Asymptotic Optimal Online Energy Distribution in the Smart Grid

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1. Introduction

A smart grid is an electrical grid that is enhanced with communications and networking, computing, and signal processing technologies [1,2]. The two-way energy and information flows, along with the smart devices, bring about new perspectives to energy management and demand response in the smart grid.

Demand side management is one of the most important problems in smart grid research, which aims to match electricity demand to supply for enhanced energy efficiency and demand profile while considering user utility, cost and price [1]. Researchers have been focusing on peak shifting or peak reduction for reducing the grid deployment and operational cost [3], [4], as well as on reducing user or energy provider's cost [5], [6]. In particular, some prior works aim to achieve a single objective, such as to improve the users' utility or reduce the cost of the energy provider [7], while others jointly consider both the user and energy provider costs, to increase the users' utility as much as possible while keeping the energy provider's cost at a relatively lower level [8]. Given the wide range of smart grid models and the challenge in characterizing the electricity demand and supply processes and the utility, cost, pricing functions, a general model that can accommodate various application scenarios would be highly desirable. Furthermore, it is important to jointly consider the utilities and costs of the key components of the system to achieve optimized performance for the overall smart grid system.

We consider real-time energy distribution in a smart grid system. As shown in Fig. 1, the distribution control center (DCC) collects real-time information from the three key components, i.e., the users, the grid, and the energy provider, makes decisions on, e.g., electricity distribution, and then sends the decisions back to the key components to control their operations. The smart meters at the user side will be responsible for the information exchange with the DCC and for enforcing the electricity schedule received from the DCC. The information flows are carried through a network infrastructure, such as a wireless network or a powerline communication system [1].

For optimizing the performance of such a complex network system, the utilities and costs of the three key components, i.e., the users, the grid, and the energy provider, should be jointly considered. In this paper, we take a holistic approach, to incorporate the key design factors including user's utility and cost, grid load smoothing, dynamic pricing, and energy provisioning cost in a problem formulation. To solve the real-time energy distribution problem, we first present an offline algorithm that can produce optimal solutions but assuming that the future user and grid information are known in advance. Based on the offline algorithm, we then develop an online algorithm that does not require any future information. As the name suggests, an online algorithm operates in an online setting, where the complete input is not known *a priori* [9]. It is very useful for solving problems with uncertainties. We find the online algorithm particularly suitable in addressing the lack of accurate mathematical models and the lack of future information for electricity demand and supply in this problem. We also prove that the online algorithm converges to the optimal offline algorithm almost surely.

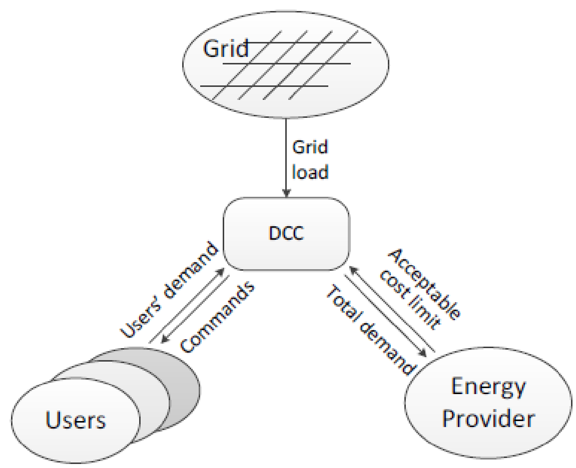

Fig. 1. Illustration of the key elements and interactions in the smart grid.

The proposed framework is quite general. It does not require any specific models for the electricity demand and supply processes, and only have some mild assumptions on the utility, cost, and price functions (e.g., convex and differentiable). The proposed algorithm can thus be applied to many different scenarios. The online algorithm also does not require any future information, making it easy to be implemented in a real smart grid system. It is also asymptotically optimal, a highly desirable property.

The proposed algorithm is evaluated with trace-driven simulation using energy consumption traces recorded in the field. It outperforms a benchmark scheme that assumes global information.

2. Problem Statement and Main Results

We aim to minimize the load variance in the grid while maximizing user satisfaction. Large load variance is undesirable for grid operation. It brings about uncertainties that affect not only user satisfaction but also the stability of the power system. Furthermore, the energy provisioning cost should be bounded and users' necessary power needs should be guaranteed.

We first consider an offline scenario where the DCC distributes the power to users during time $t = [1, 2, \cdots, T]$, and all the information on users' flexibility $\omega_i(t)$ and provider's budget $c(t)$ are assumed to be known in advance. Let $P_i(t)$ denote the power usage for user i at time t. In this paper, we use upper case P in the *offline problem*, where all the necessary constraints are known *a priori*. In the corresponding *online problem*, we use lower case p for the corresponding variables. A vector with subscript i is used to denote a time sequence, e.g., $\vec{P}_i$ for the power usage by user i for $t = \{1, 2, \cdots, T\}$. The offline problem Prob-OFF can be formulated as follows.

$$\max: \sum_{t=1}^{T}\sum_{i\in\mathbb{N}}\left[U(P_i(t),\omega_i(t)) - f\left(\sum_{i\in\mathbb{N}}P_i(t)\right)P_i(t)\right] - \frac{\alpha T}{2}\mathrm{Var}\left(\sum_{i\in\mathbb{N}}\vec{P}_i\right) \quad (1)$$

subject to:

$$P_i(t) \geq P_{i,min}(t), \forall\, i\in\mathbb{N}, t\in\{1,2,\cdots,T\} \quad (2)$$

$$C\left(\sum_{i\in\mathbb{N}}P_i(t)\right) \leq c(t), \forall\, t\in\{1,2,\cdots,T\}, \quad (3)$$

where $\mathrm{Var}(\cdot)$ is the variance of the total power, $U(\cdot)$ is the user utility function, $f(\cdot)$ is the price function, α is a nonnegative parameter to trade-off between user satisfaction and grid load variance, $p_{i,min}(t)$ is user i's minimum demand at time t, $C(\cdot)$ is the energy provisioning cost function. See [11] for details.

We show that Prob-OFF is a convex problem, as given in the following Lemma.

Lemma 1. *Prob-OFF is a convex optimization problem and has a unique solution.*

We next develop an online algorithm for energy distribution, and prove that the online solution is asymptotically convergent to the offline optimal solution, i.e., asymptotically optimal. The online energy distribution algorithm consists of the following three steps.

Step 1: For each $i\in\mathbb{N}$, initialize $\hat{p}_i(0)\in\mathbb{P}$.
Step 2: In each time slot t, the DCC solves the following convex optimization problem (termed Prob-ON).

$$\max: \sum_{i\in\mathbb{N}}U(p_i(t),\omega_i(t)) - f\left(\sum_{i\in\mathbb{N}}p_i(t)\right)\sum_{i\in\mathbb{N}}p_i(t) - \frac{\alpha}{2}\sum_{i\in\mathbb{N}}(p_i(t)-\hat{p}_i(t-1))^2 \quad (4)$$

$$\text{subject to: } p_i(t) \geq p_{i,min}(t), \forall\, i\in\mathbb{N} \quad (5)$$

$$C\left(\sum_{i\in\mathbb{N}}p_i(t)\right) \leq c(t), \forall\, t. \quad (6)$$

Let $\vec{p}^*(t)$ denote the solution to Prob-ON, where each element $p_i^*(t)$ represents the optimal power allocation to user i.
Step 3: Update $\hat{p}_i(t)$ for all $i\in\mathbb{N}$ as follows.

$$\hat{p}_i(t) = \hat{p}_i(t-1) + \frac{\alpha}{t+\alpha}\cdot(p_i^*(t)-\hat{p}_i(t-1)). \quad (7)$$

Similar to Prob-OFF, problem Prob-ON is also a convex optimization problem satisfying Slater's condition. We have the following theorem. Please see [11] for a detailed proof.

Theorem 1. *The online optimal solution converges asymptotically and almost surely to the offline optimal solution.*

3. Performance Evaluation

We evaluate the proposed online algorithm with trace-driven simulations. The simulation data and parameters are acquired from the traces of power consumption in the Southern California Edison (SCE) area recorded in 2011 [10]. We compare the online algorithm with the Optimal Real-time Pricing Algorithm (ORPA) presented in [8] as a Benchmark.

The total power consumption of the different algorithms are plotted in Fig. 2. From the aspect of smoothness, we could see clearly that the online optimal real-time energy distribution algorithm with α=1 (termed OORA(1)) achieves the best performance. The figure also shows that the online algorithm with α=0.01 (termed OORA(0.01)) also outperforms the benchmark ORPA. All the three algorithms achieve smoother total loads than the real consumption (RC). The peak reductions over RC are 35% for OORA(1), 28% for OORA(0.01), and 12.5% for ORPA. Therefore, OORA(1) achieves the largest peak reduction, while OORA(0.01) still outperforms ORPA with considerable gains. Please check out [11] for more simulation results and discussions.

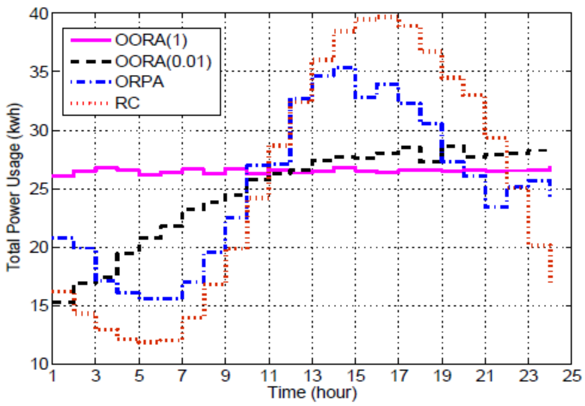

Fig. 2. Total power consumption for OORA(1), OORA(0.01), ORPA and RC.

4. Conclusion

In this paper, we present a study of optimal real-time energy distribution in smart grid. With a formulation that captures the key design factors of the system, we first present an offline algorithm that can solve the problem with optimal solutions. We then develop an online algorithm that requires no future information about users and the grid. We also show that the online solution converges to the offline optimal solution asymptotically and almost surely. The proposed online algorithm is evaluated with trace-driven simulations.

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