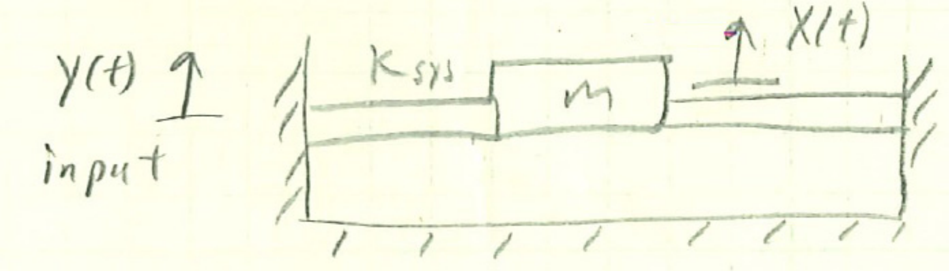
Lecture 10/29/24

Inertial Sensors (Accelerometers) Continued

1) Transient terms in the accelerometer output

So far, we have just considered the steady state operation of our MEMS SMD accelerometer. However, since it is a second order dynamic system, it has a steady state response and a transient response, and S (Sensitivity) is only defined for the steady state response. Also, we need to wait until the output signal has reached steady state to accurately determine the acceleration from the sensor's output signal.

Assuming that $\zeta = 1$, consider this model for our accelerometer:



We know that $k_{sys} \approx \frac{N_{leg}}{N_{zig}} \frac{Ewt^3}{L^3}$

Then, for the system spring model: $F_s = k_{sys}d$,

where d(t) = y(t) – x(t) or D(s) = Y(s) – X(s)

Also: d = aS, where a is a constant acceleration

We need the Laplace transform for y(t) when a is constant:

Acceleration: $a = \ddot{y}$, [a] = m/s^2

Velocity: $\dot{y} = \int_0^t a dt = at + v_o$, v_o is initial velocity

Displacement: $y = \int_o^t (at + v_o)dt = \frac{1}{2}at^2 + v_o t + y_o,$

y_o is initial displacement

Observe that a double integration is required to determine distance traveled from acceleration measurements. Any errors in acceleration measurement will accumulate over time, leading to a significant error between actual distance traveled and calculated distance traveled from acceleration measurements. Therefore, very accurate acceleration measurements are required to accurately estimate distance traveled for other than very short time periods.

For the case where $y_o = 0$ and $v_o = 0$: $y(t) = \frac{1}{2}at^2$

Let y(t) be the input to a MEMS device. It would be useful to take the Laplace transform of y(t).

From a Laplace Transform table: $\frac{t^{n-1}}{(n-1)!} \xrightarrow{L} \frac{1}{s^n}$

For $\frac{1}{2}at^2$: n – 1 = 2 → n = 3

$$\therefore \frac{1}{2}at^2 \xrightarrow{L} \frac{a}{s^3}$$

From transmissibility lectures earlier in the semester:

$$T(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Therefore X(s) = Y(s)T(s), and

$$D(s) = Y(s) - X(s)$$

= Y(s) – Y(s)T(s)

= Y(s)(1 – T(s))

$= Y(s)\left[\frac{s^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right]$, with $\zeta = 1$

Y(s) is then due to the constant acceleration: $Y(s) = \frac{a}{s^3}$

From algebra then: $D(s) = \frac{a}{s(s+\omega_n)^2}$

Using partial fraction expansion and inverse Laplace Transforms:

$$d(t) = \frac{a}{\omega_n^2} - \frac{a}{\omega_n^2}e^{-\omega_n t} - \frac{a}{\omega_n}te^{-\omega_n t}$$

$\frac{a}{\omega_n^2}$ is the steady state term: d = aS, where $S = \frac{1}{\omega_n^2} = \frac{m}{k}$

$-\frac{a}{\omega_n^2}e^{-\omega_n t}$ and $-\frac{a}{\omega_n}te^{-\omega_n t}$ are transient terms with a time constant,

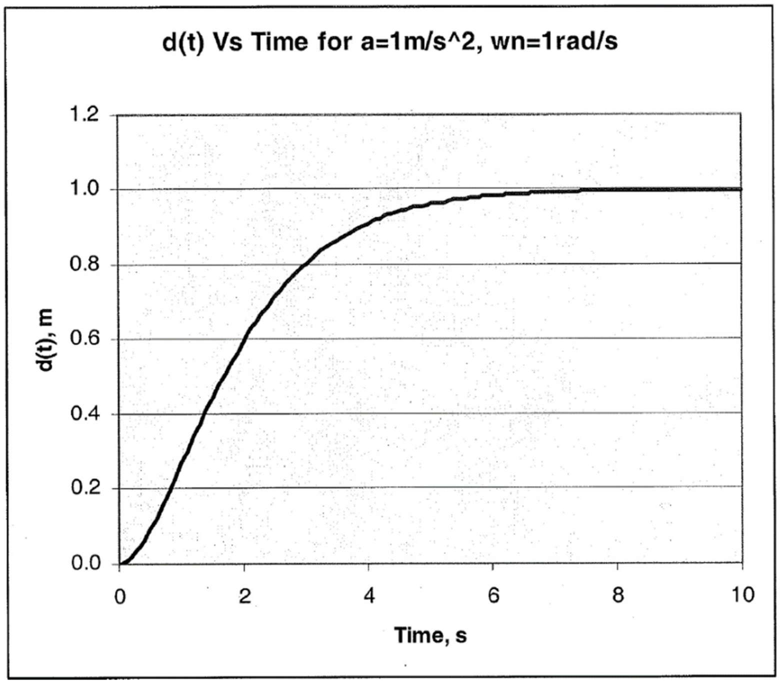
$$\tau_c = \frac{1}{\omega_n}$$

Therefore, wait at least a couple of τ_c to be approximately in steady state.

Consider a normalized example where a = 1 m/s^2 and ω_n = 1 rad/s, with ζ = 1:

Therefore, S = 1 s^2, τ_c = 1 s

Consider a plot of d(t) versus t:



Notice it took nearly 8 τ_c to approximately reach steady state.

2) Considerations for designing MEMS SMD accelerometers

Since $S = \frac{m}{k}$, and we usually want to maximize S:

make m as big as possible: large proof mass,

make k as small as possible.

Since $k \propto \frac{Ewt^3}{L^3}$, big L with small w and t would do it. However, big L means a large chip, so the springs are usually made to be short (small L), but thin (small t), in attempt to minimize k and chip size (more parts per wafer).

Packaging is a big part of the accelerometer: often it is hermetically sealed in an inert gas (N^2, Ar, etc.,) to try to make $\zeta = 1$.

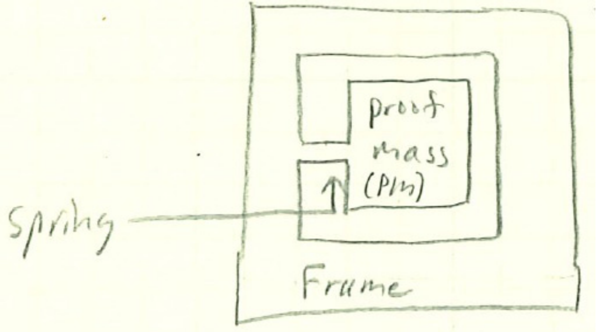
Inertial Sensors (MEMS Accelerometer Architectures)

1) Bulk micromachined accelerometer designs

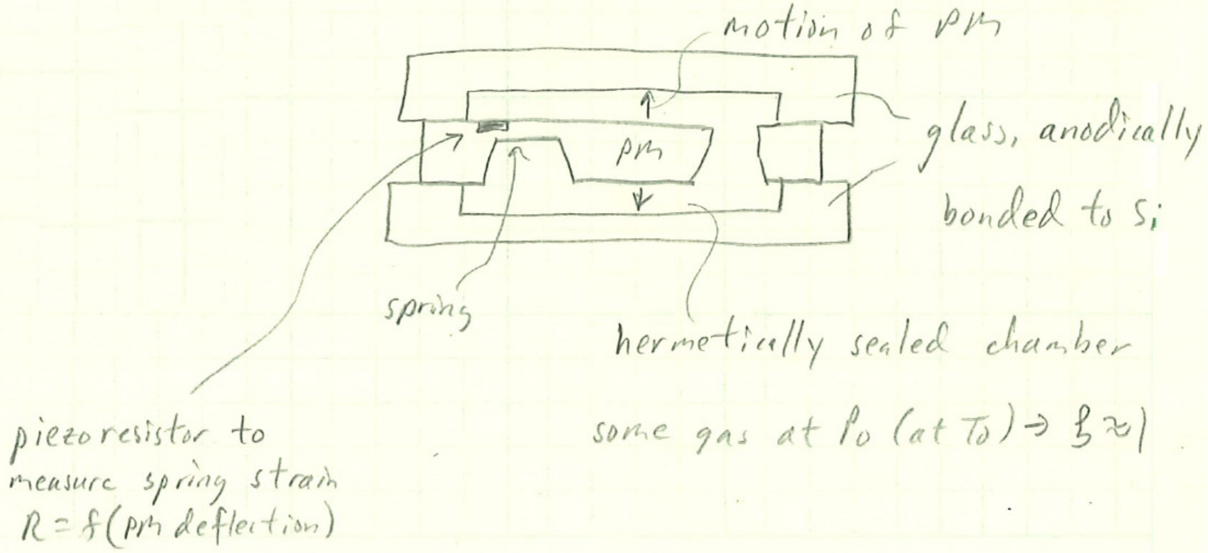
a. Single cantilevered spring design

Consider this design:

Micromachined Si part (frame, spring, and proof mass):



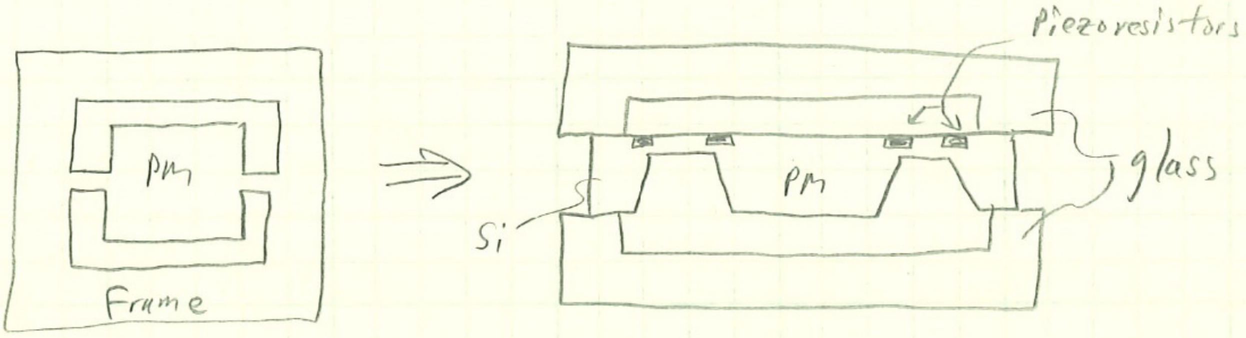
Complete accelerometer structure (cross-section):



Obviously, this simple design could have issues with lateral or torsional proof mass motion.

You should notice a similar architecture to the bossed diaphragm pressure sensor, where the diaphragm-like structure is made into a cantilevered spring and the proof mass.

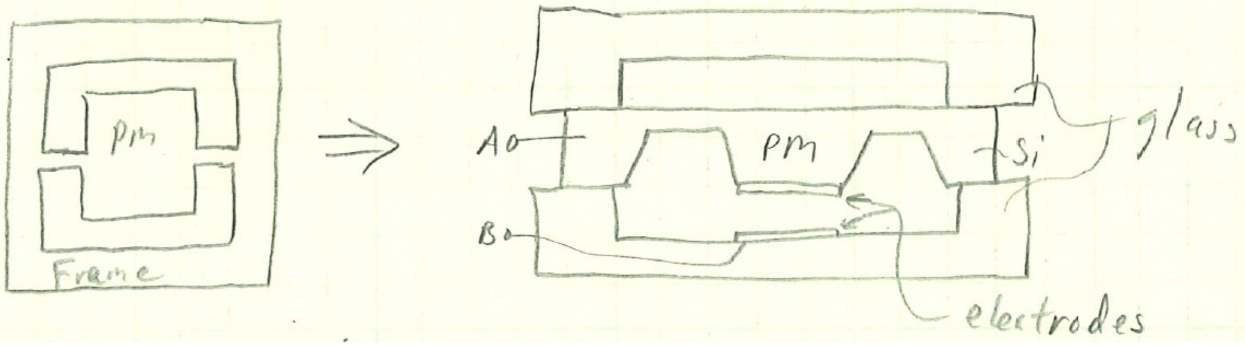
b. Double clamped beam suspension system design



Just as with the bossed pressure sensor, two PR's are in tension and two PR's are in compression (during an external acceleration event). The PR's in tension increase in R, while the PR's in compression decrease in R. Therefore, the four PR's would be connected to form a Wheatstone bridge.

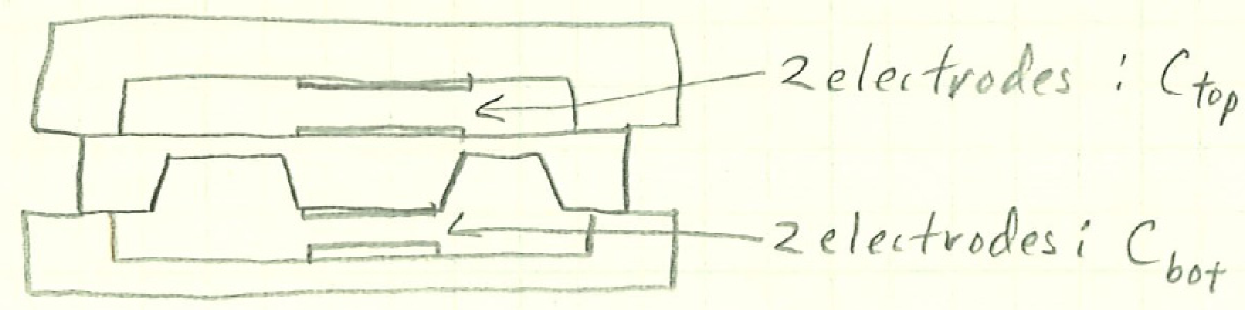
c. Single capacitance capacitive sensing accelerometer

This is also similar to pressure sensors.



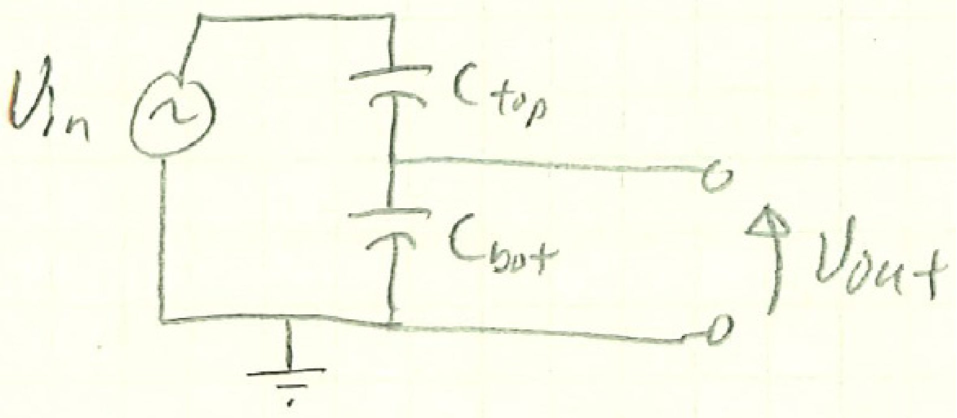
C is the capacitance between points A and B: $C = \frac{\varepsilon_o \varepsilon_r A}{d_o \pm \Delta d}$

d. Differential capacitance capacitive sensing accelerometer



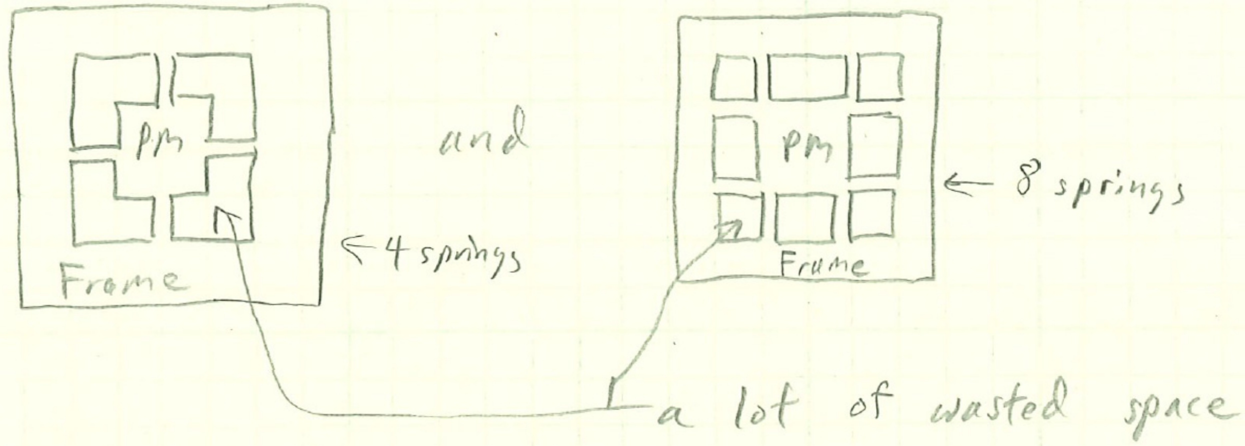
Where $C_{top} = \frac{\varepsilon_o \varepsilon_r A}{d_o \pm \Delta d}$ and $C_{bot} = \frac{\varepsilon_o \varepsilon_r A}{d_o \mp \Delta d}$.

The two capacitances can be wired into an AC voltage divider:



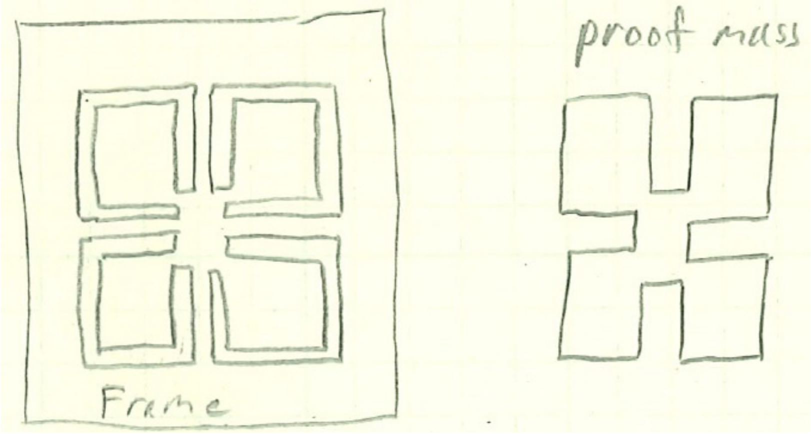
e. Other suspension system designs

Consider these spring configurations:



This wasted space causes two issues: extra cost in etching Si, and unusable real estate on the wafer. As in all real estate, space is $$.

Therefore, consider this design:



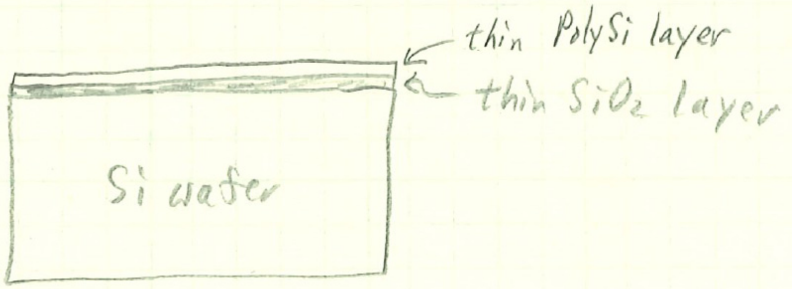
Now, much of the previously unused space is used to make a larger proof mass. However, be cautious of unexpected or undesirable bending modes with this proof mass structure.

This design yields a larger proof mass and longer springs (smaller k): which can result in a larger S.

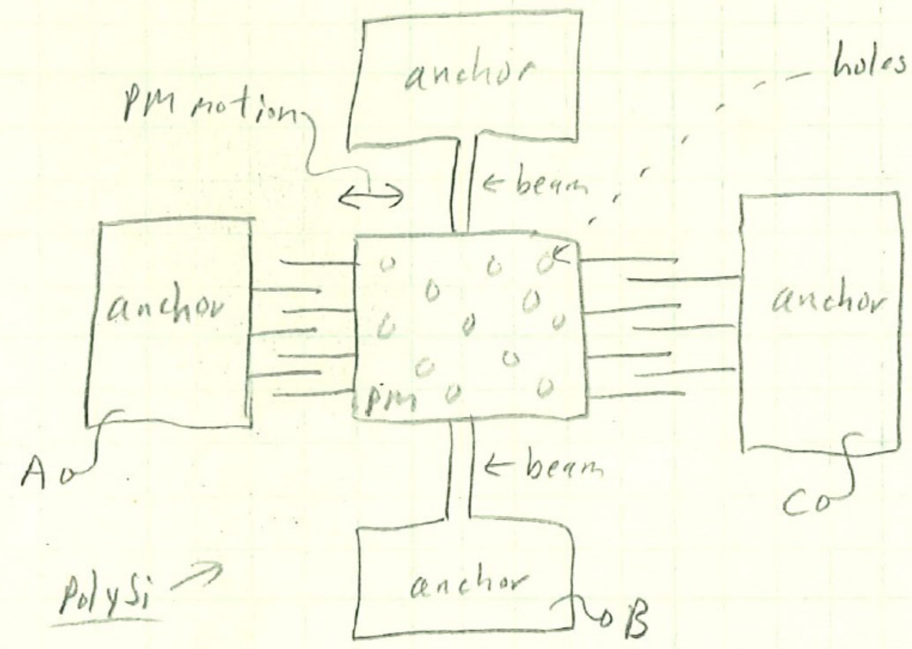
2) Surface micromachined accelerometer designs

We can build the frame, the suspension system, and the proof mass in a deposited layer of polysilicon on top of a deposited layer of silicon dioxide (sacrificial layer). This initial structure is similar to an SOI wafer, except the polysilicon layer is usually much thinner than an SOI wafer's Device Layer.

Start with:



Often, with this fabrication process, the accelerometer is designed so that the proof mass moves laterally, using interdigitated capacitive sensing to measure proof mass motion.



The holes in the proof mass (PM) allow the SiO_2 layer under the proof mass to be etched away to release the proof mass.

There are two complementary IDE capacitors: C_1 between points A and B, and C_2 between points B and C. Together, they realize a differential capacitive sensing structure, suitable for use as an AC voltage divider.

A similar lateral proof mass motion accelerometer could also be fabricated using an SOI wafer.