Lecture 11/7/24

Inertial Sensors (MEMS Gyroscopes)

From last time, the equations of motion simplified to:

$m\ddot{x} + c\dot{x} + kx = A_x \sin(\omega_n t)$ (1)

$m\ddot{y} + c\dot{y} + ky + 2m\Omega\dot{x} = 0$ (2)

Clearly, $\Omega\hat{k}$ and motion along the x-axis produces corresponding motion along the y-axis {useful if $\dot{x}$ is consistent (periodic and known)}.

1) Solve for x(t) in steady state

We will start by assuming a solution of the form:

$x(t) = X_d \cos(\omega_n t)$

Then: $\dot{x}(t) = -X_d \omega_n \sin(\omega_n t)$

And: $\ddot{x}(t) = -X_d \omega_n^2 \cos(\omega_n t)$

Therefore $m\ddot{x} + c\dot{x} + kx = A_x \sin(\omega_n t)$ becomes:

$-mX_d\omega_n^2 \cos(\omega_n t) - cX_d\omega_n \sin(\omega_n t) + kX_d \cos(\omega_n t) = A_x \sin(\omega_n t)$

Equate cos() and sin() terms:

(1) cos() terms:

$-mX_d\omega_n^2 \cos(\omega_n t) + kX_d \cos(\omega_n t) = 0$

Reduces to: $kX_d = mX_d\omega_n^2$

And finally to: $\omega_n^2 = \frac{k}{m} \rightarrow$ true but not helpful.

(2) sin() terms:

$-cX_d\omega_n \sin(\omega_n t) = A_x \sin(\omega_n t)$

Reduces to: $X_d = \frac{-A_x}{c\omega_n}$

Therefore: $x(t) = \frac{-A_x}{c\omega_n}\cos(\omega_n t)$

Then: $\dot{x}(t) = -X_d\omega_n \sin(\omega_n t) = \frac{A_x}{c}\sin(\omega_n t)$

F_x produces this motion of m along the x-axis: $x(t) = \frac{-A_x}{c\omega_n}\cos(\omega_n t)$.

Observe that as c increases: Q decreases and the amplitude of x(t) decreases.

2) Solve for the steady state motion of y(t)

From EQ (2): $m\ddot{y} + c\dot{y} + ky + 2m\Omega\dot{x} = 0$

Which can be rewritten as: $m\ddot{y} + c\dot{y} + ky = -2m\Omega\dot{x}$

Plugging in for $\dot{x}$:

$$m\ddot{y} + c\dot{y} + ky = -2m\Omega\frac{A_x}{c}\sin(\omega_n t) = A_y \sin(\omega_n t)$$

Where: $A_y = -2m\Omega\frac{A_x}{c}$

Let's assume a solution for y(t):

$y(t) = Y_d \cos(\omega_n t)$

Then: $\dot{y}(t) = -Y_d \omega_n \sin(\omega_n t)$

And: $\ddot{y}(t) = -Y_d \omega_n^2 \cos(\omega_n t)$

Therefore: $m\ddot{y} + c\dot{y} + ky = A_y \sin(\omega_n t)$ becomes:

$$-mY_d\omega_n^2 \cos(\omega_n t) - cY_d\omega_n \sin(\omega_n t) + kY_d \cos(\omega_n t) = A_y \sin(\omega_n t)$$

Equating the sin() terms:

$$-cY_d\omega_n \sin(\omega_n t) = A_y \sin(\omega_n t)$$

Therefore: $Y_d = -\frac{A_y}{c\omega_n} = \frac{2m\Omega A_x}{c^2\omega_n}$: NOTE: <u>use this for HW#9 probs 9&10</u>

So: $y(t) = \frac{2mA_x}{c^2\omega_n}\Omega \cos(\omega_n t) = G_1\,\Omega\cos(\omega_n t)$

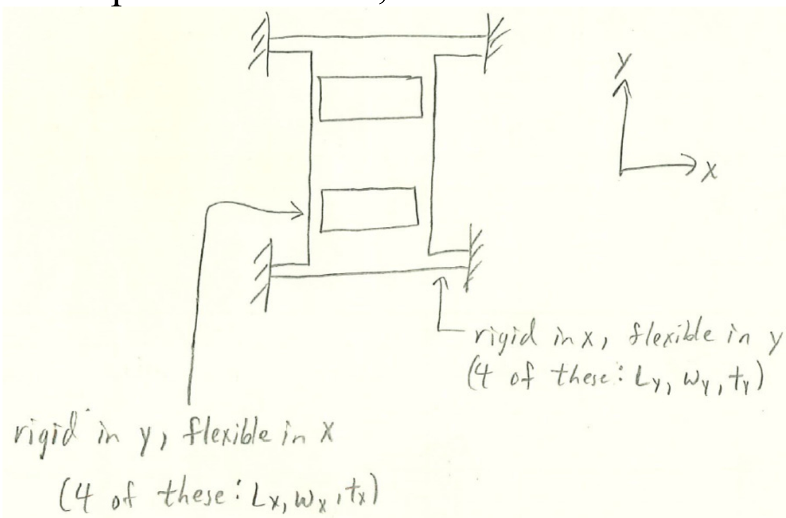
Where: $G_1 = \frac{2mA_x}{c^2\omega_n}$

With the resulting motion along the y-axis: measure y(t), multiply that measurement by $A\cos(\omega_n t)$ and then LPF the product, which results in a DC signal proportional to **Ω**.

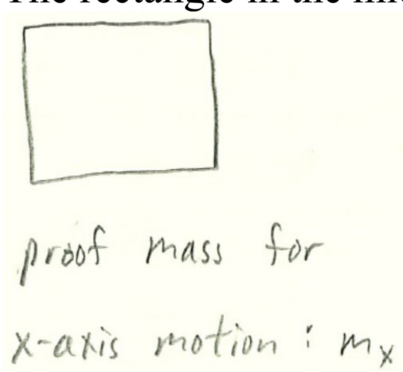
3) Realizing a possible MEMS implementation

a. Suspension system

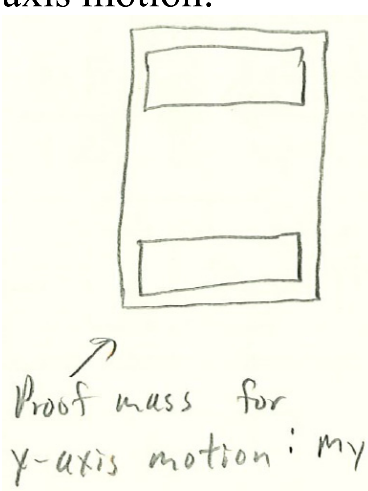
First, we need a suspension system that allows 2-D translational motion of the proof mass. So, consider this:



The rectangle in the middle is the rigid proof mass for x-axis motion:



The rectangular middle plus the x-axis springs is the proof mass for the y-axis motion:



Given that: $k_x = \frac{4Ew_x t_x^3}{L_x^3}$ and $\omega_{nx} = \sqrt{\frac{k_x}{m_x}}$

While: $k_y = \frac{4Ew_y t_y^3}{L_y^3}$ and $\omega_{ny} = \sqrt{\frac{k_y}{m_y}}$.

Select w's, L's, t's, and m's so that $\omega_{nx} = \omega_{ny} = \omega_n$.

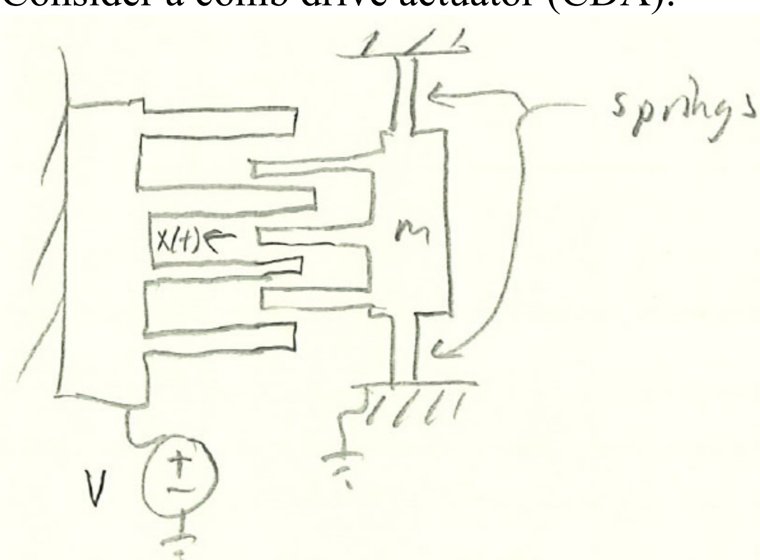
Also try to make $m_x \approx m_y$, because $\frac{\omega_n}{Q} = \frac{c}{m}$ and $c_x = c_y$ most likely.

More symmetric suspension systems are typically used, but the concept presented here is valid for discussion purposes.

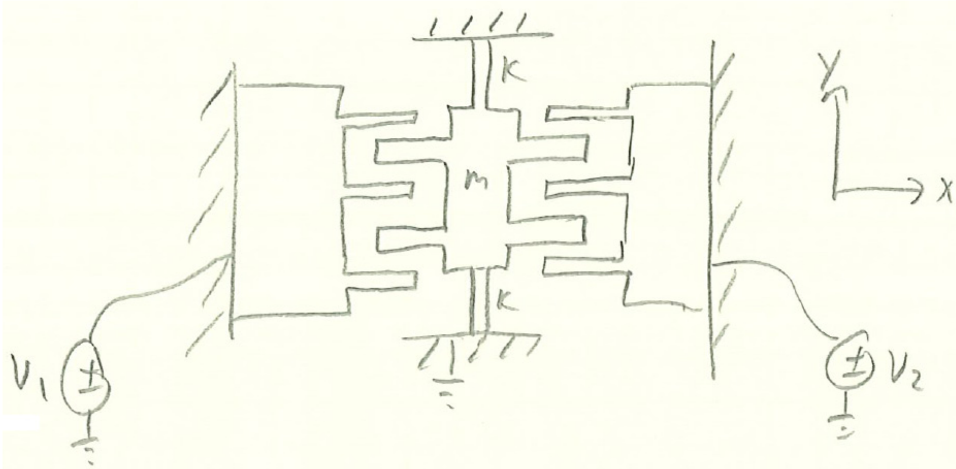
a. Generating $\vec{F}$

We need an actuator to generate $\vec{F} = A_x \sin(\omega_d t)\hat{\imath}$. Piezoelectric and electrostatic actuators have been used for this purpose.

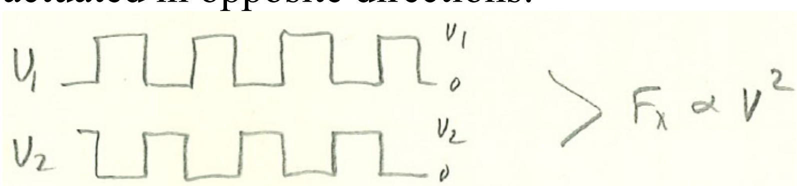
Consider a comb drive actuator (CDA):



The CDA can only pull m in one direction. So consider this:



With 2 CDA's and alternate V_1 and V_2 (180° out of phase), m can be actuated in opposite directions:



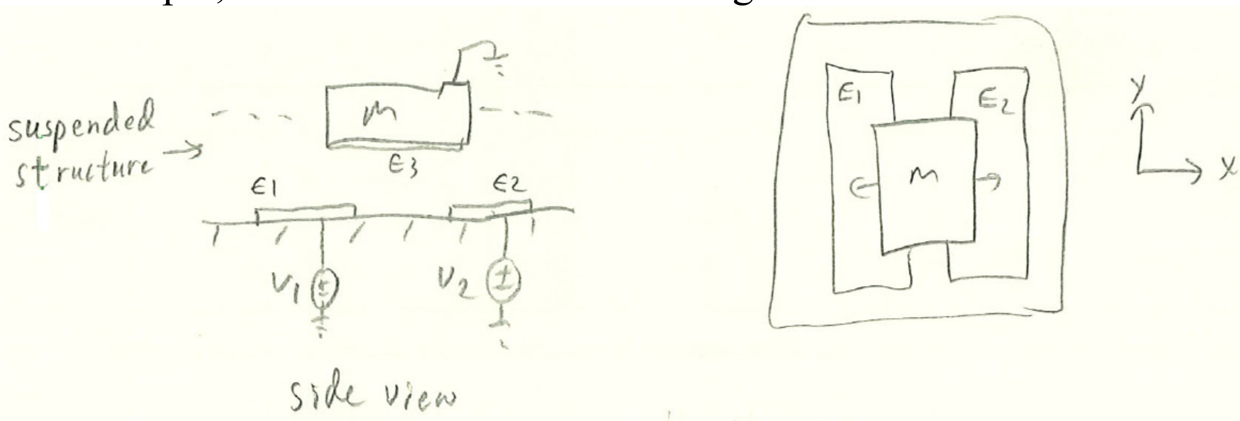
Note: $V_1 = V_{DC} + V_{AC}\cos(\omega t) + H.O.T.$

Then: $V_1^2 = V_{DC}^2 + 2V_{DC}V_{AC}\cos(\omega t) + (V_{AC}\cos(\omega t))^2 + H.O.T.$

Notice that there is a force component at ω. If Q is high enough, and V_1 and V_2 state-change pairs occur at $\omega = \omega_n$, then x(t) is "nearly" sinusoidal even though F_x is not. The higher order terms are present, though, and will affect the noise floor of the sensor: high precision MEMS gyroscopes would use a true sinusoidal F_x producing actuator.

Note: The CDA suspension system will have to be designed to allow some (ideally small) motion to occur orthogonal to x(t), due to the Coriolis acceleration, unless a y-axis force feedback controller is used to null out the y-axis motion like a closed-loop accelerometer. However, other types of electrostatic actuators could be used to avoid this issue.

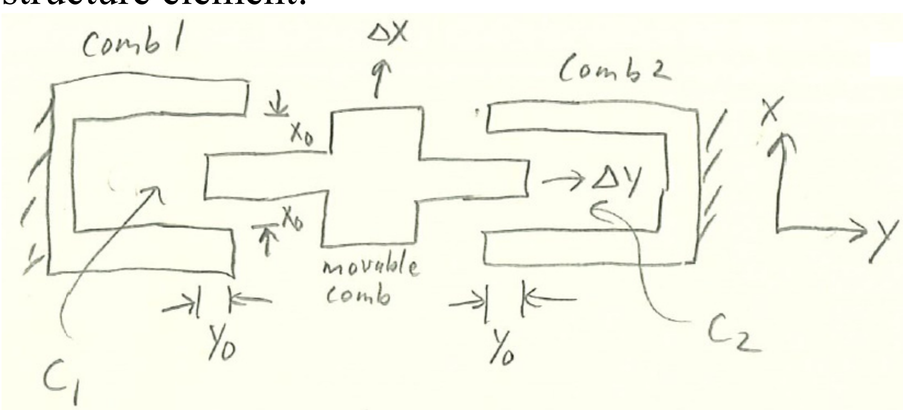
For example, consider this electrostatic tangential actuator:



This actuator attempts to increase the overlap area. As a tangential actuator, force is not a function of displacement. Observe that y-axis motion does not affect the overlap of E_1 or E_2.

a. Sensing of y(t) motion

Although many techniques are possible, consider this differential comb structure element:



All structures are "t" tall (normal to the plane of the paper).

Here, we will define C_1 and C_2 between the <u>sides</u> of the movable comb teeth and the <u>sides</u> of the fixed comb teeth. Capacitance due to the ends of the teeth is not considered here, for simplicity.

"n" comb teeth elements exist in the full comb structure. Therefore:

$$C_1 = n\varepsilon_o\varepsilon_r t(y_o - \Delta y)\left(\frac{1}{x_o + \Delta x} + \frac{1}{x_o - \Delta x}\right)$$

and

$$C_2 = n\varepsilon_o\varepsilon_r t(y_o + \Delta y)\left(\frac{1}{x_o + \Delta x} + \frac{1}{x_o - \Delta x}\right)$$

If $\Delta x \ll x_o$, then: $\left(\frac{1}{x_o+\Delta x} + \frac{1}{x_o-\Delta x}\right) \approx \frac{2}{x_o}$

Let's let $G_2 = \frac{2n\varepsilon_o\varepsilon_r t}{x_o}$, leading to:

$C_1 \approx G_2(y_o - \Delta y)$ and $C_2 \approx G_2(y_o + \Delta y)$

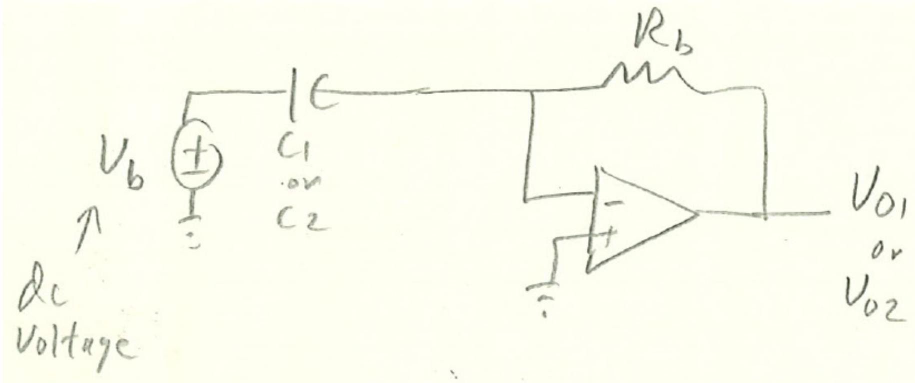
$y(t) = G_1\Omega\cos(\omega_n t)$ {from p. 3 today} where: $G_1 = \frac{2mA_x}{c^2\omega_n}$

y(t) is the Δy above, leading to:

$C_1 \approx G_2\big(y_o - G_1\Omega\cos(\omega_n t)\big)$ and

$C_2 \approx G_2\big(y_o + G_1\Omega\cos(\omega_n t)\big)$

Let's interface C_1 and C_2 through their own transimpedance amplifiers (TIA's):



Therefore, in general from the TIA: $V_o = -R_b\left(\dot{V}_b C + \dot{C} V_b\right)$.

However, here V_b is DC. Therefore $\dot{V}_b = 0$ V/s

So, $\dot{C}_1 = G_1 G_2 \omega_n \Omega \sin(\omega_n t)$, and

$\dot{C}_2 = -G_1 G_2 \omega_n \Omega \sin(\omega_n t)$,

Therefore:

$V_{01} = -V_b R_b G_1 G_2 \omega_n \Omega \sin(\omega_n t)$, and

$V_{02} = V_b R_b G_1 G_2 \omega_n \Omega \sin(\omega_n t)$,

Let's define: $V_o = V_{02} - V_{01}$

$\therefore\ V_o = 2 V_b R_b G_1 G_2 \omega_n \Omega \sin(\omega_n t)$

If we mix V_o with $V_x \sin(\omega_n t)$, and LPF to get Vout:

$$V_{OUT} = V_b V_x R_b G_1 G_2 \omega_n \Omega = \frac{4 n m A_x \varepsilon_o \varepsilon_r t V_b V_x R_b}{c^2 x_o} \Omega$$

Remember that $\vec{F} = A_x \sin(\omega_d t)\hat{\imath}$

If the actuator is a CDA, then: $A_x \approx \frac{n_x \beta b \varepsilon_o \varepsilon_r V_D^2}{d}$.

Including the equation for A_x, V_{OUT} becomes:

$$V_{OUT} = \frac{4nn_x \beta bmt\varepsilon_o^2 \varepsilon_r^2 V_D^2 V_b V_x R_b}{c^2 x_o d} \Omega$$

Which can be reduced to:

$$V_{OUT} = K\Omega$$

Where V_{OUT} is a DC voltage proportional to **Ω**.

Observe that K is made up of true constants (4, n, n_x, ε_o), parameters dependent of fabrication/packaging/material/temperature tolerances (β, b, m, t, ε_r, R_b, c, x_o, d), and signals that will be off/noisy (V_D, V_b, V_x). So, how constant is K really?

Also, a lot of assumptions, approximations, and simplifications went into deriving K.

1) A real MEMS gyroscope:

Consider this MEMS gyroscope chip fabricated with an SOI process:

Photograph of a MEMS Gyroscope

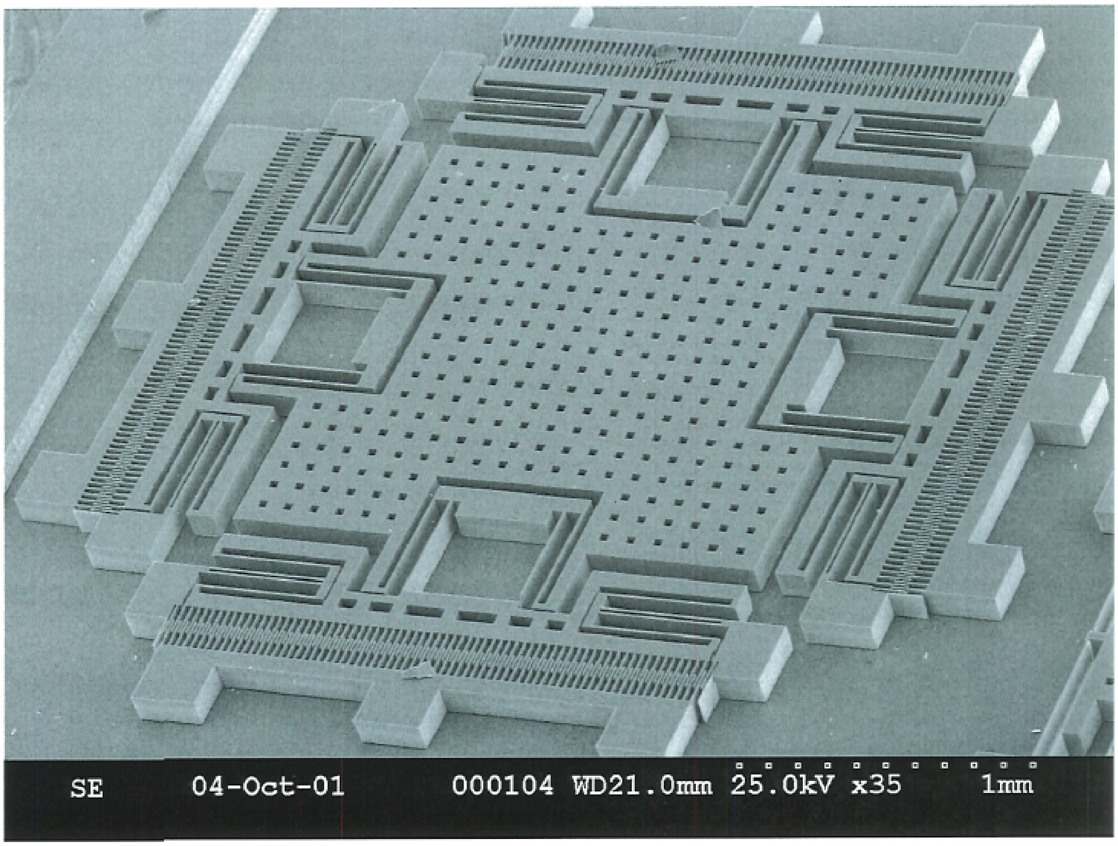


Photo courtesy of Morgan Research Corporation

Photograph of a MEMS Gyroscope

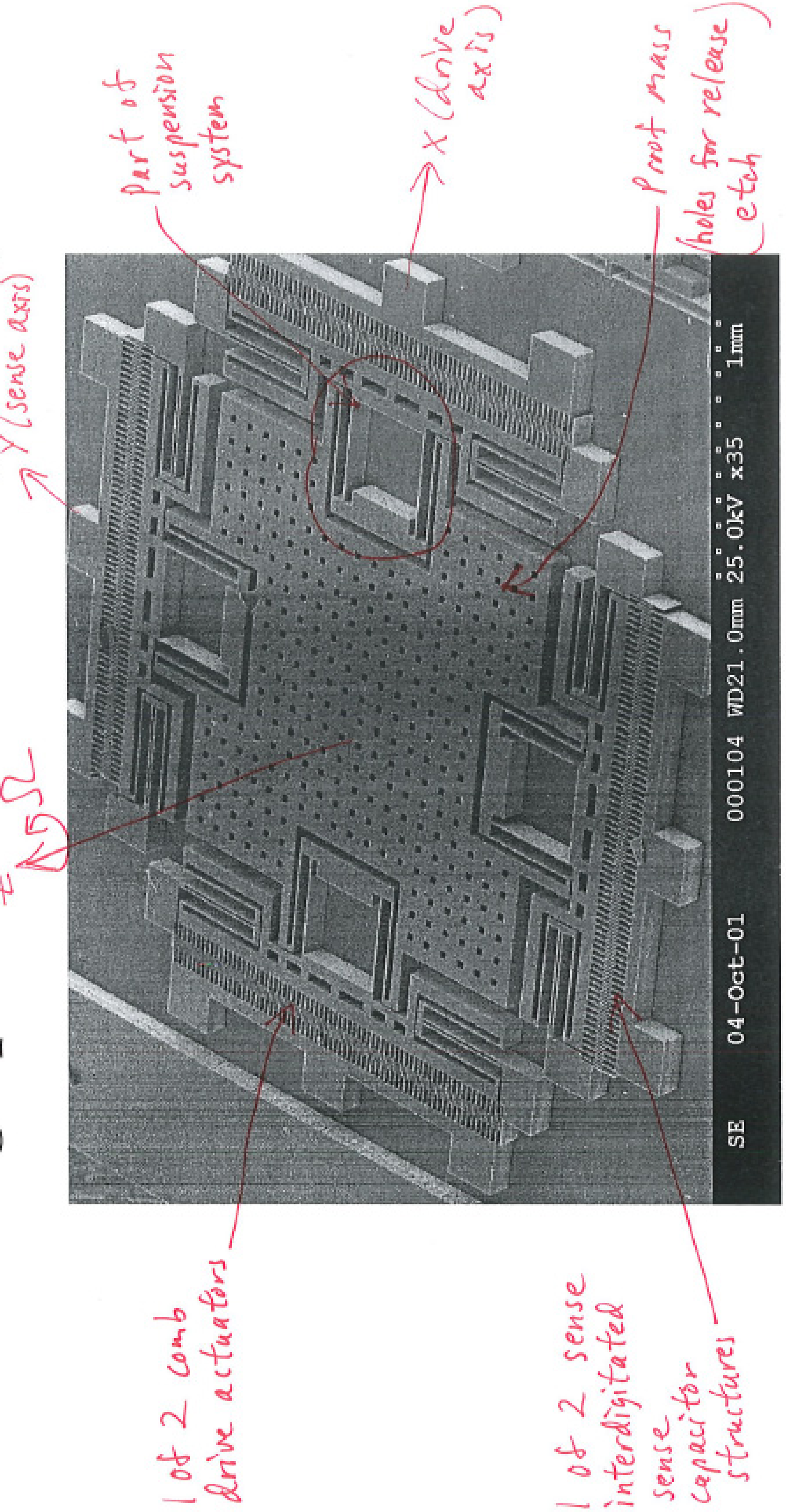


Photo courtesy of Morgan Research Corporation