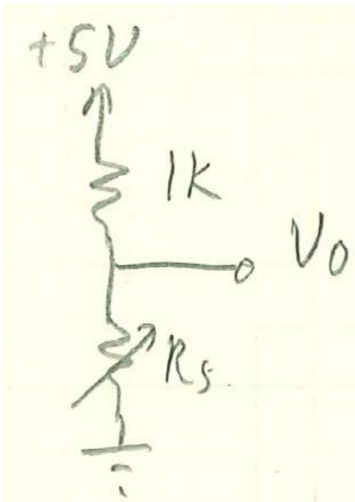
Lecture 9/17/24

<u>Resistance Sensing</u>

1. A single resistance sensor

Consider a resistive sensor, R_s, where $R_s \propto measurand$:

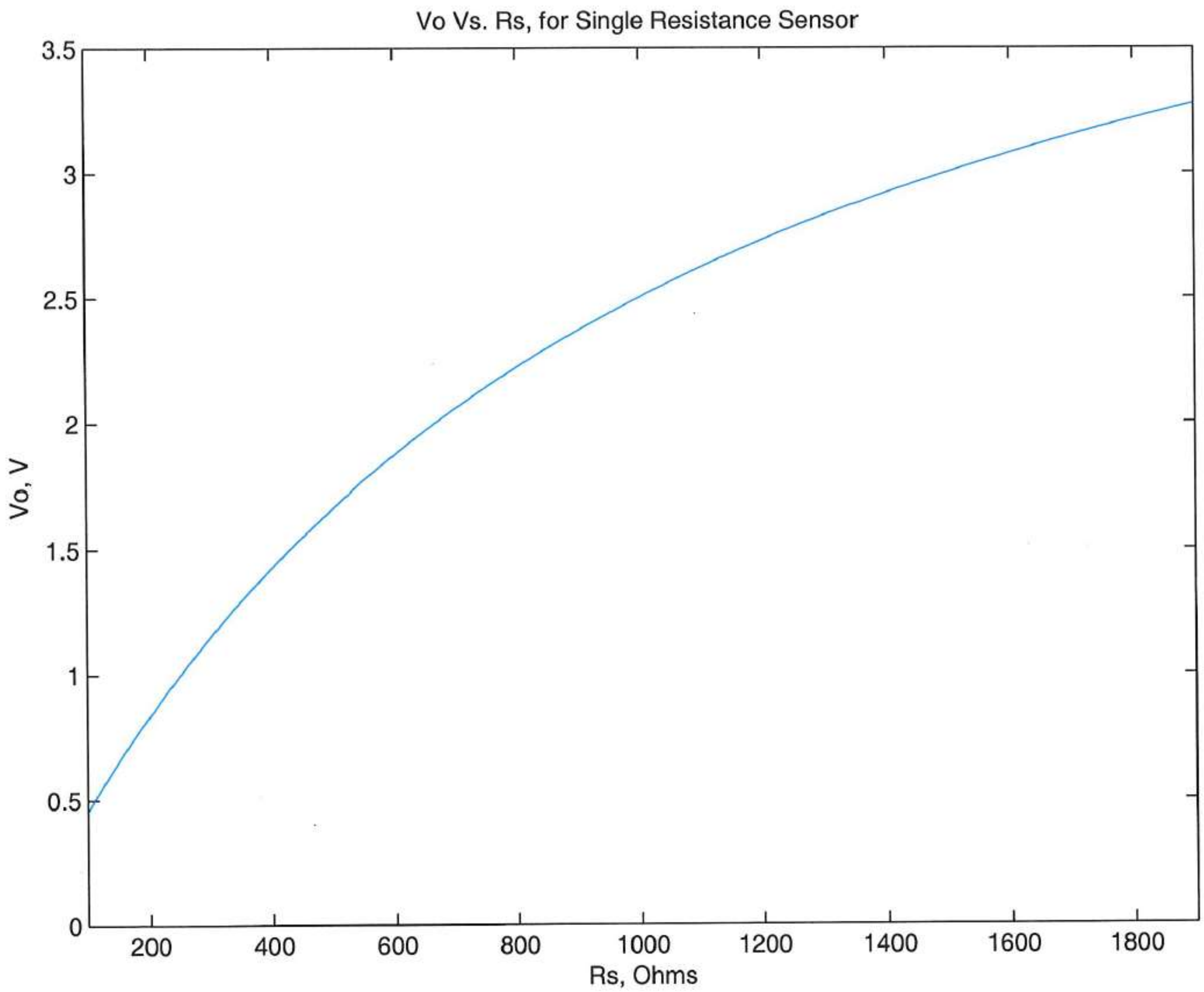


$$V_o = \frac{5R_s}{1000 + R_s}$$

Notice the V_o is a nonlinear function of R_s.

A plot of V_o vs. R_s is shown on the next page,

where $100\ \Omega \le R_s \le 1900\ \Omega$.

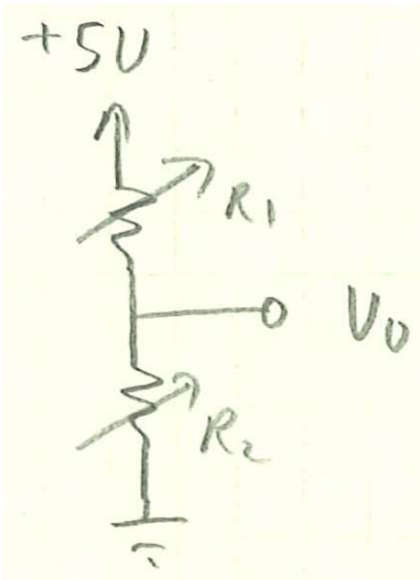


Although the interface circuit is powered off of 5V, the output voltage only ranges from about 0.4 V to 3.3V.

Although R_s might be linearly proportional to the measurand, V_o is clearly not linearly proportional to R_s. Is this a problem? It might be or it might not be, depending on the application.

2. A differential resistance sensor

Here, the sensor consists of two resistors, R_1 and R_2, similar to a potentiometer, where the measurand causes one resistor to increase in resistance while the other one decreases by the same amount.



$$V_o = \frac{5R_2}{R_1 + R_2}$$

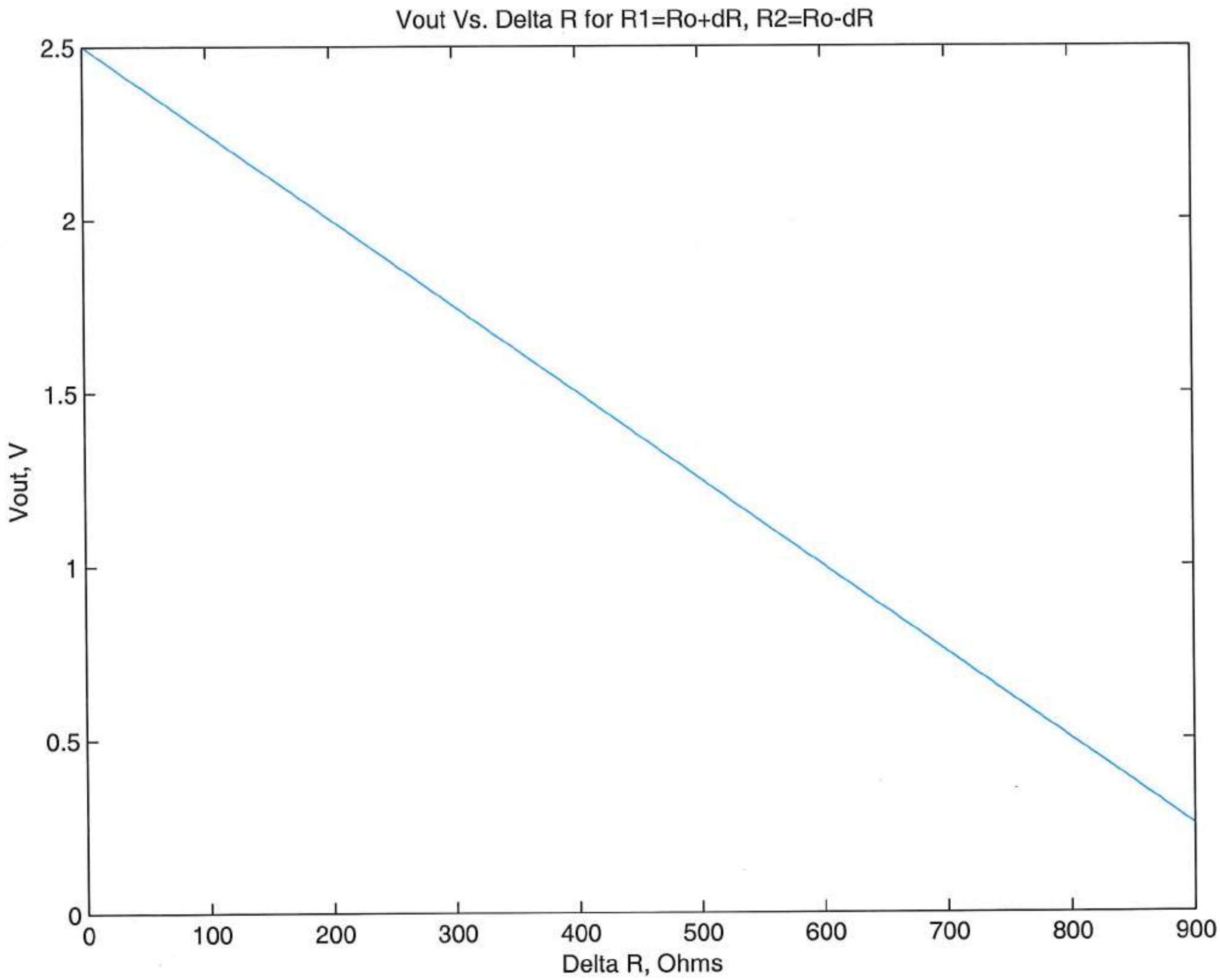
Let's let: $R_1 = R_o + \Delta R$ and $R_2 = R_o - \Delta R$,

where R_o is a constant and ΔR is a function of the measurand.

Therefore: $V_o = \frac{5(R_o - \Delta R)}{R_o + \Delta R + R_o - \Delta R} = 2.5 - 2.5\frac{\Delta R}{R_o}$

Observe that V_o is now linear function of ΔR.

Example: Let Ro = 1 kΩ and 0 Ω ≤ ΔR ≤ 900 Ω

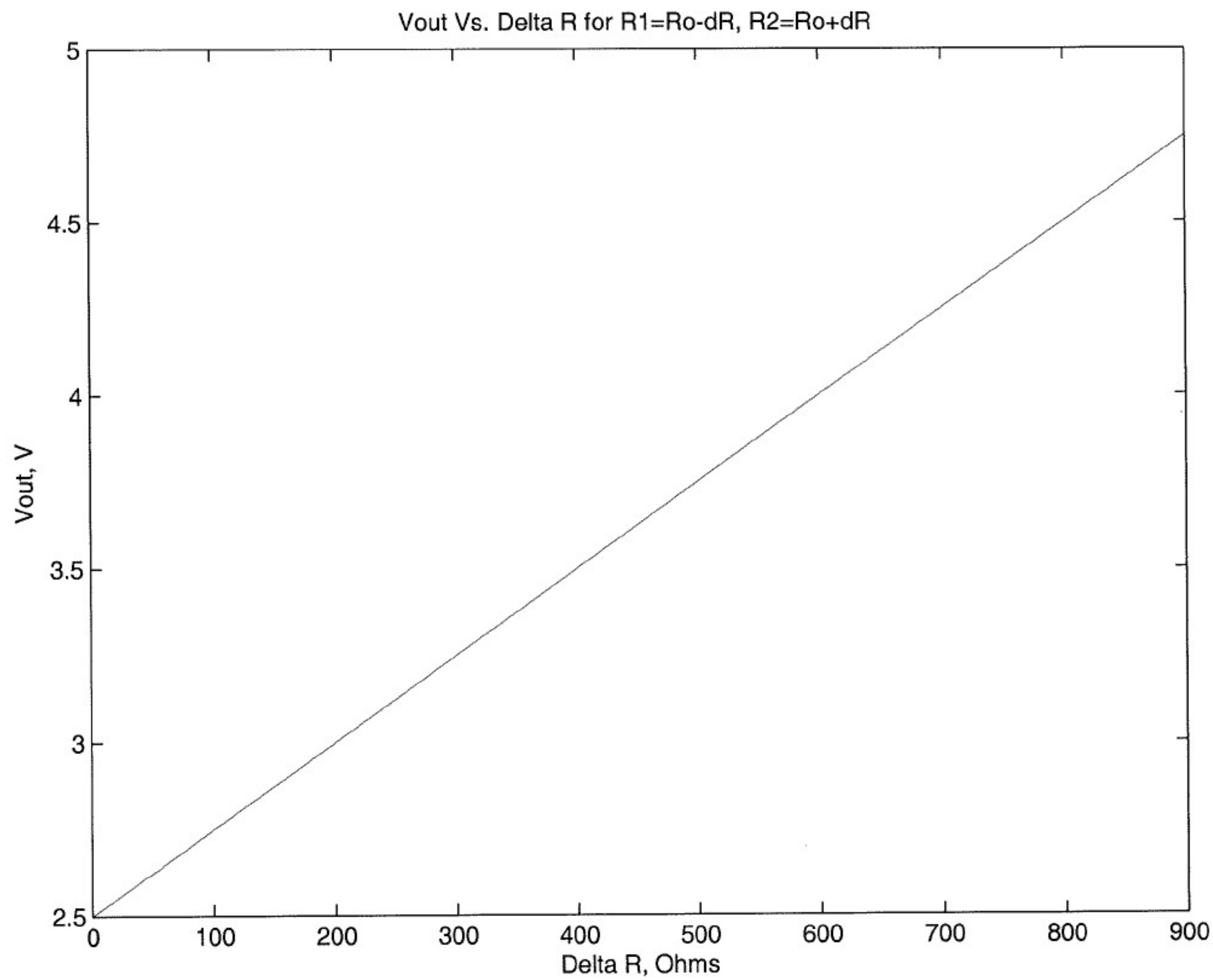


V_o is now linearly proportional to -ΔR, but it only goes from about 0.25 V to 2.5 V.

Similarly, let: $R_1 = R_o - \Delta R$ and $R_2 = R_o + \Delta R$, resulting in

$$V_o = \frac{5(R_o+\Delta R)}{R_o-\Delta R+R_o+\Delta R} = 2.5 + 2.5\frac{\Delta R}{R_o},$$

yielding:



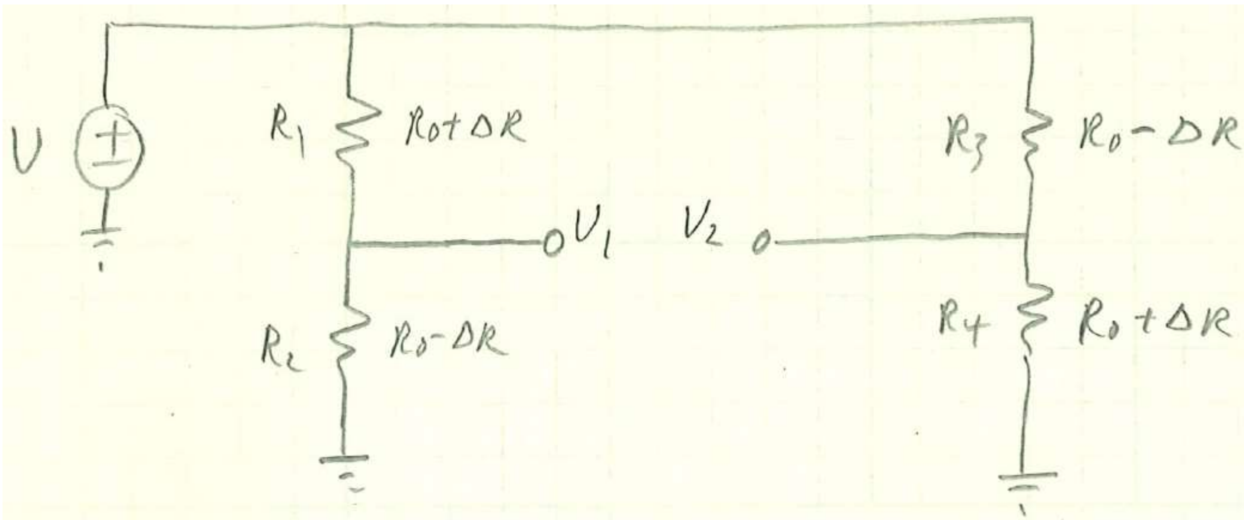
Which is still a linear response, but the slope is now positive, with V_o between about 2.5 V and 4.75 V.

3. Dual differential resistance sensor

Some resistance sensors consist of 4 resistors, arranged as two differential pairs:

$$R_1 = R_o + \Delta R$$
$$R_2 = R_o - \Delta R$$
$$R_3 = R_o - \Delta R$$
$$R_4 = R_o + \Delta R$$

Let's connect the 4 resistors as shown below:



Notice that the resistors are connected to realize two differential pairs where one is inverted compared to the other one. The is called a Wheatstone Bridge sensor configuration.

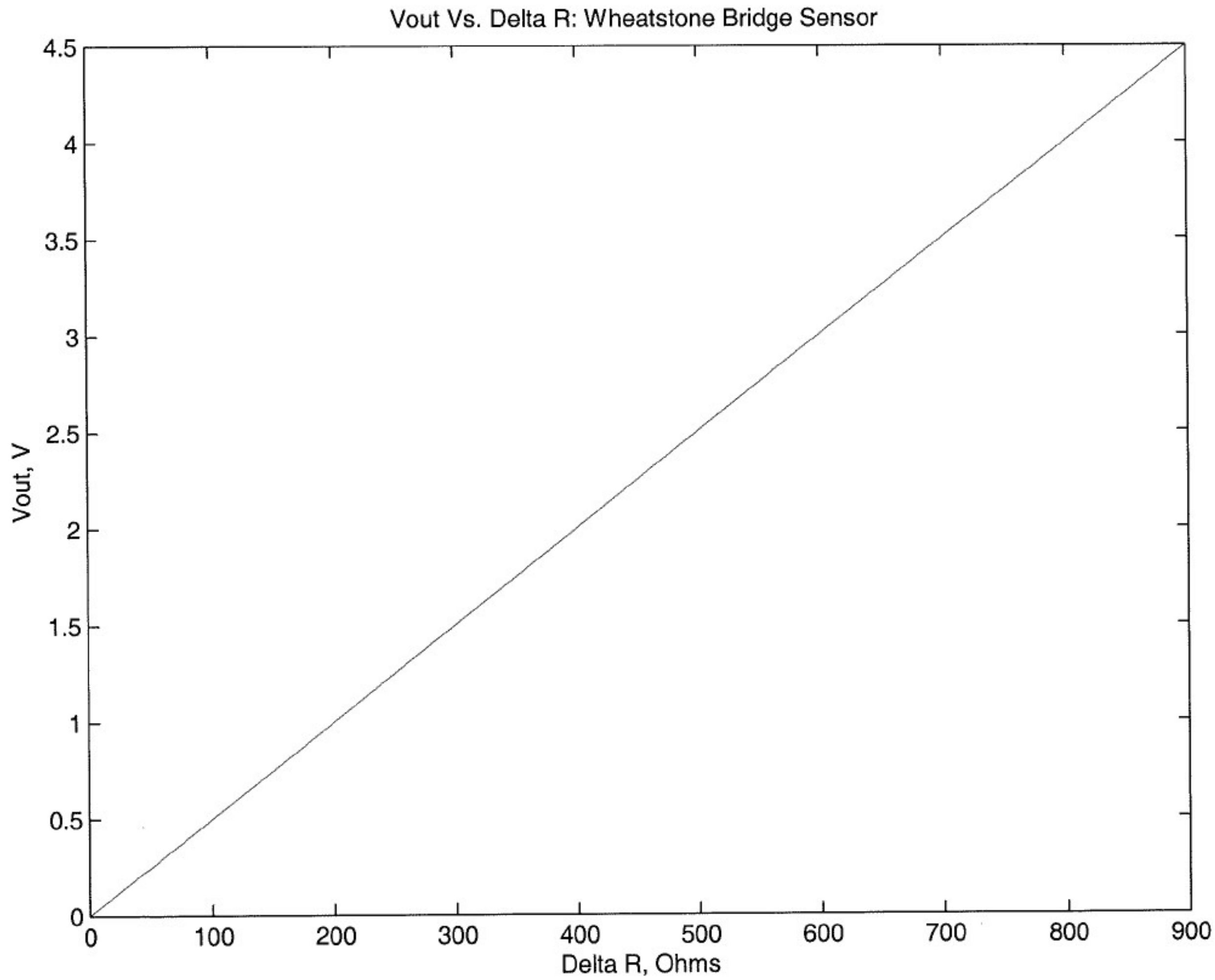
$$V_1 = \frac{V(R_o - \Delta R)}{R_o + \Delta R + R_o - \Delta R} = \frac{V(R_o - \Delta R)}{2R_o}$$

$$V_2 = \frac{V(R_o + \Delta R)}{R_o + \Delta R + R_o - \Delta R} = \frac{V(R_o + \Delta R)}{2R_o}$$

Let's define: $V_o = V_2 - V_1 = \frac{V(R_o+\Delta R)}{2R_o} - \frac{V(R_o-\Delta R)}{2R_o} = V\frac{\Delta R}{R_o}$

Example: $R_o = 1\ k\Omega$, $V = 5\ V$, $0\ \Omega \leq \Delta R \leq 900\ \Omega$

yielding:



V_o is a linear function of ΔR.

Notice that this configuration has a larger V_o range (0 V to 4.5 V) than with the 2-resistor differential resistance sensor, i.e. this is a more sensitive sensor.

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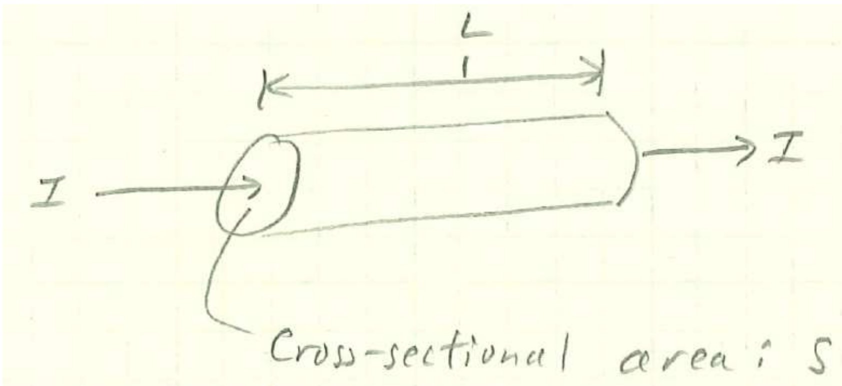
Resistive Sensors

1. Resistance of a section of conductor

Consider this section of a conductor:



Resistance: $R = \rho \frac{L}{S}$

where $\rho \equiv$ resistivity, a material property

$[\rho] = \Omega$-cm

2. Temperature effects

ρ varies with temperature:

For metals: $\rho[T] \approx \rho_o(1 + \alpha_T T + \beta_T T^2)$

where $\rho_o \equiv$ a resistivity at some reference temperature, such as 0°C
α_T and $\beta_T \rightarrow$ temperature coefficients
$\alpha_T \equiv$ linear temperature coefficent of resistivity (TCR)
$[\alpha_T] = 1/°C$ and $[\beta_T] = 1/(°C)^2$

For metals: $\alpha_T \sim 10^{-3}\ ^oC^{-1}$ and $\beta_T \sim 10^{-7}\ ^oC^{-2}$

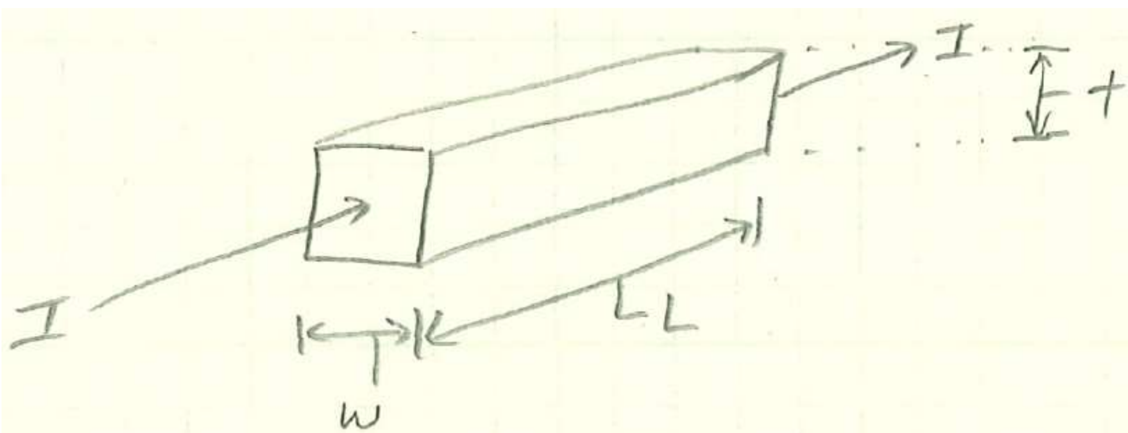
Since $\alpha_T >> \beta_T$, an approximation is often used: $\rho[T] \approx \rho_o(1 + \alpha_T T)$

Example: at 20°C for Al: $\rho_o = 2.65\text{x}10^{-6}$ Ω-cm and $\alpha_T = 4.3\text{x}10^{-3}\ ^oC^{-1}$, where T is relative to the reference temperature.

This property can be used to make a metal temperature sensor (more on this later in the course).

3. Strain effects

Consider this section of a conductor:



For most materials, if you axially stretch along L, the cross-sectional area (wt) will shrink.

Let S = wt ≡ cross-sectional area of the conductor

Since $R = \rho\frac{L}{S}$, if L ↑ and S ↓, then R ↑

This leads to Poisson's Ratio: a ratio of the tendancy of a material to get thinner in a transverse direction when subjected to axial stretching.

Poisson's Ratio: $\nu = -\frac{transverse\ strain}{axial\ strain} = -\frac{\varepsilon_{trans}}{\varepsilon_{axial}}$

[ν] = dimensionless

Typical values for ν: 0.1 to 0.4

Examining the effects of strain:

$$R = \rho \frac{L}{wt} \qquad (1)$$

$$dR = \frac{L}{wt} d\rho + \frac{\rho}{wt} dL - \frac{L\rho}{w^2 t} dw - \frac{L\rho}{wt^2} dt \qquad (2)$$

$$\therefore \frac{(2)}{(1)} \equiv \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dw}{w} - \frac{dt}{t}$$

$$\frac{dL}{L} \equiv axial\ strain = \varepsilon_1$$

$$\frac{dw}{w} \equiv transverse\ strain = \varepsilon_w = -\nu\varepsilon_1$$

$$\frac{dt}{t} \equiv transverse\ strain = \varepsilon_t = -\nu\varepsilon_1$$

$$\therefore \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_1 + \nu\varepsilon_1 + \nu\varepsilon_1 = \frac{d\rho}{\rho} + \varepsilon_1 + 2\nu\varepsilon_1$$

Note: Young's Modulus: $E = \frac{stress}{strain} = \frac{\sigma}{\varepsilon}$

[E] = Pa = N/m^2 = [σ] = [P]

Let's define Gauge Factor = GF, where:

$$GF = \frac{dR/_R}{\varepsilon_1} = \frac{d\rho/_\rho}{\varepsilon_1} + (1 + 2\nu) = \frac{dR/_R}{dL/_L} = \frac{\Delta R/_R}{\Delta L/_L}$$

Note: in textbook on p. 86: it should be $\frac{\Delta R/_R}{\Delta L/_L}$

$\frac{d\rho/_\rho}{\varepsilon_1}$ → a change in resistivity due to strain → Piezoresistive Effect (PE)

$1 + 2\nu$ → a change in resistance due to a change in shape → Geometric Effect (GE)

$$GF = \frac{\%\ change\ in\ resistance}{\%\ change\ in\ length} = PE + GE$$

A sensor that makes use of the GE is called a <u>strain gauge</u>.

A sensor that makes use of the PE is called a <u>piezoresistor</u>.

For metals: GE > PE

For semiconductors: PE > GE

<u>From Table 5.1 in testbook</u>

Material	GF
Metal foils	2-5
Thin film metals	2
Single crystal Si	-125 to +200
Polysilicon	±30

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a. Strain Guages

Example: A certain metal strain guage has a nominal resistance of 1 kΩ, and has a GF = 2. If it experiences a 1% axial strain, what does the resistance become?

Solution

$$\varepsilon_1 = \frac{\Delta L}{L}\Big|_{1\%} = \frac{0.01}{1} = 0.01$$

$$GF = \frac{\Delta R/R}{\varepsilon_1} \rightarrow \Delta R = R\varepsilon_1 GF = (1000)(0.01)(2) = 20\ \Omega$$

$$R_{new} = R + \Delta R = 1000 + 20 = 1020\ \Omega$$

b. Piezoresistors

Single Crystal Si

P-type: GF up to +200

N-type: GF down to -125

Note: a negative GF means that the resistance decreases with applied strain (tensile strain)

$\frac{d\rho/\rho}{\varepsilon_1} = PE$: what causes the piezoresistive effect?

Answer: The applied strain affects the majority charge carriers in the semiconductor material:

P-Type: strain ↑ : mobility of the holes ↓ : ρ ↑

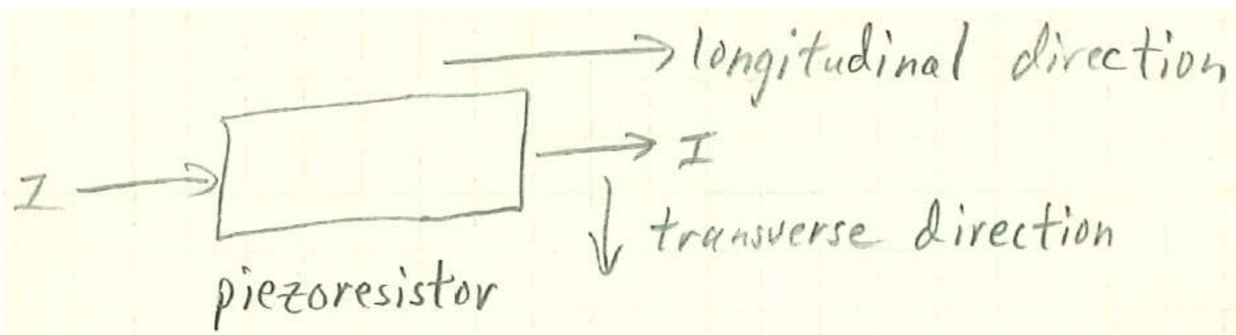
N-Type: strain ↑ : mobility of the electrons ↑ : ρ ↓

Note: this effect is highly dependent on crystallographic orientation, doping level, and temperature → pretty complicated

$$\frac{d\rho}{\rho} = \pi_l \sigma_l + \pi_t \sigma_t$$

Where: $\pi_l = longitudinal\ piezoresistive\ coefficient$
$\pi_t = transverse\ piezoresistive\ coefficient$
$\sigma_l = longitudinal\ stress$
$\sigma_t = transverse\ stress$

The longitudinal direction is defined as the direction parallel to the current flow through the piezoresistor.



π_l and π_t are a function of crystal orientation, doping, and temperature.

Polysilicon

Polysilicon is polycrystaline Si, therefore the piezoresistive effect averages over all directions

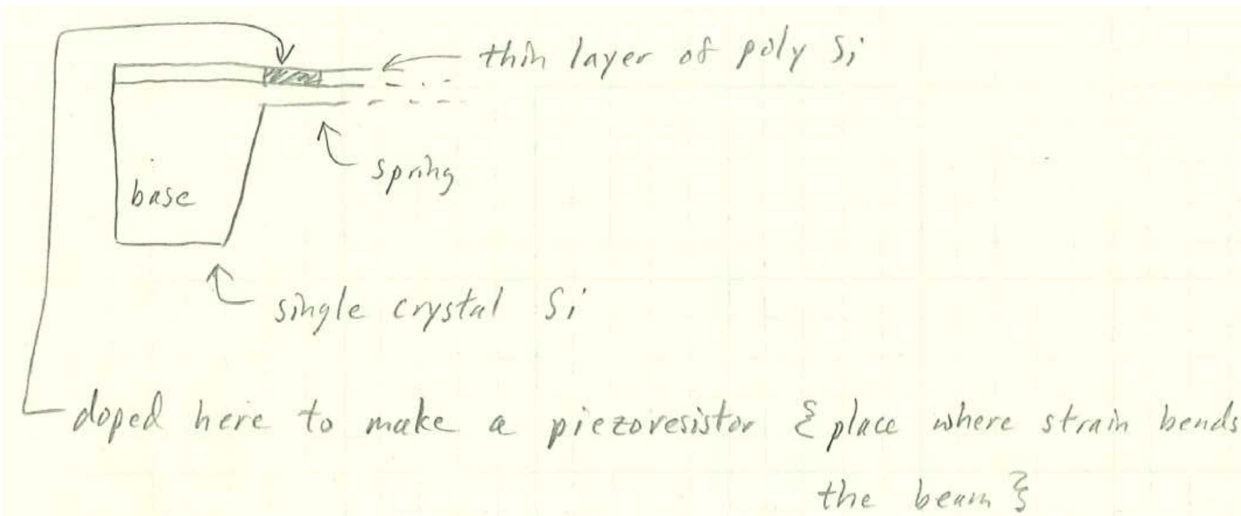
$$\therefore GF|_{\substack{poly \\ Si}} < GF|_{\substack{single \\ crystal \\ Si}}$$

P-type poly Si: GF ~ +30
N-type poly Si: GF ~ -30

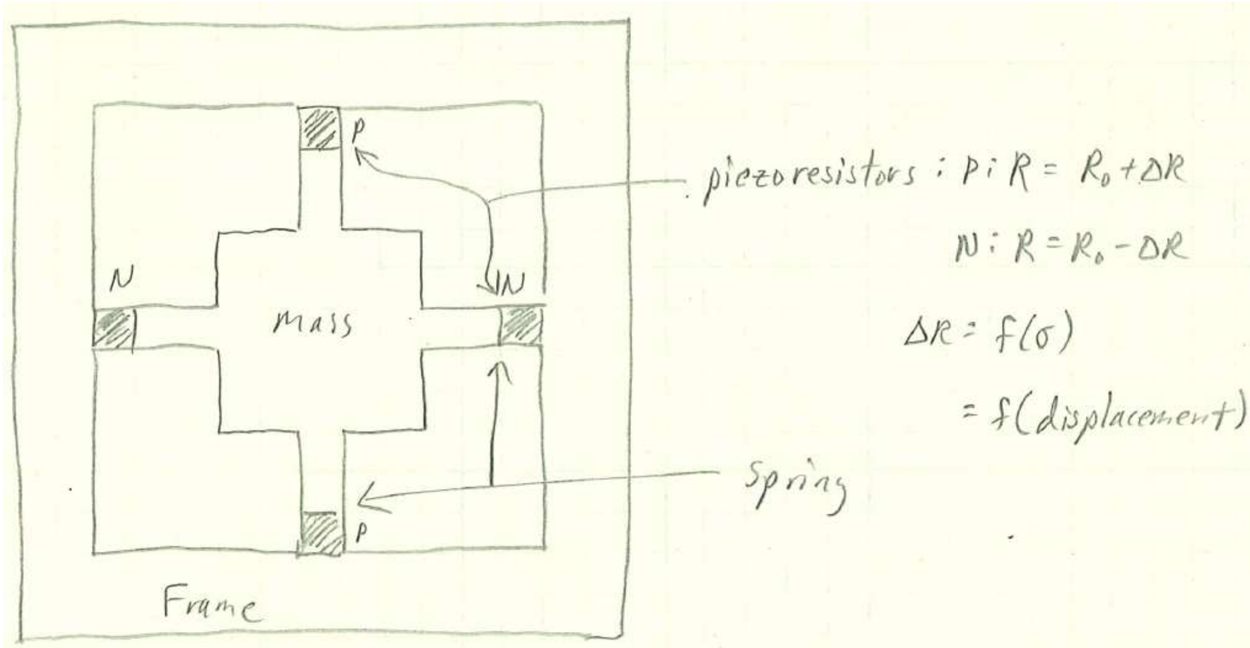
Polysilicon can be deposited as a thin film (up to a few μm), such as by LPCVD, and selectively doped to by N-type or P-type.

Both N-type and P-type polysilicon piezoresistors can be realized on the same chip → useful for realizing a Wheatstone bridge type sensor.

The piezoresistor's resistance changes with strain, such as on a spring element:



Consider this device:



With the four piezoresistors above, externally or internally connect them to realize a Wheatstone bridge confuguration.