1. PROBLEM DEFINITION

The multi-criteria optimal location query (MOLQ) updating problem is defined based on MOLQ problem [1]. We first define the object insertion and deletion operations over object sets as two types of object updating operations over a set of object sets. Then, given a universal set of object sets, \( E = \{P_1, \ldots, P_n\} \), the query result MOLQ\((E, \sigma, \Delta)\), and a set of changes on \( E \), the MOLQ updating problem is defined as a process that finds the result of MOLQ after the changes have been applied to \( E \). The changes on \( E \) are abstracted by a set of object insertion or deletion operations on \( E \). The updates on a particular object (e.g., the changes in its location, type weight, or object weight) are equivalent to deleting the object from \( E \) and then adding it back with new attributes.

1.1 Object Updating to a Set of Object Sets

Object Insertion Operation. Given a set of object sets \( E = \{P_1, \ldots, P_n\} \) and an object \( q \) (\( \exists P_i \in E, q \in P_i \)), we assume there exists an object set \( P_j \in E \), in which the objects are in the same type with \( q \). Then we define the process of inserting \( q \) to \( E \) as follows:

\[
E' = E \uplus q = \{P_1, \ldots, P_j \cup \{q\}, \ldots, P_n\}
\]  
(1)

If an object set \( Q = \{q_1, \ldots, q_k\} \) is given to insert into \( E \), the insertion process is equivalent to sequentially inserting each object in \( Q \) to \( E \). Note that there must exist one and only one object set in \( E \), which contains objects in the same type with the newly inserted object. Objects in \( Q \) can be in different types. We define the process that inserts an object set \( Q \) to \( E \) as follows (\( \uplus \) is left associative):

\[
E' = E \uplus Q = E \uplus \{q_1\} \uplus \cdots \uplus \{q_k\}
\]  
(2)

Object Deletion Operation. The object deletion is an inverse operation of the object insertion \( \uplus \). Given a set of object sets \( E = \{P_1, \ldots, P_n\} \) and an object \( p_i \in P_i \), the object deletion operation that removes \( p_i \) from \( E \) is defined as follows:

\[
E' = E \ominus p_i = \{P_1, \ldots, P_i \setminus \{p_i\}, \ldots, P_n\}
\]  
(3)

Deleting a set of objects \( Q = \{q_1, \ldots, q_k\} \) from \( E \) can be completed by removing each object in \( Q \) from \( E \). Every objects in \( Q \) must be contained by an object set in \( E \). Objects in \( Q \) can be in different types. The formal definition of deleting \( Q \) from \( E \) can be presented as (\( \ominus \) is left associative):

\[
E' = E \ominus Q = E \ominus \{q_1\} \ominus \cdots \ominus \{q_k\}
\]  
(4)

1.2 Multi-criteria Optimal Location Query Updating Problem

In this subsection, we focus primarily on the definition of MOLQ updating problem. We assume that the MOLQ has been addressed over a set of object sets. Each object is assigned with object weight and type weight. However, for any reasons, there are changes applied to a small number of objects, and the MOLQ is required to be re-evaluated over the updated object sets. So, the MOLQ updating problem can be defined as follows: given a set of object sets \( E = \{P_1, \ldots, P_n\} \), a type weight function \( f_i \), and an object weight function \( \sigma = \{f_1', \ldots, f_m'\} \), where \( f_i' \) is applied to an object \( p_i'' \in P_i \), let \( l = MOLQ(E, f_i', \sigma) \) be the answer to the MOLQ query, \( Q \) be a set of objects updated to \( E \), then the multi-criteria optimal location query updating problem is to find an optimal location \( l' \), which minimizes the total weighted distance from \( l' \) to one object in each type after updating \( Q \) to \( E \).

If \( Q \) is inserted into \( E \), then

\[
MOLQ(E \uplus Q, f_i', \sigma) = l', \quad \text{if } l' \text{ satisfies the condition}
\]

\[
\min\{\text{Min}\(_D\)(l', E \uplus Q, f_i', \sigma) \mid l'' \in \mathbb{R}\}
\]

If \( Q \) is deleted from \( E \), then

\[
MOLQ(E \ominus Q, f_i', \sigma) = l', \quad \text{if } l' \text{ satisfies the condition}
\]

\[
\min\{\text{Min}\(_D\)(l', E \ominus Q, f_i', \sigma) \mid l'' \in \mathbb{R}\}
\]

It is worth noting that any change on an object is equivalent to deleting the object from \( E \) and adding it back with new attributes. The object insertion operation only applies to \( Q \) if \( \forall q_i \in Q \), there must exist an object set \( P_i \in E \), which contains objects in the same type of \( q_i \). The object deletion operation only applies to \( Q \) if every object in \( Q \) must be in an object set of \( E \). The query result over updated object sets could be \( l \) or a better location.

2. REFERENCES