

# Distributed Online Algorithm for Optimal Real-Time Energy Distribution in the Smart Grid

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**Abstract**—In recent years, the smart grid has been recognized as an important form of the Internet of Things (IoT). The two-way energy and information flows in a smart grid, together with the smart devices, bring about new perspectives to energy management. This paper investigates a distributed online algorithm for electricity distribution in a smart grid environment. We first present a formulation that captures the key design factors such as user's utility, grid load smoothing, and energy provisioning cost. The problem is shown to be convex and can be solved with a centralized online algorithm that only requires present information about users and the grid in our prior work. In this paper, we develop a distributed online algorithm that decomposes and solves the online problem in a distributed manner, and prove that the distributed online solution is asymptotically optimal. The proposed distributed online algorithm is also practical and mitigates the user privacy issue by not sharing user utility functions. It is evaluated with trace-driven simulations and shown to outperform a benchmark scheme.

**Index Terms**—Convex optimization, demand response, distributed algorithm, Enernet, Online algorithm, smart grid.

## I. INTRODUCTION

THE term Internet of Things (IoT) was arguably first coined in an online article by Kevin Ashton in 1999 [2], referring to uniquely identifiable objects that are organized in an Internet-like structure. There have been considerable interests and advances in IoT systems such as Radio-Frequency Identification (RFID) and wireless sensor networks in the past decade [3]. In recent years, the smart grid has been recognized as an important form of IoT, where power grid is integrated with information networks. The incorporation of communication, control and computation intelligence in the smart grid, and the deployment of smart meters (SMs) and smart facilities enable real-time sensing, monitoring, and automatic control of electricity generation, distribution, and consumption. The integration of the Internet and smart grid will lead to the so-called Energy Internet: the Enernet [4].

According to the National Institute of Standard and Technology (NIST) standard [5], the smart grid model includes seven domains: Customer, Market, Service provider, Operations, Bulk generation, Transmission, and Distribution. Each domain

functions differently, interactively, and cooperatively. However, in some cases, different domains may share some actors and applications. For instance, the Distribution and Customer domains probably contain SMs. On the other hand, an integrated utility may have actors in many domains: a distribution system operator could have applications in both Operation and Market domains [5].

In this paper, we consider a real-time energy distribution in a certain area with the smart grid system. As shown in Fig. 1, the system considered in this paper includes three large domains: Customer, Power Grid Operator (PGO), and Energy Distributor (ED). The Customer domain here is similar to the one in the NIST model, which represents power users including resident, industrial, and others. The PGO performs as Market, Service Provider, and Operations do in the seven-domain model. The ED includes the generation, transmission, and distribution domains. It generates power to meet local demand and stores excessive power. It also transmits power from outside when there is not enough local generation and storage. This way, we simplify the seven domains to three large domains or utilities. The SM in the Customer domain is responsible for information exchange with the PGO and for scheduling the electrical appliances on the user side. The information flows are carried through a communications network infrastructure, such as a wireless network or a powerline communication system [6]–[8]. With both energy and communication connections among the domains, the PGO can exchange information with the Customer and ED and thus it controls the energy operation of the entire area.

Demand side management is one of the most important problems in smart grid research, which aims to match electricity demand to supply for enhanced energy efficiency and demand profile while considering user utility, cost, and price [6]. Researchers have been focusing on peak shift or peak reduction for reducing grid deployment and operational cost [9], [10], as well as user or energy provider's cost [11], [12]. In particular, some prior works have jointly considered both user and energy provider costs, to maximize users' utility while keeping energy provider's cost at a lower level [1], [13]. Furthermore, privacy is also emphasized in demand side management in practice. Some researchers investigate the privacy problem in the smart grid from many aspects and show that an individual's daily life can even be reconstructed with collection of data on power usage [14], [15].

Given the wide range of smart grid models and the challenge in characterizing the electricity demand and supply processes and the utility/cost/pricing functions, a general model that can accommodate various application scenarios would be highly desirable. Furthermore, it is important to jointly consider the

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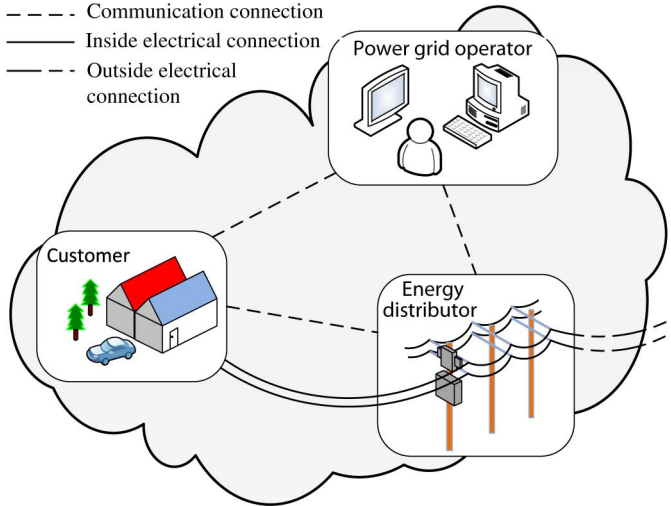


Fig. 1. Illustration of the key components and their interactions in the smart grid: the PGO, ED, and Customer domains.

utilities and costs of the key components of the system to achieve optimized performance for the overall smart grid system. For optimizing the performance of such a complex IoT system, the utilities and costs of the three key components, i.e., Customer, PGO, and ED, should be jointly considered.

In this paper, we take a holistic approach to incorporate the key design factors including Customer's utility, grid load smoothing, and energy provisioning cost in a problem formulation. To solve the real-time energy distribution problem, we first introduce a centralized offline solution and then a centralized online algorithm (COA) from our prior work [16], which is variance-sensitive without requiring any future information of the system. Furthermore, we propose a distributed online algorithm (DOA), which first decomposes the master problem into several sub-problems and then solves them locally at each user and the PGO with the online approach. We also investigate a communications protocol to facilitate the information exchange for the iterative DOA, which can be built on existing or emerging smart grid communication standards [7], [8].

The proposed framework is quite general. It does not require any specific models for the electricity demand and supply processes, and only has some mild assumptions on the utility and cost functions (e.g., convex and differentiable). The proposed algorithm can thus be applied to many different scenarios. It inherits the advantages of online algorithms that requires no future information for a convergent solution, and the advantages of distributed algorithms, which solves the problem in a distributed manner with local information. Although user power usages are still exchanged with the PGO, the DOA mitigates the privacy problem since it does not require disclosure of user's utility function and its parameters. The proposed algorithm is easy to be implemented in a real smart grid system. The distributed computation allows scalability for handling large systems. The DOA inherits the variance-sensitive nature from the online algorithm, while converging to the offline optimal solution almost surely, a highly desirable property. The proposed algorithm is evaluated with trace-driven simulation using energy consumption traces recorded in the field. It outperforms a benchmark scheme that

is also distributed online but with no control for grid load smoothing.

The remainder of this paper is organized as follows. We present the system model in Section II. The problem formulation with both centralized offline and online solutions are introduced in Section III. The DOA is developed and analyzed in Section IV. The communications protocol is discussed in Section V. We present the simulation studies in Section VI and review related work in Section VII. Section VIII concludes this paper.

## II. SYSTEM MODEL

### A. Network Structure

We consider a power distribution system in the smart grid environment where the PGO supports the power usage of all users in the Customer domain. The users could be residential, commercial, and industrial energy consumers. Each user deploys an SM to monitor and control the energy consumption of the electrical appliances [6]. All SMs are connected to the PGO through the information infrastructure such as a wireless or wireline local area network. During each distribution time cycle, SMs and PGO exchange status and control information to maximize users' utility, to minimize the PGO's generating cost, and to smooth the total power variance. The ED then transmits and distributes electricity to the users accordingly.

The relevant time period for the operation is divided into  $T$  time slots, indexed by  $t \in \mathbb{T} = \{1, 2, \dots, T\}$  and  $\mathbb{T}$  is the set of all the time slots. Usually, the operation time period is a 1-day cycle based on the daily periodic nature of electricity usage, while the time slot duration could be 1 h, 0.5 h, or 15 min, according to users' power demand pattern/timescale in consideration of varying demand in different time of the day, as well as the amount of users in an area in consideration of communications cost.

We denote the power consumption of user  $i$  at time slot  $t$  as  $p_i(t)$  and let  $\mathbb{N} = \{1, 2, \dots, N\}$  be the set of users. We also define a set  $\mathbb{P}$  of energy consumption at each time slot  $t$  for each user as

$$\mathbb{P} = [p_{i,\min}(t), p_{i,\max}(t)], \quad \text{for all } t \in \mathbb{T}, i \in \mathbb{N} \quad (1)$$

where  $p_{i,\min}(t)$  is the minimum power demand and  $p_{i,\max}(t)$  is the maximum power demand of user  $i$  at time  $t$ , as the users are assumed to be rational. That is,  $\mathbb{P}$  includes all the possible value of power requested and consumed, that is,  $p_i(t) \in \mathbb{P}$  for all  $i$  and  $t$ . It is noted that  $\mathbb{P}$  is defined to be a nonnegative set, because even today, few users are able to generate enough power for themselves in a short time.

### B. User Utility Function

We assume that each user behaves independently in the power grid. They have their own preferences and time schedules for using different electrical appliances. For instance, users may set their air conditioner at different temperatures, and different users may use their washer and dryer at different times of the day. Also, the user demand may vary as weather condition

changes. Usually, the power consumption is larger in a hot summer (or a cold winter) day than that in a mild day in the spring (or autumn). Furthermore, different users may have different reactions to different pricing schemes [12]. Therefore, it is nontrivial to characterize the diverse user preference with a precise mathematical model.

In prior work, user preference is usually represented by an *utility function* [11]. Similarly, we adopt a function  $U(p_i(t), \omega_i(t))$  to represent user  $i$ 's satisfaction on power consumption in this paper. Here  $U(\cdot)$  is a general, strictly increasing, concave function of the allocated power  $p_i(t)$ , although the quadratic utility function is also popular in the literature [11]–[13]. The other parameter  $\omega_i(t)$  of the utility function indicates user  $i$ 's level of flexibility at time  $t$ . It is a “sorting” parameter for users and thus can be normalized to be within the interval  $[0, 1]$  [17]. A larger  $\omega_i(t)$  indicates a higher level of flexibility or power consumption. For example, a user with  $\omega_i(t)$  close to 1 will probably consume more power than others. Different users can have different  $\omega_i(t)$ , and  $\omega_i(t)$  can vary over time.

In a centralized scheduling scheme, the PGO will require the  $\omega_i(t)$ 's from all users in every updating interval. The user utility function and preference are private information, which can be used possibly to reconstruct many aspects of users' daily life and infringe their privacy [14], [15]. Information about a user's utility function and its parameters should be protected. To this end, a distributed algorithm that does not require exchanging privacy information would be appealing.

### C. Energy Provisioning Cost Function

For ED, when demand is in the normal level, the generation cost increases only slowly as the demand grows. However, it will cost much more when the load peak is approaching the grid capacity, because PGO has to ask ED to transmit more power from outside to avoid a blackout, which incurs considerable power loss on the transmission line. Therefore, we could use a general increasing and strictly convex function to approximate the *cost function* for energy provisioning.

Similar to [12] and [13], we choose a quadratic function to model the ED's cost, as

$$C(g(t)) = a \cdot g^2(t) + b \cdot g(t) + c \quad (2)$$

where  $a > 0$  and  $b, c \geq 0$  are preselected for the power grid and  $g(t)$  denotes the total amount of electricity generated by the ED at time slot  $t$ . ED has to provide sufficient power for users while reducing its cost.

In addition, we assume a maximum generating capacity  $g_{\max}(t)$  for ED at time slot  $t$ . Thus, we have the following constraint for  $g(t)$ :

$$\sum_{i \in \mathbb{N}} p_i(t) \leq g(t) \leq g_{\max}(t), \quad \text{for all } t \in \mathbb{T}. \quad (3)$$

The constraint indicates that  $g(t) \in \mathbb{G} = [g_{\min}(t), g_{\max}(t)]$ , where  $g_{\min}(t) = \sum_{i \in \mathbb{N}} p_i(t)$  and  $\mathbb{G}$  is a closed positive set.

Because the cost function  $C(\cdot)$  is strictly convex and increasing,  $C(\cdot)$  is reversible, so that the energy provisioning cost  $C(g(t))$  is also bounded in a closed set, i.e.,  $C(g(t)) \in \mathbb{C}$  for all  $t$ . In other words, by adjusting the amount of power generation, the ED can control its provisioning cost.

## III. PROBLEM FORMULATION AND CENTRALIZED SOLUTIONS

In this section, we summarize the problem formulation and the centralized offline and online algorithms presented in our prior work [16]. This is just for the sake of completeness and we do not claim contribution for this part. The proposed DOA is presented in Section IV and evaluated in Section VI.

### A. Problem Formulation

We consider three core parts in the smart grid environment: Customer, ED, and PGO in the model. Under certain constraints, we aim to achieve the triple goals of 1) maximizing users' utility, 2) minimizing ED's cost, and 3) smoothing the total power load of the grid.

We first consider an offline scenario where the PGO has global information on users' flexibility  $\omega_i(t)$  and ED's total generated power  $g(t)$  for the entire period (i.e., future information is known). Let  $P_i(t)$  denote the power usage for user  $i$  at time  $t$ , for  $t \in \mathbb{T}$ . We use upper case  $P$  in the *offline problem*. In the corresponding *online problem*, which is examined in Section III-C, we use lower case  $p$  for the corresponding variables. A vector with subscript  $i$  is used to denote a time sequence, e.g.,  $\vec{P}_i$  for the power usage by user  $i$  for  $t \in \mathbb{T}$ . The offline problem (termed Prob-OFF) can be formulated as follows. For  $P_i(t) \in \mathbb{P}$ ,  $g(t) \in \mathbb{G}$  for all  $i \in \mathbb{N}, t \in \mathbb{T}$ , we have the offline problem Prob-OFF as [16]

$$\begin{aligned} \text{maximize: } & \Theta(\vec{P}_1, \dots, \vec{P}_N) \\ & = \sum_{t \in \mathbb{T}} \left[ \sum_{i \in \mathbb{N}} U(P_i(t), \omega_i(t)) - C(g(t)) \right] \\ & \quad - \frac{\alpha T}{2} \text{Var} \left( \sum_{i \in \mathbb{N}} \vec{P}_i \right) \quad (4) \\ \text{subject to: } & \sum_{i \in \mathbb{N}} P_i(t) \leq g(t), \quad \text{for all } t \in \mathbb{T} \quad (5) \end{aligned}$$

where  $\text{Var}(\cdot)$  is the variance function defined as

$$\text{Var} \left( \sum_{i \in \mathbb{N}} \vec{P}_i \right) = \frac{1}{T} \sum_{t \in \mathbb{T}} \left( \sum_{i \in \mathbb{N}} P_i(t) - \frac{1}{T} \sum_{k \in \mathbb{T}} \sum_{i \in \mathbb{N}} P_i(k) \right)^2.$$

The objective function (4) consists of three parts. The first part represents users' satisfaction and preference. The second part represents ED's cost for energy provisioning. The third part represents the load variance of the grid. It is integrated with a parameter  $\alpha > 0$ , to enable a trade-off between the grid and the users' benefits. All the users' demand and generating power

should be included in the sets  $\mathbb{P}$  and  $\mathbb{G}$  as we have discussed in Sections II-A and II-C.

### B. Centralized Offline Algorithm

In Problem Prob-OFF (4), the user power consumption  $P_i(t)$ 's are independent. Hence the grid load variance term can be rewritten as  $\text{Var}(\sum_{i \in \mathbb{N}} \vec{P}_i) = \sum_{i \in \mathbb{N}} \text{Var}(\vec{P}_i)$ . It can be verified that Prob-OFF is a convex optimization problem because function  $U(\cdot)$  is concave and  $C(\cdot)$  and  $\text{Var}(\cdot)$  are both convex. Also, due to convexity of the variance function  $\text{Var}(\cdot)$ , we can show that Prob-OFF has a unique solution [16]. If we carefully define sets  $\mathbb{P}$  and  $\mathbb{G}$ , the Slater's condition can be satisfied as well, which indicates that the KKT conditions are sufficient and necessary for the optimality of Prob-OFF [18]. By solving the KKT conditions, we can derive the optimal energy allocation for each of the users at each time slot.

In Prob-OFF, all information are assumed to be known *a priori*. Because of this, its solution is optimal. However, since it requires future information for computing the grid load variance [i.e., the third part in (4)], we cannot solve the KKT conditions at each time slot in practice.

### C. Centralized Online Algorithm

We now present the online algorithm for energy distribution, and show the main result that the online solution is asymptotically convergent to the offline optimal solution, i.e., *asymptotically optimal*. The online energy distribution algorithm consists of the following three steps.

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#### Algorithm 1: Centralized Online Algorithm

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**Step 1:** For each  $i \in \mathbb{N}$ , initialize  $\hat{p}_i(0) \in \mathbb{P}$ .

**Step 2:** In each time slot  $t$ , the PGO solves the following convex optimization problem (termed Prob-ON). For  $p_i(t) \in \mathbb{P}$ ,  $g(t) \in \mathbb{G}$  for all  $i \in \mathbb{N}$ ,

$$\begin{aligned} \text{maximize: } & \sum_{i \in \mathbb{N}} U(p_i(t), \omega_i(t)) - C(g(t)) - \\ & \frac{\alpha}{2} \sum_{i \in \mathbb{N}} (p_i(t) - \hat{p}_i(t-1))^2 \end{aligned} \quad (6)$$

$$\text{subject to: } \sum_{i \in \mathbb{N}} p_i(t) \leq g(t) \text{ for all } t \in \mathbb{T}. \quad (7)$$

Let  $\vec{p}^*(t)$  denote the solution to Prob-ON, where each element  $p_i^*(t)$  represents the optimal power allocation to user  $i$ .

**Step 3:** Update  $\hat{p}_i(t)$  for all  $i \in \mathbb{N}$  as follows and go to Step 2.

$$\hat{p}_i(t) = \hat{p}_i(t-1) + \frac{\alpha}{t + \alpha} \cdot (p_i^*(t) - \hat{p}_i(t-1)). \quad (8)$$


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Comparing (4), the variance term is approximated by  $\sum_{i \in \mathbb{N}} (p_i - \hat{p}_i(t-1))^2$  in (6). Similar to problem Prob-OFF, Prob-ON is also a convex optimization problem satisfying

Slater's condition. The KKT conditions can be derived as follows:

$$\begin{cases} U'(p_i^*(t), \omega_i(t)) - \alpha(p_i^*(t) - \hat{p}_i(t-1)) - \lambda^*(t) = 0 \\ -C'(g(t)) + \lambda^*(t) = 0 \\ \lambda^*(t) (\sum_{i \in \mathbb{N}} p_i^*(t) / g(t) - 1) = 0 \\ \lambda^*(t) \geq 0 \quad \forall t \end{cases} \quad (9)$$

where  $\lambda^*(t)$  is the Lagrange multiplier. In (9), only information for time slot  $t$  is needed to solve the equations. This allows us to solve the problem in each time slot without needing any future information. The following theorem states that the offline solution converges to the optimal Prob-OFF solution, which is obtained by assuming all future information is available. See [16] for the proof.

*Theorem 1:* The centralized online optimal solution converges asymptotically and almost surely to the centralized offline optimal solution [16].

Although the formulation in [16] is slightly different with our problem in this paper, the conditions of the theorem are still satisfied in our model. Therefore, the theorem still holds true. It presents a strong result, based on which we could solve Prob-ON instead of Prob-OFF but with an equally good result.

However, Prob-ON is still solved in a centralized manner, which means that at each time slot, PGO still requires the accurate utility functions of all users with their preference parameters  $\omega_i(t)$ , which are important user privacy information. It will be appealing to develop a distributed algorithm that can preserve user privacy, but still achieve the optimal performance. The DOA will also provide scalability and have low control and communication overhead.

## IV. DISTRIBUTED ONLINE ALGORITHM

In this section, we first decompose problem Prob-OFF in a distributed manner, so that the PGO and every user can solve the subproblems independently without requiring global information. We then present a distributed offline algorithm for the decomposed problem. Finally, we show that the distributed offline problem can also be solved with an online approach, and the distributed online solution is asymptotically convergent to that of the centralized offline problem. Therefore, we can eliminate the need to share users' utility functions and their parameters.

### A. Decomposition and Distributed Offline Algorithm

First, the offline objective function (4) can be rewritten as

$$\begin{aligned} \Theta = & \sum_{t=1}^T \left[ \sum_{i \in \mathbb{N}} (U(P_i(t), \omega_i(t)) \right. \\ & \left. - \frac{\alpha}{2} \left( P_i(t) - \frac{1}{T} \sum_{k=1}^T P_i(k) \right)^2 \right) - C(g(t)) \right] \end{aligned} \quad (10)$$

where the first two terms are functions of  $P_i(t)$  and  $\omega_i(t)$  (i.e., information available at user  $i$ ) and the third term is a function of the total load  $g(t)$  (i.e., information available at the PGO).

However, we cannot decompose the problem in this simple way, because constraint (5) involves both user information  $P_i(t)$  and PGO information  $g(t)$ .

To decompose the problem, we first derive the Lagrangian for Prob-OFF as<sup>1</sup>

$$\begin{aligned}
& L^T(\vec{P}(t), g(t), \lambda^T(t)) \\
&= \sum_{t=1}^T \left[ \sum_{i \in \mathbb{N}} \left( U(P_i(t), \omega_i(t)) - \frac{\alpha}{2} \left( P_i(t) - \frac{1}{T} \sum_{k=1}^T P_i(k) \right)^2 \right) \right. \\
&\quad \left. - C(g(t)) - \lambda^T(t) \left( \sum_{i \in \mathbb{N}} P_i(t) - g(t) \right) \right] \\
&= \sum_{t=1}^T \left[ \sum_{i \in \mathbb{N}} \left( U(P_i(t), \omega_i(t)) - \frac{\alpha}{2} \left( P_i(t) - \frac{1}{T} \sum_{k=1}^T P_i(k) \right)^2 \right. \right. \\
&\quad \left. \left. - \lambda^T(t) P_i(t) \right) \right] + \sum_{t=1}^T [\lambda^T(t) g(t) - C(g(t))] \quad (11)
\end{aligned}$$

where  $\lambda^T(t)$  is the Lagrange multiplier. In (11), functions of  $P_i(t)$  and  $g(t)$  are decoupled. For each  $P_i(t) \in \mathbb{P}$ , define

$$\begin{aligned}
S_i^T(\lambda^T(t)) = \max \left\{ \sum_{t=1}^T [U(P_i(t), \omega_i(t)) \right. \\
\left. - \frac{\alpha}{2} \left( P_i(t) - \frac{1}{T} \sum_{k=1}^T P_i(k) \right)^2 - \lambda^T(t) P_i(t) \right\}. \quad (12)
\end{aligned}$$

For  $g(t) \in \mathbb{G}$ , define

$$R^T(\lambda^T(t)) = \max \left\{ \sum_{t=1}^T [\lambda^T(t) g(t) - C(g(t))] \right\}. \quad (13)$$

We can reformulate problem Prob-OFF to the Lagrange dual problem as follows [18].

$$\begin{aligned}
& \text{minimize : } D^T(\lambda^T(t)) \\
& \text{subject to : } \lambda^T(t) \geq 0 \quad (14)
\end{aligned}$$

where

$$\begin{aligned}
D^T(\lambda^T(t)) &= \max \left\{ L^T(\vec{P}(t), g(t), \lambda^T(t)) \right\} \\
&= \sum_{i \in \mathbb{N}} S_i^T(\lambda^T(t)) + R^T(\lambda^T(t)). \quad (15)
\end{aligned}$$

<sup>1</sup>The superscript [e.g.,  $L^T(\cdot)$  or  $\lambda^T(\cdot)$ ] indicates the functions and Lagrange multiplier of the distributed offline problem. The superscript is removed for the corresponding functions and Lagrange multiplier of the distributed online problem.

This way, problem Prob-OFF is decomposed into two parts: (i) the first one is an optimization problem  $S_i^T(\lambda^T(t))$  defined in (12) for each user to solve and (ii) the other one is also an optimization problem  $R^T(\lambda^T(t))$  defined in (14) for the PGO to solve. Since they are both concave and have linear constraints, *strong duality* holds for careful selections of  $P_i(t)$  and  $g(t)$ , which guarantees the zero gap between Prob-OFF and the dual problem  $D^T(\lambda^T(t))$ .

### B. Distributed Online Subproblem

Although we can apply several methods from convex optimization to solve problems (12) and (14) in a distributed way, such an approach is still not practical because the offline problem and solution require future information to be known *a priori*. We next develop an online distributed algorithm to further eliminate such need for future information.

Observe that in (11), the only term that needs future information other than that at time  $t$  is  $\frac{1}{T} \sum_{k=1}^T P_i(k)$ , i.e., the average of  $P_i(t)$  over  $\mathbb{T}$ , which is also a term in subproblem (12) for users. Therefore, if the average of  $P_i(t)$  can be revealed with accumulated historic information, we will be able to solve (14) in an online manner. Similar to the idea of transforming problem Prob-OFF into Prob-ON, we use  $\hat{p}_i(t-1)$  to approximate the average in the DOA and show that the solution obtained this way is still asymptotically optimal.

We first present the distributed online subproblems by rewriting the distributed offline optimization problems for users and PGO according to (12)–(15). At each time slot  $t$ , for each user  $i$ , define

$$\begin{aligned}
S_i(\lambda(t)) &= \max \left\{ U(p_i(t), \omega_i(t)) \right. \\
&\quad \left. - \frac{\alpha}{2} (p_i(t) - \hat{p}_i(t-1))^2 - \lambda(t) p_i(t) \right\} \quad (16)
\end{aligned}$$

$$R(\lambda(t)) = \max \{ \lambda(t) g(t) - C(g(t)) \}. \quad (17)$$

The objective function for  $\lambda(t)$  is

$$\begin{aligned}
& \text{minimize : } D(\lambda(t)) \\
& \text{subject to : } \lambda(t) \geq 0 \quad (18)
\end{aligned}$$

where

$$D(\lambda(t)) = \sum_{i \in \mathbb{N}} S_i(\lambda(t)) + R(\lambda(t)). \quad (19)$$

This way, we derive the distributed online subproblems for users and the PGO to solve. Note that the dual decomposition is only able to decompose the online problem and we still need to show that the distributed online problem is optimal and convergent. The following theorem shows that the distributed online subproblems can be solved and the solutions are asymptotically optimal.

*Theorem 2:* The optimal solution to the distributed online subproblems converges asymptotically and almost surely to the offline optimal solution.

*Proof:* Substituting (16) and (17) into (19), we have

$$\begin{aligned}
D(\lambda(t)) &= \sum_{i \in \mathbb{N}} S_i(\lambda(t)) + R(\lambda(t)) \\
&= \max \left\{ \sum_{i \in \mathbb{N}} (U(p_i(t), \omega_i(t)) \right. \\
&\quad \left. - \frac{\alpha}{2} (p_i(t) - \hat{p}_i(t-1))^2 - \lambda(t) p_i(t)) \right. \\
&\quad \left. + \lambda(t) g(t) - C(g(t)) \right\} \\
&= \max \left\{ \sum_{i \in \mathbb{N}} (U(p_i(t), \omega_i(t)) \right. \\
&\quad \left. - \frac{\alpha}{2} (p_i(t) - \hat{p}_i(t-1))^2 \right. \\
&\quad \left. - C(g(t)) - \lambda(t) \left( \sum_{i \in \mathbb{N}} p_i(t) - g(t) \right) \right\} \\
&= \max \{ L(\vec{p}(t), g(t), \lambda(t)) \}. \tag{20}
\end{aligned}$$

Comparing function  $L(\vec{p}(t), g(t), \lambda(t))$  with the centralized online problem Prob-ON (6) and its constraint (7), it can be seen that function  $L(\vec{p}(t), g(t), \lambda(t))$  is actually the Lagrangian of Prob-ON and  $\lambda(t)$  is the Lagrange multiplier. Also function  $D(\lambda(t))$  is then the dual function for Prob-ON. Similar to the Prob-OFF case, the Slater's condition holds true here again by careful choices of  $p_i(t)$  and  $g(t)$ . Therefore, the distributed online subproblems (16) and (17) have the same solution as the centralized online problem Prob-ON.

On the other hand, Theorem 1 has proved that the solution of Prob-ON is optimal and asymptotically convergent to the offline optimal solution. We then conclude that the solution of the distributed online subproblems is also optimal and converges asymptotically to the optimal offline solution. ■

The proof of Theorem 2 clarifies the relationship among problems Prob-OFF, Prob-ON, and the distributed online subproblems. Actually, we can also achieve the distributed online decomposition from Prob-ON by dual decomposition as we did for Prob-OFF. Theorem 2 also presents an effective means of solving the online distribution problem in a practical manner. We next present the DOA.

### C. Distributed Online Algorithm

Following Theorem 2, we can solve the dual problem (18) to acquire the optimal online solution. Because of constraint (7),  $S_i(\lambda(t))$  and  $R(\lambda(t))$  are coupled by the Lagrange multiplier  $\lambda(t)$ ;  $\lambda(t)$  is associated with both the user utility maximization problem (16) and the ED cost minimization problem (17). As the dual variable, it is also a key parameter for solving the dual problem.

In our case, the dual function  $D(\lambda(t))$  is differentiable. So, we can apply the following gradient method to acquire the dual variable  $\lambda(t)$  at each time slot  $t$  [19].

$$\lambda_t(k+1) = \left[ \lambda_t(k) - \delta \left( g_t(k) - \sum_{i \in \mathbb{N}} p_{i,t}^*(k) \right) \right]^+ \tag{21}$$

where  $\delta$  is the step-size,  $[\cdot]^+$  is the projection onto the nonnegative orthant,  $\lambda_t(k)$  is the  $k$ th update of  $\lambda(t)$ , and  $g_t(k)$  and  $p_{i,t}^*(k)$  are the solutions to (16) and (17), respectively.

At each time slot  $t$ , this method requires that PGO and the user's exchange  $\lambda_t(k)$  and  $p_{i,t}^*(k)$  for a number of times to obtain the convergent  $\lambda(t)$ , the power that will be generated  $g(t)$ , and the energy  $p_i(t)$  allocated to each user  $i$ . We then present the DOA, Algorithm 2, to solve the dual problem (18) as well as problem Prob-ON. The algorithm consists of two parts:

- 1) a three-step Algorithm 2.a for all users;
- 2) a three-step Algorithm 2.b executed by the PGO.

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#### Algorithm 2.a: Distributed Online Algorithm for Users

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**Step 1:** For each user  $i \in \mathbb{N}$ , initialize  $\hat{p}_i(0) \in \mathbb{P}$ .

**Step 2:** In time slot  $t$ , the SM of each user does the following:

- 1) Receives the updated  $\lambda_t(k)$  from the PGO;
- 2) Solves problem (16) for user utility maximization;
- 3) Transmits the solution  $p_{i,t}^*(k)$  to the PGO for energy demand;
- 4) Repeats 1) to 3) until  $|\lambda_t(k+1) - \lambda_t(k)| < \varepsilon$ , where  $\varepsilon > 0$ .

**Step 3:** Update  $\hat{p}_i(t)$  for all  $i \in \mathbb{N}$  as (8).

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#### Algorithm 2.b: Distributed Online Algorithm for the PGO

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**Step 1:** For each  $i \in \mathbb{N}$ , initialize  $p_{i,t}^*(0) \in \mathbb{P}$ . Choose an arbitrary  $\lambda_t(0) \geq 0$ .

**Step 2:** In each time slot  $t$ , the PGO does the following:

- 1) Solves problem (17) to obtain  $g_t(k)$ ;
- 2) Receives  $p_{i,t}^*(k)$  from all the users;
- 3) Updates the value of  $\lambda_t(k)$  using (21) and broadcasts it to the users;
- 4) Repeats 1) to 3) until  $|\lambda_t(k+1) - \lambda_t(k)| < \varepsilon$ , where  $\varepsilon > 0$ .

**Step 3:** Sends  $g(t)$  to ED for energy generation for time slot  $t$  and distributes  $p_i^*(t)$  to user  $i$  for all  $i \in \mathbb{N}$ .

---

Note that for each time  $t$ , we have a terminating condition that  $|\lambda_t(k+1) - \lambda_t(k)| < \varepsilon$  for the inner loop, where  $\varepsilon$  is a positive real number small enough to indicate the convergence of  $\lambda_t(k)$ . A smaller  $\varepsilon$  will produce a more precise  $\lambda(t)$ . But the computation will also take more time. The other factor affecting the convergence of  $\lambda_t(k)$  is the step-size  $\delta$  in (21). For the gradient method, a small  $\delta$  guarantees the convergence of  $\lambda_t(k)$  but may require more iterations. In fact, the terminating condition could be rewritten as

$$\left| \delta \left( g_t(k) - \sum_{i \in \mathbb{N}} p_{i,t}^*(k) \right) \right| < \varepsilon.$$

Therefore,  $\delta$  and  $\varepsilon$  should be carefully selected for Algorithm 2 to achieve fast convergence within one time slot. This is especially important for large scale systems with a large population of users in the Customer domain, where the information exchanged increases fast for more users. However, we conjecture that the

communications will not be a big issue under today's advanced wired and wireless communication infrastructures. We will evaluate the effect of  $\delta$  on the convergence of  $\lambda_t(k)$  in Section VI-B.

In Algorithm 2, we see an interaction between users and the PGO realized by the dual variable  $\lambda(t)$ . It not only is the necessary parameter to solve both (16) and (17) but also connects users and the PGO decisions. The PGO has no information about user utilities, while  $\lambda(t)$  instead conveys information from users to the PGO. By updating  $\lambda(t)$  as in (21), the new value contains new information from both users and the PGO. Thus, by using Algorithm 2, the online problem can be solved in a distributed manner with comparable optimality to the COA. Furthermore, from Theorems 1 and 2, the distribution solution from Algorithm 2 will also converge asymptotically to the offline optimal solution.

It is worth noting that no information on user utility and preference parameter is transmitted between the users and the PGO. Consider practical data communication networks for the smart grid, less-transmitted data bring about higher security, reliability, and sufficiency. This also helps simplify the communication protocol designs for the grid. Furthermore, the computational load is offloaded from the PGO to the SMs at each user's site; the computation at the PGO is greatly simplified, leading to resource and time savings, so that a larger number of users can be supported. In conclusion, the DOA 2 could be useful in practice.

## V. COMMUNICATION NETWORK PROTOCOL

Information exchange is an important element of the emerging smart grid. Communications between SMs and the PGO are essential for both control and distribution [7], [8]. The DOA is also based on such information exchanges. As more advances are made in smart grid, there is a compelling need for network architectures, standards, and protocols for communications in smart grid. We hereby introduce a basic protocol for communications network support in the smart grid for the proposed DOA, which is simple but sufficient to support the real-time online power distribution algorithm and can be built upon existing or future smart grid communication standards [7], [8].

In the distributed online energy distribution algorithm, the users and the PGO need to exchange  $\lambda_t(k)$  and  $p_{i,t}^*(k)$  several times at each time  $t$ , to achieve a satisfactory  $p_i^*(t)$  for users and  $g(t)$  for the ED. The ED should update periodically the grid information to the PGO and the emergency report should be timely. The PGO also collects other information from the ED, such as the actual grid load (AGL). After the DOA is executed, the users obtain their own power consumption and the PGO sends the total energy usage to the ED. Moreover, the PGO is able to send other control information to the EP or users for, e.g., regulation, accounting, emergency response, and alerts.

Fig. 2 illustrates the information flows in the network system, where we have three large entities in the system: the PGO is the core controller and users and the ED are also important participants. Fig. 2 illustrates the communications at time  $t$ . For other time slots, the communications protocols are almost the same. We take user  $i$  as an example, because other users have similar interactions with the PGO.

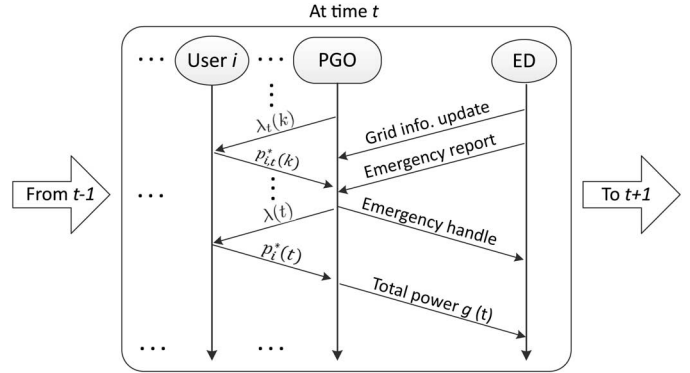


Fig. 2. Information flows in the power distribution network.

With Algorithm 2, at each updating slot, the PGO solves the subproblem, receives  $p_{i,t}^*(k)$  from the users and updates  $\lambda_t(k)$  to the users; the users receive the updated  $\lambda_t(k+1)$ , use it to solve the distributed optimization problem for users and update the new solution to the PGO. The iteration process terminates when the terminating condition is satisfied. Then the PGO will inform the ED to transmit the power request and distribute to the users. Meanwhile, the ED updates the power grid information to the PGO and sends alarms when emergency events happen. The PGO returns corresponding commands for the ED to execute.

## VI. PERFORMANCE EVALUATION

### A. Simulation Configuration

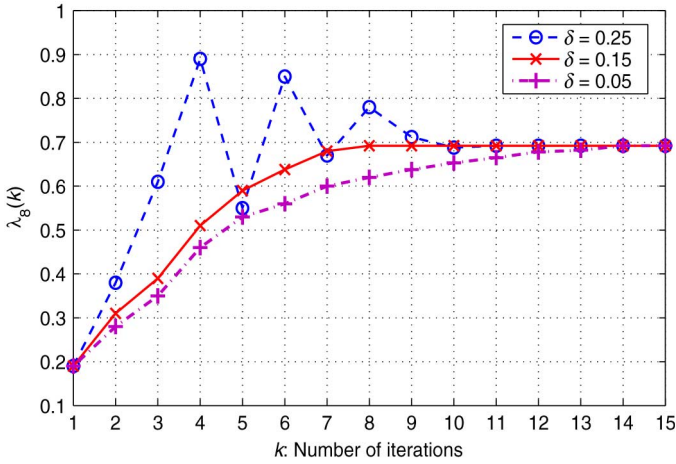
In this section, we evaluate the proposed DOA with trace-driven simulations. The simulation data and parameters are acquired from the recorded power consumption in the Southern California Edison (SCE) area in 2011 [20]. We first study the performance of DOA on convergence comparing to the COA described in Section III-B. We then compare the distribution solutions between DOA and COA, as well as with an existing scheme as benchmark.

Consider a power distribution system in a small area with  $N = 20$  users and 15-min updating periods. For COA, the 15-min interval is sufficient to obtain the required user information and execute the centralized optimization algorithm. The 15-min interval is also short enough to show the users' change of demand, although with DOA, shorter time slots are also practical. We will show results within a 24-h time pattern for an evaluation of the daily operations.

We choose users' utility function from a function set  $\mathcal{U}$  in which the functions are generated as widely used quadratic expression (see [11], [12]) with  $\omega_i(t) \in (0, 1)$  randomly selected.

$$U(p_i(t), \omega_i(t)) = \begin{cases} \omega_i(t)p_i(t) - \frac{1}{8}p_i(t)^2, & \text{if } 0 \leq p_i(t) \leq 4\omega_i(t) \\ 4\omega_i(t), & \text{if } p_i(t) \geq 4\omega_i(t). \end{cases} \quad (22)$$

We also assume that user's energy demand  $p_i(t)$  is selected from the set of  $\mathbb{P} = [1.0, 3.0]$  for all  $i$ . The maximum generating power  $g_{\max}(t)$  is set to the maximum total power demand of all the users, i.e.,  $g_{\max}(t) = \sum_{i \in \mathcal{N}} p_{i,\max}(t)$ , which implies that the

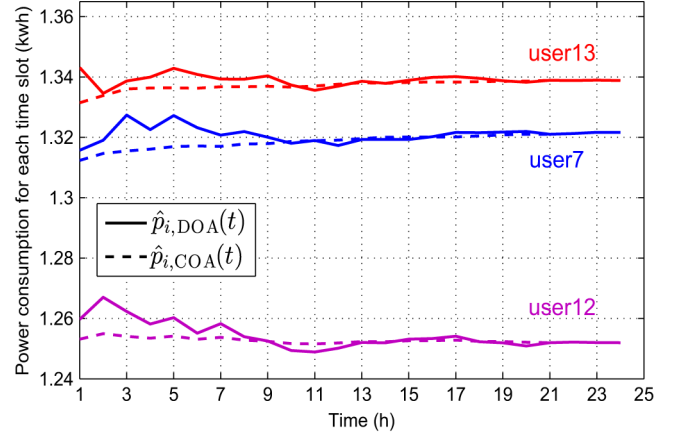
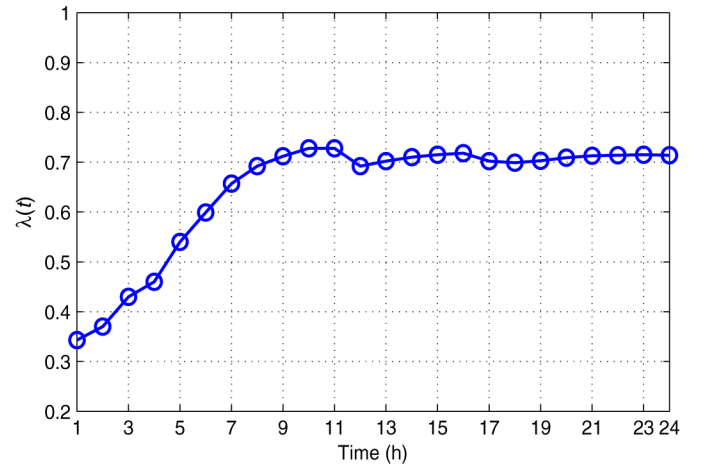
Fig. 3. Convergence of  $\lambda_8(k)$  when  $\varepsilon = 0.2$ .

generating power is equal to the power demand. The initial value of  $\lambda(t)$  in Algorithm 2 is picked randomly from the set and the termination condition  $\varepsilon$  is chosen as 0.2. The parameters in the energy provisioning cost function (2) are set as  $a = 0.05$  and  $b = c = 0$ . These parameters are carefully determined after studying the characteristics of the SCE trace. For parameter  $\alpha$  in the updating function (8), we take  $\alpha = 1$  in the following simulations. In [16], we have shown that  $\alpha = 1$  is a proper value for fast convergence.

### B. DOA Performance Evaluation

As shown in Section IV, DOA is based on the convergence of  $\lambda_t(k)$ . We first show the convergence of  $\lambda_t(k)$  in a time slot  $t$ . The gradient method applied in Algorithm 2 [see the updating function (21) for  $\lambda_t(k)$ ] requires that the positive step-size  $\delta$  be sufficiently small to guarantee the convergence of  $\lambda_t(k)$ . However, small  $\delta$  may slow down the convergence. For a fixed  $\varepsilon$ , which indicates the same tolerance for the convergent  $\lambda_t(k)$ , Fig. 3 illustrates the evolution of  $\lambda_8(k)$  as a function of  $k$  for the same user at the eighth time slot with different step-sizes  $\delta$ . It is observed that the series of  $\lambda_8(k)$  with larger  $\delta$  of 0.25 has large perturbation than the other two series of smaller  $\delta$ . Also, the series of  $\lambda_8(k)$  with the smallest  $\delta$  of 0.05 has the slowest speed of convergence. Although the  $\lambda_8(k)$  with  $\delta$  of 0.25 converges faster than the one of 0.05, it is slower than the one of 0.15. This implies that increasing  $\delta$  cannot guarantee faster convergence of  $\lambda_t(k)$ , because a larger  $\delta$  may make  $\lambda_t(k)$  not convergent. In practice, a proper  $\delta$  is important for convergence and thus the efficiency of DOA. It can be decided after several simple experiments. From Fig. 3, we also observe a fast convergence of  $\lambda_t(k)$  in about 10 times of information exchanges. We set  $\delta$  to 0.15 in all the following simulations.

We then show the convergence of  $\hat{p}_i(t)$  from both COA and DOA.  $\hat{p}_i(t)$  is a key variable in the online algorithm. Its convergence indicates that the gap between the online and the offline solutions becomes zero [see the updating function (8)]. In Fig. 4,  $\hat{p}_{i,COA}(t)$  and  $\hat{p}_{i,DOA}(t)$  for three users are both convergent. For COA, we see a fairly fast convergence with a very short transient period. For DOA, it shows slower convergence with larger variance before stable values are achieved. This is because

Fig. 4. Convergence of  $\hat{p}_i(t)$  for DOA and COA for users of different levels of flexibility.Fig. 5. Evolution of  $\lambda(t)$  for a 24-h period.

comparing to COA, DOA has another iteration function brought about by (21) for updating  $\lambda_t(k)$ . The initial value  $\lambda_t(0)$  is set randomly, so it requires extra time for the convergence of  $\hat{p}_{i,DOA}(t)$ . Also in Fig. 4, we find the coincidence of two curves for the several last time slots. This can be explained by Theorem 2, which indicates that DOA and COA deliver identical solutions. It can be also observed in Fig. 4 that both algorithms achieve convergence for users of different levels of consumption, where user 13 has a  $\omega(t)$  larger than that of users 7 and 12.

It is common that the power usage of users may not have large perturbations within one time slot. As discussed,  $\lambda(t)$  is related to the users and the power grid. It is natural to assume that the  $\lambda(t)$ 's of consecutive time slots are correlated. If such a correlation could be revealed, DOA can be further improved. Therefore, we plot the variable  $\lambda(t)$  in Fig. 5. We observe a convergent trend of  $\lambda(t)$  for the 24-h period. However, it is not clear whether it is convergent or not at this time. Because the initial value of  $\lambda_t(k)$  is selected randomly, we can confirm our assumption that  $\lambda(t)$  and  $\lambda(t+1)$  are highly correlated. Thus, set  $\lambda_t(0)$  as  $\lambda(t-1)$  would reduce the iteration steps and speed up convergence in time slot  $t$ .

Furthermore, the power consumption of users and the grid load are usually closely related for consecutive days. Therefore,



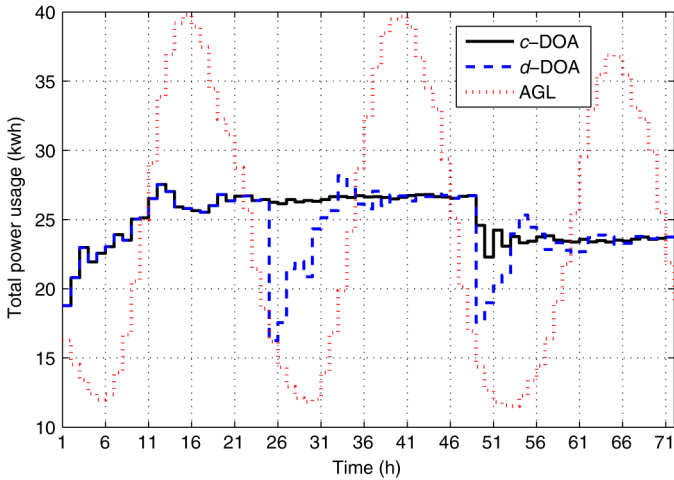


Fig. 6. AGL and total power consumption by  $d$ -DOA and  $c$ -DOA of three consecutive days.

we can use the final results/parameters from the previous day as a starting point for the present day, which leads to a better performance. We plot the grid load of three consecutive days by applying DOA separately on *daily* basis ( $d$ -DOA) and by applying DOA *consecutively* ( $c$ -DOA), as discussed in Fig. 6. For the first 2 days, the grid loads are almost the same. We find that  $c$ -DOA achieves an obviously better convergence performance over  $d$ -DOA in Day 2 because the initial values of Day 2 are set to the final values of Day 1. Although the third day has a lower grid load,  $c$ -DOA still achieves a better convergence and smoothness performance over  $d$ -DOA, because the initial values for  $d$ -DOA are randomly chosen. This way, we can enhance the proposed algorithm to achieve fast convergence and reduce communication requirements. In the remaining simulations, the enhanced DOA algorithm is used whenever possible.

### C. Comparison with Other Algorithms

One important benefit of DOA is the variance control it offers, which is inherited from COA. In Figs. 7 and 8, we plot the AGL and total power consumption by DOA, COA, and the state-of-the-art algorithm proposed in [13], which is a dynamic pricing algorithm (DPA) based on utility maximization. DPA considers both users and the ED as we do in our paper, but it has no consideration on the load variance. The AGL in Fig. 7 is the summation of 20 independent users' consumptions generated by the average real load in the SCE trace on a hot day (i.e., September 1, 2011) [20]; while the AGL in Fig. 8 is based on a typical day in the same SCE trace when the grid load is the average case (i.e., October 5, 2011) [20]. We show that these two figures have a direct comparison of our energy distribution algorithms.

From both Figs. 7 and 8, we first observe that DOA needs several time slots to converge to COA. This is caused by the effect of  $\epsilon$ , as discussed earlier. On the other hand, we also observe a larger gap between the DOA and the COA curves for the hot day in Fig. 7 than that for a typical day in Fig. 8. This is because the typical day has a much lower peak demand. This confirms that under average condition, DOA has very good performance, which is close to COA.

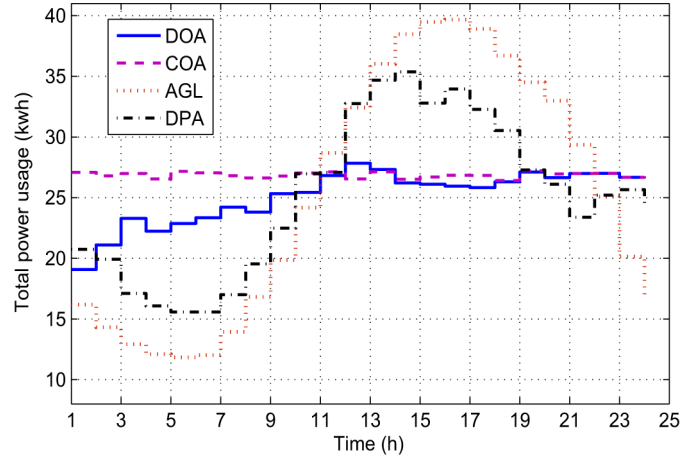


Fig. 7. AGL and total power consumptions achieved by DOA, COA, and DPA for a hot day.

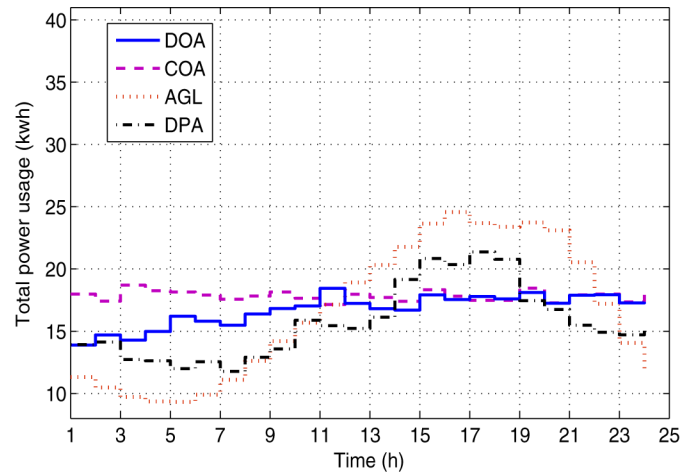


Fig. 8. AGL and total power consumptions achieved by DOA, COA, and DPA for an average day.

On the other hand, peak reduction is another objective of our algorithm. Peak refers to the highest point of the grid load curve and for different curves, the amount of peak reduction is represented by the normalized percentage, which is calculated as the ratio of the difference of the peak between the actual load curve and the controlled load curve, and the peak of actual curve. We have three controlled curves here: COA, DOA, and DPA. The peak reduction percentages for COA, DOA, and DPA are 29.8%, 31.7%, and 10.9%, respectively, for the hot day, and 23.9%, 23.9%, and 12.9%, respectively, for the typical day. We can see that DOA achieves almost the same peak reduction as COA in both cases, which are superior than DPA. Note that both DOA and COA have better performance on peak reduction of the hot day over the typical day; while DPA has the opposite result. This is because it does not consider variance reduction.

Finally, we compare several performance metrics for the three schemes (i.e., DOA, COA, and DPA) together with the actual trace results (i.e., the AGL based on the worse condition in the hot day) in Table I. These metrics are usually used as optimization objectives in prior work (see Section VII). As defined in (23),  $\bar{V}$ ,  $\bar{U}$ , and  $\bar{PK}$  denote the averages (across all users) of the grid load

TABLE I  
SIMULATION RESULTS OF SEVERAL PERFORMANCE METRICS FOR DOA, COA,  
DPA, AND AGL

System size $N$	Algorithm	$\bar{V}$	$\bar{U}$	$c(T)$	$\bar{PK}$
200	DOA	3.3	3.49	1.62	1.39
	COA	0.02	3.52	1.69	1.35
	DPA	24.5	3.66	1.74	1.61
	AGL	53.5	3.86	1.86	1.97
500	DOA	9.2	3.51	9.54	1.41
	COA	0.05	3.54	10.1	1.37
	DPA	62.6	3.64	10.5	1.73
	AGL	113	3.88	14.0	2.27
1000	DOA	18.1	3.47	38.1	1.55
	COA	0.10	3.53	40.2	1.41
	DPA	125	3.69	42.2	1.99
	AGL	266	3.88	54.1	2.63

variance, users' utility, users' cost, and the peak of the total grid load, respectively, while  $c(T)$  is the total energy provisioning cost for the entire period. The simulation results are listed for systems with 200, 500, and 1000 users.

$$\begin{cases} \bar{V} = \frac{1}{N} \sum_{i \in \mathcal{N}} \text{Var}(\mathbf{p}^*) \\ \bar{U} = \frac{1}{N} \sum_{t=1}^T \sum_{i \in \mathcal{N}} U(p_i^*(t), \omega_i(t)) \\ \bar{PK} = \frac{1}{N} \max_{t \in [1:T]} \sum_{i \in \mathcal{N}} p_i^*(t) \\ c(T) = \sum_{t=1}^T C(\sum_{i \in \mathcal{N}} p_i^*(t)) / 1000. \end{cases} \quad (23)$$

For  $\bar{V}$ , the best performer is COA, which is closely followed by DOA. This is consistent with the curves in Fig. 7. For  $\bar{U}$ , we observe a slightly better performance for DPA without the variance control. For energy provisioning cost  $c(T)$ , the three algorithms yield similar results, because they all include the function  $C(\cdot)$  as a part of objective function. For the peak  $\bar{PK}$ , we see the same result as in Fig. 7, with COA achieving the best and DOA following COA tightly.

Overall, the DOA proposed in this paper achieves better results than DPA. Although COA is slightly better than DOA, its centralized manner in energy distribution limits its usage in practice for large scale systems. It also has the disadvantage of requiring user's privacy information. DOA successfully mitigates these problems with the distributed approach. In summary, DOA is a practical method with a highly competitive performance comparing to the optimum, especially on variance control and peak reduction, for online energy distribution in the smart grid.

## VII. RELATED WORK

Smart grid, characterized with the two-way flows of electricity and information, is envisioned to replace the existing power grid in the future [21], [22]. A comprehensive review on smart grid technologies and research can be found in [6], where major topics on smart grid is discussed in three areas: infrastructure, management, and protection.

Within the three areas, demand side management or demand response has been attracting considerable research efforts [9], [11], [12], [23]–[28]. Researchers work mainly on demand profile shaping, user utility maximization, and cost reduction. For example, machine learning is used in [11] to develop a

learning algorithm for energy costs reduction and energy usage smoothing, while [23] aims to balance the users' cost and waiting time. A constrained multiobjective optimization problem is formulated in [24] to minimize energy consumption cost and maximize a certain utility among a group of users. Lyapunov optimization is adopted in [26]–[28] to stabilize the energy storage and user utility while reducing the operation cost of a microgrid. Lyapunov optimization is also used in [25] to optimally schedule the usage of all the energy resources in the system and minimize the long-term time-averaged expected total cost of supporting all users load demand. In these works, convex programming, machine learning, and game theory are mostly used. In some other works, online algorithms [29], which are widely used in wireless communications and networking, is also utilized [13], [16]. In [13], the authors propose an DPA based on utility maximization in a distributed way. Reference [16] presents a COA that achieves the optimal energy distribution and variance control without any future information.

Furthermore, for practical considerations, user's privacy is emphasized more and more by many authors [14], [15]. In [14], the author studies how high-resolution user electricity information can be used to reconstruct a user's daily life and preference. In [15], the authors examine privacy in smart grid from definition to different concerns in detail.

Our work is inspired by considering the above two aspects for the energy distribution in smart grid. In power systems, it is possible to use online algorithms to detect and control the grid load variance in real time. Also, the online algorithm can be decomposed into subproblems for users to solve locally. Motivated by this two observations, we propose an energy distribution DOA to achieve utility maximization, load smoothing, and privacy protection. The proposed DOA is quite effective as shown in Section VI.

## VIII. CONCLUSION

In this paper, we presented a study of optimal distributed online energy distribution in the smart grid. With a formulation that captures the key design factors of the system, we extend our prior work of a COA by decomposing the problem into many subproblems that can be solved in a distributed manner, thus protecting users' privacy and achieving scalability. We also show that the distributed online solution converges to the optimal offline solution asymptotically. The proposed DOA is evaluated with trace-driven simulations and outperforms a benchmark scheme.

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