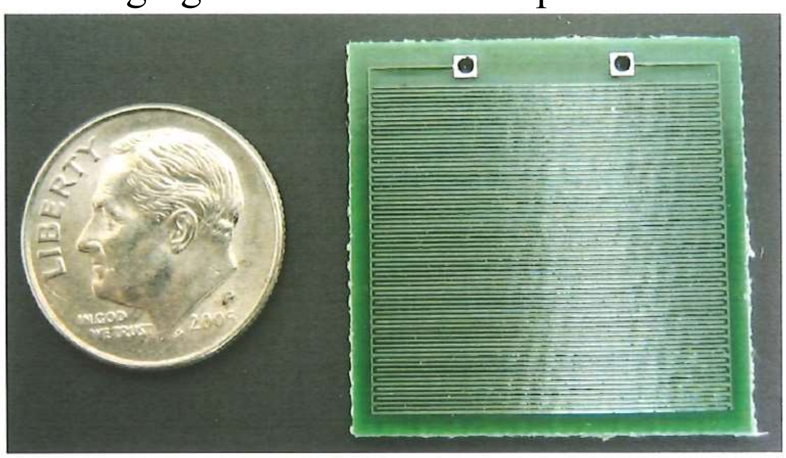
When the electrode width is equal to d, the sensing range above the sensor is approximately 1.25d to 1.5d.

Applications for capacitive fringing field sensors:

1. Detecting the presence of liquid water ($\varepsilon_r|_{air} \approx 1$ and $\varepsilon_r|_{water} \approx 80$ at room temperature)
2. Measuring the moisture content of many materials
3. Measuring the level of water and other liquids
4. Detecting ice: above ~ 10 KHz, $\varepsilon_r|_{water} >> \varepsilon_r|_{ice}$
5. Measuring relative humidity

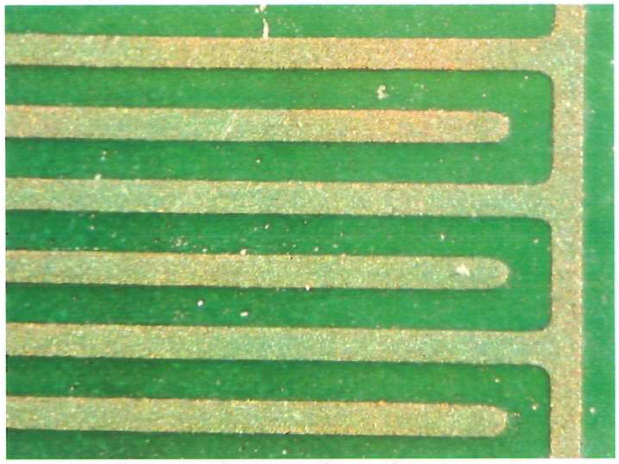
Meso scale versions, such as in PCB technology, can have relatively large capacitances → 100s of pF.

PCB Capacitive Fringing Field Sensor Example:

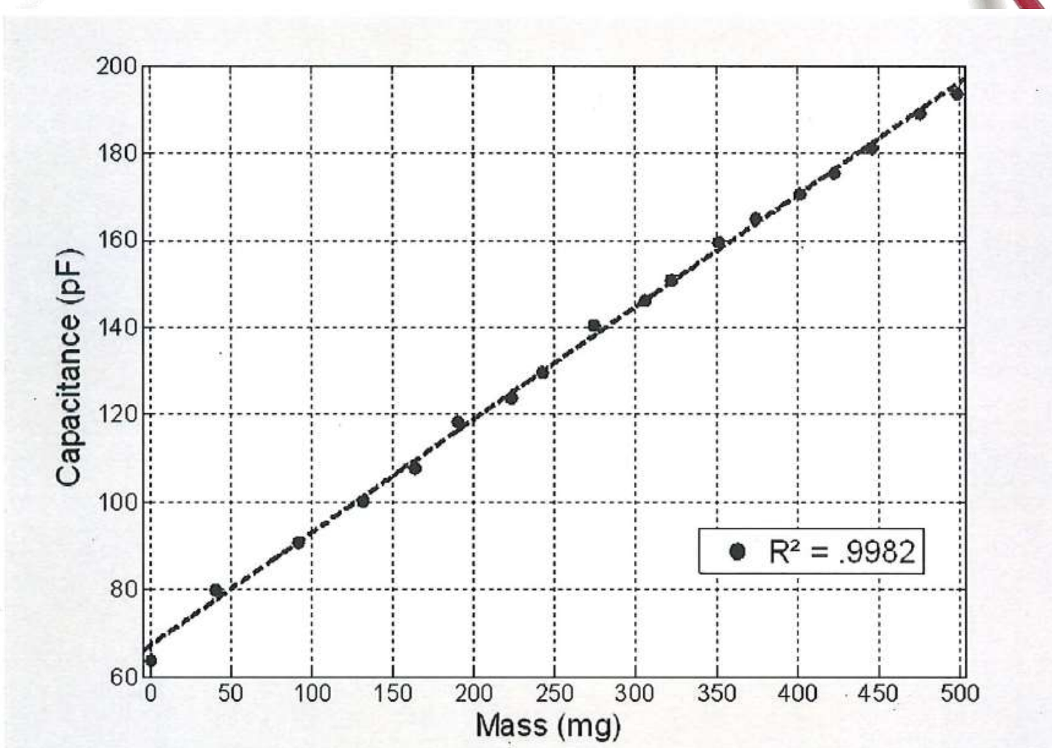


Photo

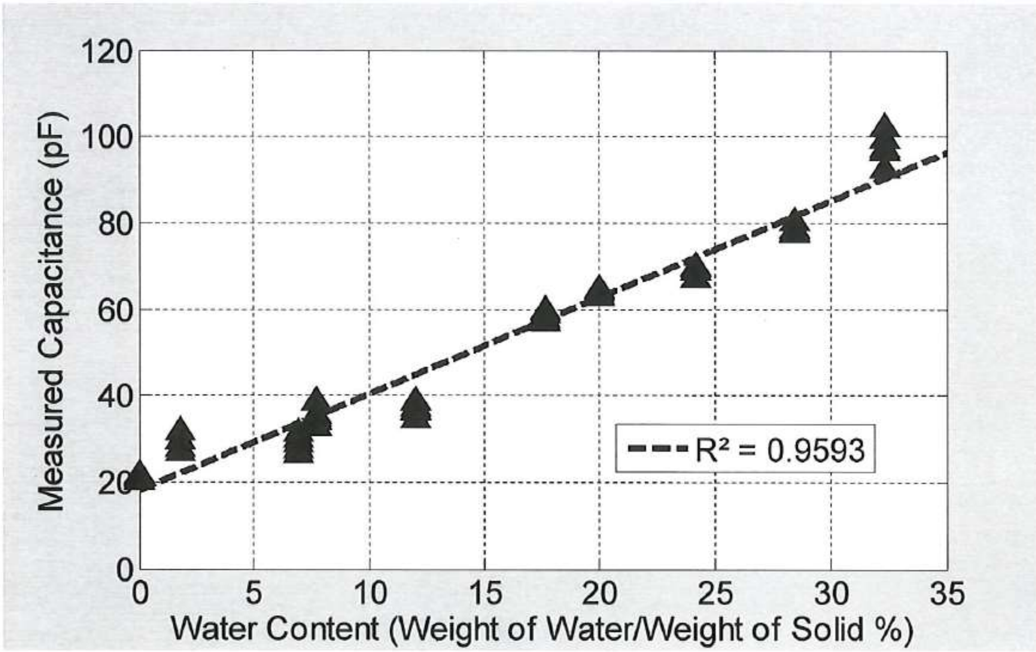
Device: 25.4 mm x 25.4 mm
70 interdigitated fingers (~150µm wide)
22.4 mm electrode overlap
63.9pF capacitance in air
321.3pF capacitance when submerged in water



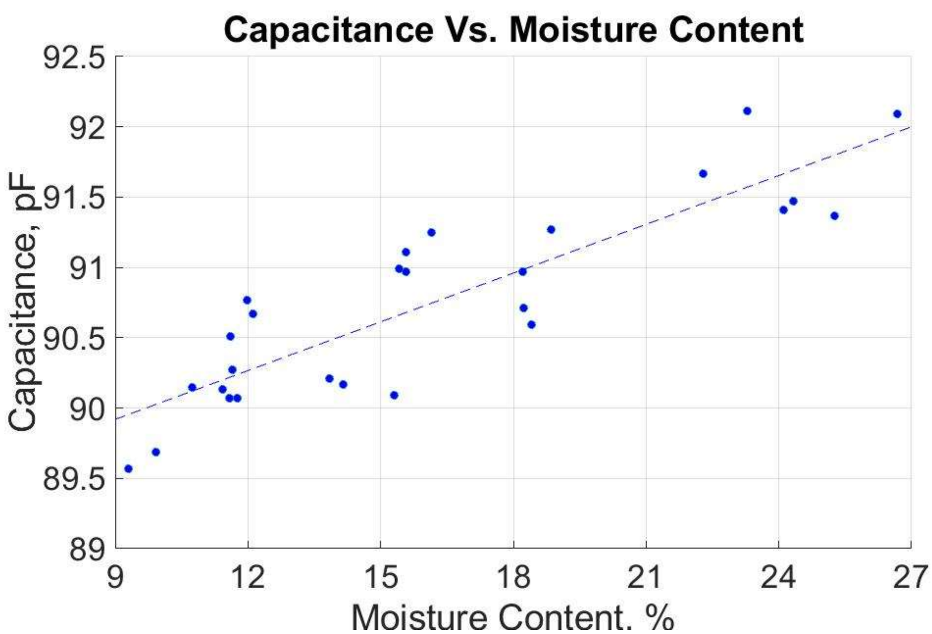
Close up photo or electrode structure



Mass of water drop sensor response



Measured moisture content of soil



Capacitance vs. moisture content of corn